

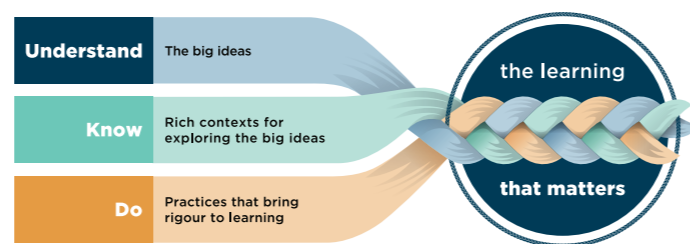
# Purpose statement for Mathematics and Statistics in the New Zealand Curriculum

## Ānō me he whare pūngāwerewere.

To say that something is like a whare pūngāwerewere – a spiderweb – is to recognise its intricacy, complexity, interconnectedness, and strength due to its many threads. The learning area of mathematics and statistics weaves together the effort and creativity of many cultures that over time have used mathematical and statistical ideas to understand their world.

Being numerate in Aotearoa New Zealand today relies upon understanding diverse cultural perspectives and privileging te ao Māori and Pacific world-views. Like mathematics and statistics, mātauranga Māori is a body of knowledge with a history and a future. When we afford mana ōrite to mātauranga mathematics and statistics and mātauranga Māori while retaining their distinctiveness, ākonga can draw from both in ways that are beneficial to both spheres of knowledge. For example, they will understand how ethical questions posed by measurement, quantification, and stories told about data take unique forms in Aotearoa New Zealand.

Learning in mathematics and statistics builds both literacy and numeracy. Mathematics and statistics contribute to ākonga literacy by developing their skills in oral and written communication, meaning-making, and the use of specific vocabulary and symbols. Statistics and probability, in particular, support the understanding of tables, graphs, and diagrams as well as critical thinking about the quality of data and stories told about it.



Ākonga become numerate as they “develop their ability to apply mathematical and statistical knowledge and skills purposefully across all learning areas and in their lives to achieve their goals” ([Literacy & Communication and Maths Strategy](#)).

As they progress, ākonga can use their mathematical and statistical knowledge and skills to contribute to their communities, Aotearoa, the Pacific, and beyond as informed citizens. Mathematical and statistical models can help identify misinformation and disinformation and are essential to resolving collective global challenges, including securing human rights and social justice, adapting to a changing climate, and building an equitable, sustainable future.

As the above whakatuakī tells us, connections between different concepts, knowledge, and practices are central to mathematics and statistics. Kaiako weave together the elements of Understand, Know, and Do to ensure ākonga learn mathematics and statistics as a connected body of knowledge.

## Important considerations for teaching mathematics and statistics

Effectively teaching mathematics and statistics requires a strengths-based approach in which all students have the opportunity to learn and progress at the curriculum learning phase of their peers. This means that it is important for our teaching to be ambitious, drawing on progress outcomes for an appropriate level of challenge while also providing opportunities for ākonga to demonstrate understanding beyond this.

When designing the scope and sequence of a mathematics and statistics programme, kaiako need to plan for providing multiple opportunities to progress. Learning happens best when mathematics and statistics are taught on a daily basis, using a series of purposeful contextualised and decontextualised tasks.

When thinking about these multiple opportunities, kaiako can ask: What opportunities do ākonga have to:

- learn new mathematics and statistics concepts and practices?
- understand the interrelated nature of concepts in mathematics and statistics?
- solve relevant, contextualised tasks?
- practise the mathematics and statistics that they have learned?

When planning contextualised tasks, kaiako can ask:

- What are the cultural contexts that will resonate with my ākonga?
- How can I support ākonga to engage with both the mathematical and statistical knowledge inherent in the context and the context’s whakapapa, significance, and tikanga?

As they prepare, kaiako can work through the learning tasks themselves and ask:

- How can I help ākonga find the joy in this learning?
- How can I help them see its relevance to their lives?
- What routes might ākonga take when working through the tasks?
- What are the possible next steps for teaching?

Features of this draft	Their treatment in the 2007 NZ Curriculum
<b>Understand: Big ideas</b>	Although big ideas were described in the 2007 learning area statement, they were not clear in the achievement objectives.
<b>Know: Contexts</b>	In response to research and feedback, the six new ‘contexts’ have split the three ‘combined’ strands of the 2007 learning area, allowing for easier interweaving. Geometry has been renamed Space to make it more encompassing of what it explores.
<b>Do: Practices</b>	The new practices existed in different versions in older curricula. In response to research and feedback from the sector, they have been updated.
<b>Progress steps</b>	Progress steps are being developed based on current research and learning progression frameworks. See the progress step for ‘During the first six months’ as an example of what these will look like.

# Overview

## Understand Big ideas

*Kotahi te kōhao o te ngira e kuhuna ai te miro mā, te miro pango, te miro whero. (Pōtatau Te Wherowhero)*

*There is but a single eye of the needle through which white, black, and red threads must pass together, yet each thread keeps its own colour and integrity while adding its strength and beauty to the others.*

### **Mātauranga Māori and mathematics and statistics help make sense of the world.**

Mātauranga Māori and mātauranga mathematics and statistics consist of different systems for viewing, understanding, and organising the world and how we operate in it. The interfaces between them offer opportunities for meaningful inquiry and for mathematical and statistical insights that uphold the integrity of each.

*Nō ngā tūpuna, tuku iho, tuku iho.*

*The human ideas that have been passed down from generation to generation over time can help people today develop their thinking.*

### **Mathematics and statistics have a continuous, evolving human history.**

Mathematics and statistics have been constructed over thousands of years across the globe, as people have grappled with notions of quantity, numerical representation, measurement, dimension, and pattern. They continue to be constructed from ideas drawn from many cultures. In Aotearoa New Zealand, they are informed by their location in Te Moana Nui a Kiwa.

*Kei hopu tōu ringa ki te aka tāepa, engari kia mau ki te aka matua.*

*Do not catch hold of the loose vine but lay hold of the main vine. You can use the strength of the aka matua (main vine) for the sure footing you will need to reach for new ideas and to climb to new heights.*

### **Mathematics and statistics are elegant, explorative, creative, and powerful.**

Mathematics and statistics use quantification, transformation, prediction, data, patterns, and relationships to find out, design, and explain. They offer a unique system of symbols, terms, conventions, and interconnected ideas that can be used to explore and think about the world critically, logically, and creatively.

*Whiria te kaha tūātinitini, whiria te kaha tūāmanomano.*

*Together we can use our strengths to achieve more. All learning contributes specific threads that people can use to weave a rope strong enough to get them where they want to go, do what they want to do, and be what they want to be.*

### **Mathematics and statistics help us in our everyday lives and decisions and are key to many areas of knowledge and practice.**

The concepts, skills, and processes of mathematics and statistics are used in everyday life and activities, including in individual and collective decision making about health, community, work, and finance. Being able to use mathematics and statistics to evaluate claims and options and make decisions helps people to participate fully in society. Mathematics and statistics are fundamental to many endeavours across engineering, science and technology; sport and recreation; social sciences; and the arts.

*Ko te pae tawhiti whāia kia tata, ko te pae tata whakamaua kia tina.*

*Seek to bring distant horizons closer and cherish those that you have attained. There will always be pae tawhiti, the 'not yet', but we can move ever closer to that for which we strive.*

### **Mathematics and statistics reward persistence and positivity.**

The interconnected nature of mathematics and statistics offers multiple ways to approach and solve problems. This means that experimentation and making mistakes play an important role in learning mathematics and statistics for everyone. Mathematics and statistics are open to all, no matter their background or identity.

# Overview (continued)

## Know Contexts

### Mātauranga tau | Number

Number allows us to describe and compare quantities and to operate on these quantities, including using them to estimate, calculate, reason, and justify.

### Taurangi | Algebra

Algebra allows us to generalise, to represent patterns, and to use symbols and graphs to express mathematical relationships.

### Inenga | Measurement

Measurement provides the tools and concepts for quantifying phenomena in the physical world by estimating, measuring accurately, and using appropriate units.

### Mokowā | Space

Space provides the tools and concepts for visualising, representing, and reasoning about the shape, position, orientation, and transformation of objects.

### Tauanga | Statistics

Statistics provides the tools, concepts, and methods for understanding phenomena through data and for examining and evaluating data-based information. Mana whenua have rangatiratanga over their data. All data are taonga to be treated with respect and kept safe.

### Tūponotanga | Probability

Probability provides the tools and concepts for quantifying chance, dealing with uncertainty and expectation, making predictions, and identifying how likely events are to occur.

## Do Mathematical and statistical practices

### Te whakatauirā me te tūhura | Modelling and investigating

Modelling and investigating processes are used to describe, explore, understand, and evaluate situations mathematically or statistically. They begin with a question or focus of interest and proceed in systematic but flexible ways, using mathematical and statistical concepts to make sense of solutions, stories about data, and conclusions in context.

### Te whakaata | Representing

Representing enables a deeper and more flexible understanding of mathematical and statistical ideas. Representations can be used to compare, explore, simplify, illustrate, prove, and justify as well as to look for patterns, variations, and trends. Representations can use words or symbols and be physical, virtual, graphical, diagrammatic, mental, or abstract. They can be drawn from mātauranga Māori, Pacific cultures, various places, and different periods of history.

### Te honohono | Connecting

The whakataukī for mathematics and statistics, Anō me he whare pūngāwerewere, expresses the intricacies and significance of connection in mathematics and statistics. There are important connections between ideas in mathematics and statistics, between mathematics and statistics and other learning areas, and between mathematics and statistics and a wide range of contexts within and beyond school. Connections can be made with mātauranga Māori and between different cultural, linguistic, and historical contexts. The more connected mathematics and statistics knowledge is for ākonga, the stronger it will be.

### Te whakatau whānui | Generalising

Generalisations are the building blocks for exploring relationships and possibilities in mathematics or statistics. They enable more advanced mathematical or statistical thinking because they express ideas that are always mathematically and statistically true. They can be used to make and test conjectures, explain ideas and patterns, develop justifications and proofs, and predict outcomes.

### Te whakaaro arorau | Reasoning

Working mathematically and statistically centres on logical reasoning. Reasoning can be inductive or deductive, drawing on evidence or on known rules and relationships. Reasoning is the way we make predictions, build arguments, unpack stories from data, make inferences, and quantify and accept uncertainty.

### Te whakawhiti whakaaro | Communicating

Communicating in mathematics and statistics uses words, symbols, representations, visualisations, and conventions to explain thinking and share ideas and findings. Effective communication pays attention to the needs, capacities, and cultural context of its audience and promotes rigour, mathematical argumentation, shared understanding, participation, and working together.

# Progress outcome, typically by the end of **year 3** (Foundation)

*Te tupu pāhautea i te taiao ako whakahihiko* | *Thriving in environments rich in literacy and numeracy*

## Understand

*Through building knowledge about contexts and drawing on mathematical and statistical practices, I am deepening my understanding that:*

- Mātauranga Māori and mathematics and statistics help make sense of the world.
- Mathematics and statistics have a continuous, evolving human history.
- Mathematics and statistics are elegant, explorative, creative, and powerful.
- Mathematics and statistics help us in our everyday lives and decisions and are key to many areas of knowledge and practice.
- Mathematics and statistics reward persistence and positivity.

## Know

### Mātauranga tau | Number

#### Whole numbers

*I know:*

- Our number system is based on groups of ten, groups of 100, groups of 1000, and so on (base ten)
- Numbers can be put together and taken apart in different ways (composing and decomposing)
- Multiplication and division involve recognising and working with groups, the number of groups, and the total
- Numbers can include groups that I can use to find patterns, by, for example, skip counting or using pairs of numbers that add to 10
- Instead of counting I can use number patterns and an understanding of addition, subtraction, multiplication, and division to solve problems.

*I know how to:*

- estimate discrete and continuous quantities (i.e., separate items and amounts of something) and explain my estimate
- recognise, read, write, and order whole numbers up to 10 000
- group, partition, and recombine whole numbers up to 1 000
- solve addition and subtraction with two- and three-digit numbers
- solve multiplication problems involving two one-digit numbers, and a one- and a two-digit number
- solve division problems involving whole numbers, a one-digit divisor, and no remainders
- use the rules and patterns for operations that I have learned in algebra when working with numbers.

#### Rational numbers

*I know:*

- Fractions are a way to show pieces of a whole
- We need to know what the whole is to understand the size of a fraction
- The bottom number of a fraction (denominator) shows how many pieces a whole has been equally split into, and the top number (numerator) shows how many of those parts the fraction represents
- The bigger the bottom number (denominator) of a fraction, the smaller the pieces
- When fractions have the same denominator, I can add them together by seeing how many pieces I have in total and putting the total over the size of the pieces (the denominator)
- When a fraction has a 1 as the numerator, it is called a unit fraction (in the same way that we call 1 a 'unit')
- I can find a unit fraction by dividing a whole into the number of pieces in the denominator and taking one of those pieces
- A whole can be an area, a measurement, or a set of objects
- The same amount, like a half or a quarter, can be shown by different (equivalent) fractions because the relationship between the numerator and the denominator is the same.

*I know how to:*

- recognise, read, write, represent, and order halves, thirds, quarters, fifths, sixths, and eighths
- identify or find an equivalent fraction involving halves and quarters
- add unit fractions with like denominators
- find a unit fraction of a whole.

## Know

### Taurangi | Algebra

#### Patterns

*I know:*

- Patterns are made of numeric or spatial elements in a sequence governed by a rule; sometimes they repeat and sometimes they continue on
- I can describe a pattern using a rule; I can also start with a rule and create a pattern from it
- The same pattern structure can be found in many forms (e.g., numbers, shapes, colours, and rhythm)
- Identifying the rule of a pattern brings predictability and allows me to make generalisations
- To find the rule for a pattern, I need to work out what the unit of the pattern is (what is repeated or what changes)
- An algorithm is a sequence of rules that can be followed.

*I know how to:*

- find another element of a pattern, given part of it
- describe a rule that explains how a pattern works
- follow and create algorithms that sort numbers into categories such as odd and even, or multiples of 3.

#### Equations, expressions, and number properties

*I know:*

- Addition and subtraction, and multiplication and division, 'undo' or 'reverse' each other (inverse property)
- In addition and multiplication, I can change the order of the numbers and the answer stays the same (commutative property)
- If I add or subtract 0 from a number, or multiply or divide by 1, the answer will be the number I started with (identity property)
- Patterns in the way numbers are organised can help solve equations
- The equals sign is relational; it shows the two sides of an equation are balanced or the same
- I can keep things equal by doing the same operation to both sides of an equation.

*I know how to:*

- generalise the properties of addition and subtraction
- recall addition facts to 20 and related subtraction facts, and extend and apply these facts to computation with larger numbers
- recall multiplication and corresponding division facts for twos, fives, and tens
- solve open number sentences and true and false number sentences.

### Inenga | Measurement

*I know:*

- We can measure the properties of things (length, area, volume, capacity, mass, duration, and turn)
- Measuring starts at the beginning of the object being measured
- The measurement unit size has to remain the same
- When measuring, measurement units are repeated with no gaps or overlaps
- The measurement is the total number of units used.

*I know how to:*

- estimate and then reliably measure length, area, volume, capacity, and mass, using standard metric units
- use rulers, scales, square grids, and cubes to measure
- choose the right type of measurement unit for the property I am measuring
- tell the time on an analogue and digital clock to hours, half hours, and quarter past or quarter to the hour
- find out how far something has been turned, using half and quarter turns as benchmarks.

## Know

### Mokowā | Space

*I know:*

- Two- and three-dimensional shapes have features that can be observed and described using geometric language
- Two-dimensional shapes can be joined or partitioned to form new shapes
- Flip (reflection), turn (rotation), and slide (translation) are found in objects and structures and can be used to create patterns
- Two-dimensional shapes can have both reflection and rotation symmetry or no symmetry at all
- There are patterns and regularities in shapes that can be used to compare, classify, and predict
- Maps are two-dimensional representations of locations and places in the real world
- Landmarks and other aspects of the real world are shown on maps using symbols; the relative position of something can be described in terms of the positions of other things on the map
- Things can be turned (rotated); they may then look different, but they are the same.

### Shape

*I know how to:*

- visualise and recognise two- and three-dimensional shapes
- visualise partitioning shapes and recombining smaller shapes to make two-dimensional shapes
- identify horizontal and vertical lines of symmetry in two-dimensional shapes
- represent two- and three-dimensional shapes, using drawing, models, and digital tools
- compare and classify two- and three-dimensional shapes, and justify my classifications.

### Position and orientation

*I know how to:*

- mentally rotate a two-dimensional shape and identify what the outcome of the rotation will be
- predict and justify what will happen to two-dimensional shapes if you rotate, reflect, or translate them
- represent the relative positions of objects by making a model or a simple map
- interpret simple maps to locate objects, and move position by following directions and pathways
- turn myself and other objects through quarter, half, and full turns, clockwise and anticlockwise
- create and follow an algorithm consisting of a set of step-by-step instructions for moving an object to a different location.

### Tauanga | Statistics

*I know:*

- Data is information about the world and comes in many forms
- Primary data is data I collect; secondary data is data that has been provided by someone else
- Sorting and organising categorical variables (sorting objects into groups) helps to make sense of data and to answer investigative questions
- Data visualisations are visual representations of all possible values of one or more variables
- Displays of data can tell a story
- Summary investigative questions are questions about our class or another whole group
- Categorical variables are classified into groups.

### Data-based investigations

Ākonga experience a kaiako-led statistical enquiry cycle (PPDAC – Problem, Plan, Data, Analysis, & Conclusion) to undertake data-based investigations.

*I know how to:*

Problem	<ul style="list-style-type: none"> <li>• use kaiako-led summary investigative questions about a variable relevant to me and my class</li> </ul>
Plan	<ul style="list-style-type: none"> <li>• ask survey and data-collection questions, with support</li> </ul>
Data	<ul style="list-style-type: none"> <li>• collect, sort, and record categorical variables to support answering investigative questions</li> </ul>
Analysis	<ul style="list-style-type: none"> <li>• create kaiako-led data visualisations (picture graphs, pictographs, bar graphs, and tables)</li> <li>• describe what data visualisations show</li> </ul>
Conclusion	<ul style="list-style-type: none"> <li>• answer investigative questions with kaiako support, by choosing statements from my findings that best describe the data</li> <li>• share my findings with others.</li> </ul>

### Critical thinking in statistics

*I know how to:*

- identify relevant features in others' data visualisations
- explain and question statements about data visualisations, with reference to the data.

## Know

### Tūponotanga | Probability

#### *I know:*

- Probability is about the chance of outcomes occurring
- An event has more than one possible outcome
- I cannot predict the outcome of an event with certainty.

#### *I know how to:*

Problem and prediction	<ul style="list-style-type: none"><li>• recognise activities and everyday events in my own life that involve chance</li><li>• anticipate what might happen using the language of probability and terms such as certain, likely, possible, unlikely, and impossible</li></ul>
Plan	<ul style="list-style-type: none"><li>• identify possible categorical outcomes of an event</li></ul>
Data	<ul style="list-style-type: none"><li>• categorise possible outcomes, using digital tools, and explain how and why I have done this</li></ul>
Analysis	<ul style="list-style-type: none"><li>• display outcomes of chance situations by making models or using lists, tally charts, or digital tools</li></ul>
Conclusion	<ul style="list-style-type: none"><li>• identify different outcomes in simple chance situations and explain my thinking</li><li>• explain that different outcomes are possible in a chance situation.</li></ul>

### Critical thinking in probability

#### *I know how to:*

- decide whether a simple statement about the likelihood of outcomes is true or false.

### Progress step during the first six months

The indicators on the right are important. Make sure ākonga have many opportunities to learn in relation to them and to demonstrate them during their first six months at school. If they are not evident, this requires action.

#### *I know how to:*

- instantly recognise the total number in a group of up to six objects
- partition and recombine sets of up to 10 in different ways
- copy, continue, create, and describe a repeating pattern
- compare two objects by a property (e.g., length, weight, and volume)
- classify and sort objects by observable features (e.g., colour and shape)
- compose smaller shapes to complete a clearly outlined target shape, and decompose single shapes into smaller shapes, by trial and error.

# Progress outcome, typically by the end of **year 3**

## Do

### Te whakatauirā me te tūhura | Modelling and investigating

*In my learning in mathematics and statistics, I can:*

- ask a question that can be investigated using mathematics or statistics
- understand and explain in my own words what a question means mathematically or statistically
- find entry points for addressing a question using mathematics or statistics, identifying relevant prior knowledge, givens, assumptions, constraints, and relationships
- formulate a mathematical or statistical solution pathway and carry it out
- monitor and evaluate progress, adjusting pathways if needed
- make sense of mathematical or statistical outcomes or conclusions in light of a given situation and context.

### Te whakaahua | Representing

*In my learning in mathematics and statistics, I can:*

- use representations to compare, explore, simplify, illustrate, prove, justify, and find patterns, variations, and trends
- use representations to learn new ideas
- use representations to explain ideas to others
- use representations to investigate conjectures and support arguments
- select physical, virtual, graphical, or diagrammatic representations appropriate for what I am working on
- use visualisation to mentally represent and manipulate things and ideas.

### Te honohono | Connecting

*In my learning in mathematics and statistics, I can:*

- recognise and explain connections between ideas and between different representations of ideas
- connect new ideas to things I already know
- recognise and explore mathematical and statistical ideas in a range of contexts, including matters that are important in my community
- use my mathematics and statistics learning to connect to ideas in other learning areas and to cultural, linguistic, and historical contexts.

### Te whakatau whānui | Generalising

*In my learning in mathematics and statistics, I can:*

- recognise, explore, conjecture, and draw conclusions about mathematical or statistical patterns
- use mathematical or statistical relationships that I know to propose new relationships and to identify similarities, differences, and new connections across my learning
- look for patterns and regularities that might be applied in another situation or always be true as I work with mathematical and statistical ideas
- use mathematical and statistical symbols to express generalisations.

### Te whakaaro arorau | Reasoning

*In my learning in mathematics and statistics, I can:*

- build statements and explanations inductively from observations or data, considering the context
- build statements and explanations deductively from mathematical and statistical knowledge, definitions, and rules
- make conjectures, using reasoning and counter examples to decide if they are true or not
- critically reflect on statements based on mathematical and statistical ideas, distinguishing correct logic from flawed logic and asking questions to clarify and understand
- use evidence, reasoning, and proofs to explain why I agree or disagree with statements or why something will always be true
- develop collective understandings with others by sharing ideas and comparing, contrasting, and building on them.

### Te reo matatini o te pāngarau | Communicating

*In my learning in mathematics and statistics, I can:*

- use mathematical and statistical language to describe my thinking
- record, share, and explain my thinking
- communicate using representations that are appropriate for the idea I am talking about
- choose an effective format for communicating an idea or argument
- answer questions, engage in discussion, and respond to others' ideas in constructive ways
- present concise and coherent explanations and arguments for an idea, solution, or process.

## Learning experiences

In the digital version of the refreshed New Zealand Curriculum, there will be classroom resources and kaiako support linked to each progress outcome.

# Progress outcome, typically by the end of **year 6**

*Te kaingākaunui me te hiamō ki te ako | Expanding horizons of knowledge and collaboration*

## Understand

*Through building knowledge about contexts and drawing on mathematical and statistical practices, I am deepening my understanding that:*

- Mātauranga Māori and mathematics and statistics help make sense of the world.
- Mathematics and statistics have a continuous, evolving human history.
- Mathematics and statistics are elegant, explorative, creative, and powerful.
- Mathematics and statistics help us in our everyday lives and decisions and are key to many areas of knowledge and practice.
- Mathematics and statistics reward persistence and positivity.

## Know

### Mātauranga tau | Number

#### Whole numbers

*I know:*

- In our number system each place value is a power of 10, and this continues infinitely
- When I am multiplying two- or three-digit numbers, I can split one of the numbers into parts that add up to the number, then multiply each of the parts by the other number and add the results to get the answer (distributive property)
- In division, I might know how many shares I am making (partitive), or I might know the size of the shares (quotative); either way, to work out the answer I undo multiplication
- There are many different situations where the relationships between things are multiplicative and so problems can be solved with multiplication or division; these situations include equal groups, rates, comparisons, part-whole relationships, combinations, areas, and volumes.

*I know how to:*

- check if my answer is reasonable using estimation when working with whole numbers
- recognise, read, write, order, partition, recombine, and represent whole numbers up to 1 000 000
- add and subtract whole numbers
- multiply two- and three-digit whole numbers
- divide whole numbers by one- or two-digit divisors
- find factors and multiples of numbers to 100
- use the rules and patterns for operations that I have learned in algebra when working with numbers.

#### Rational numbers

*I know:*

- Fractions are numbers and can be put on a number line to show their relative size; one point on the number line can have different fraction 'names' that represent the same amount (equivalent fractions)
- Whole numbers are fractions with a denominator of 1

- Fractions can be a measure; when they are, they show parts of a measurement unit
- Fractions can describe a proportional relationship between two amounts
- Fractions can be an operator; when they are, they resize (enlarge or shrink) a number through multiplication
- Fractions with larger numerators than denominators are improper fractions; they can also be written as a mixed number (a whole number and a fraction)
- In simplified fractions, the numerator and denominator have no common factors; if the denominator of a simplified fraction is 1, then I should write it as a whole number
- The place-value system continues to the right, beyond units or ones, to create decimals (tenths, hundredths, thousandths); the decimal point is placed between the units (or ones) column and the tenths column, so we can read numbers as wholes and parts
- Decimals are a particular set of fractions that have powers of ten as their denominators
- Decimals make calculation with parts of numbers easier because the place-value system means we can add and subtract them in the same way as whole numbers
- A percentage is the number of hundredths of a whole
- Benchmark fractions ( $0, \frac{1}{10}, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ , and 1) are useful for estimating.

*I know how to:*

- compare and order fractions with the same or related denominators, including mixed numbers and improper fractions, and represent them on a number line
- recognise, read, write, represent, compare, and order decimals to three places
- convert between decimals (to two places), fractions, and percentages
- add and subtract decimal numbers to two places
- find a fraction or a percentage of a whole number
- multiply fractions by whole numbers
- represent a fraction in simplified form.

## Know

### Taurangi | Algebra

#### Patterns

**I know:**

- In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern's rule than the relationship between successive elements
- Tables and graphs provide a clear way to organise the positions and elements of a pattern to reveal a relationship or a rule for the general term
- In a pattern, when I know the relationship between a position and the corresponding element, I can find any element if I know the position, and any position if I know the element.

**I know how to:**

- connect elements of patterns that do not repeat to their ordinal position
- use tables, XY-graphs, and diagrams to find relationships between successive elements of patterns
- develop an explicit generalisation about a linear pattern using words
- create and use algorithms in digital tools to experiment with patterns in numbers
- use a rule to make predictions.

#### Equations, expressions, and number properties

**I know:**

- The equals sign and inequalities (greater than and less than) show relationships, and I can use these relationships to solve problems
- The distributive property says that multiplying a number by two numbers added together is the same as doing each multiplication separately and adding the results together
- In multiplication and addition, numbers can be grouped and combined in any way to solve problems (associative property)
- When I multiply or divide a number by 1, the number is unchanged (identity property)
- Multiplying a number by 0 is calculating that many 'lots of' 0, so the answer will always be 0
- Dividing by 0 is impossible because you cannot share out a number into 'lots of' 0 and you cannot split a number into 'lots of' 0.

**I know how to:**

- generalise properties of multiplication and division
- recall multiplication facts to  $10 \times 10$  and corresponding division facts, and apply these facts to computation problems with larger numbers
- solve open number sentence and true and false number sentences using equality and inequality relationships.

### Inenga | Measurement

**I know:**

- A measurement might contain wholes and parts of units
- Different measurement tools and scales use different-sized units
- When I use a scale, the point on the scale where the measurement ends tells me how much there is
- The metric measurement system is based on powers of ten, like our place-value number system
- When I record a measurement, I have to give the unit in order to be clear about what the measurement means
- Covering things gives area, filling things gives capacity or volume
- Two-dimensional shapes have a line around the outside, and this line is the perimeter
- Angles are a measure of turn and can be measured in degrees.

**I know how to:**

- estimate and then accurately measure length, area, volume, capacity, mass, and duration, using appropriate metric units or a combination of units
- read measurement tools and interpret scales accurately
- visualise a property of something so I can estimate and measure it
- visualise and find the perimeter and area of rectangles and the volume of rectangular prisms
- use analogue and digital clocks, twelve- and twenty-four-hour time measurements, and longer measures of duration (e.g., days, weeks, months, years, decades, centuries, and millennia)
- describe an angle using these benchmarks: 90 degrees is a quarter turn, 180 degrees is a half turn, and 360 degrees is a full turn.

## Know

### Mokowā | Space

*I know:*

- Two- and three-dimensional shapes can be defined by their geometric properties; there are rules for defining groups of shapes
- Properties of two- and three-dimensional shape can be used to compare, classify, predict, and identify relationships among shapes
- Two-dimensional shapes can be rotated around their centre or vertex to create an image
- Models of three-dimensional shapes can be created by combining two-dimensional shapes and surfaces; nets are used to create these models by folding connected two-dimensional shapes
- Three-dimensional shapes appear differently from different viewpoints; these viewpoints can be represented in models and diagrams
- Lines of symmetry can be horizontal, vertical, and diagonal
- Resizing a shape creates a bigger or smaller image of the shape (e.g., by doubling or halving)
- Locations can be described using a grid reference or coordinate system on a landscape represented as a map; the map involves scale, so the representations on it are proportional to the distances or lengths in reality.

### Shape

*I know how to:*

- classify plane shapes and prisms using their properties
- visualise three-dimensional shapes and work out what they would look like from different directions
- visualise partitioning and combining of two- and three-dimensional shapes
- visualise the perimeter and area of two-dimensional shapes
- represent three-dimensional shapes from different viewpoints using drawings, models, and digital tools
- visualise and predict what cuboids would look like when they are opened out to make nets
- create nets and construct cuboids.

### Position and orientation

*I know how to:*

- visualise and recognise line and rotational symmetry, and transformations in shapes and patterns
- mentally manipulate two- and three-dimensional shapes and identify the result of the manipulation
- describe relationships among two- and three-dimensional shapes in a construction or pattern
- use grid references and the language of direction (compass points), distance (m, km), and turn (degrees) to locate and describe positions and pathways
- use a single step to reflect, rotate, translate, or resize two-dimensional shapes, and represent the outcomes of the transformations using drawings, models, and digital tools
- compare the image of a transformed two-dimensional shape with the original and describe the transformation.

### Tauanga | Statistics

*I know:*

- Investigative questions are questions we ask about data
- Numerical variables can be counted or measured; discrete variables are counted; continuous variables are measured
- Multivariate datasets have more than one variable
- A conjecture involves thinking about what data will show before it is analysed
- A variety of data visualisations for the same data can give different information
- A comparison investigation compares similarities and difference for a variable across two or more groups
- A time-series investigation considers a variable that changes over time
- Publishers of data provide information about data in a data dictionary.

### Data-based investigations

Ākonga use the statistical enquiry cycle (PPDAC) with kaiako support to undertake data-based investigations.

*I know how to:*

Problem	<ul style="list-style-type: none"> <li>• pose with kaiako support summary investigative questions for one variable</li> <li>• participate in forming investigative, comparison, or time-series questions about school-related matters</li> <li>• make conjectures about what I expect to find out</li> </ul>
Plan	<ul style="list-style-type: none"> <li>• make a plan to collect data for observational studies, including who to collect from, when and where to collect, what to measure, and how to measure</li> <li>• consider whether the data collection may affect the people from whom it is being collected</li> </ul>
Data	<ul style="list-style-type: none"> <li>• collect and systematically record categorical and numerical variables to support answering investigative questions</li> <li>• store data using methods such as data cards, recording tables, or spreadsheets</li> <li>• use information from data dictionaries to find out about variables in secondary datasets</li> <li>• check data collected for errors</li> </ul>
Analysis	<ul style="list-style-type: none"> <li>• use digital statistical-analysis tools to create data visualisations</li> <li>• describe features, patterns, or trends in context</li> <li>• Identify similarities and differences between my findings and those of other groups</li> </ul>
Conclusion	<ul style="list-style-type: none"> <li>• answer investigative questions choosing statements from findings that best describe the data</li> <li>• explain and make sense of findings</li> <li>• think beyond the data I have and reflect on my conjectures.</li> </ul>

## Know

### Tauanga | Statistics *(continued)*

#### Critical thinking in statistics

*I know how to:*

- interrogate others' survey or data-collection questions
- identify and explain features and errors in others' data visualisations
- interrogate statements others make about data.

### Tūponotanga | Probability

*I know:*

- A probability experiment is a repeated set of trials, each with a possible outcome; all the possible outcomes are called events
- It is not possible to know the exact probability of something occurring in most everyday situations; therefore results may vary in different trials
- Outcomes that are certain are described by 1 or 100%, and outcomes that are impossible are described by 0 or 0%
- The results of repeated trials can be used to determine the relative frequency of an event (a fraction in which the numerator is the number of times the event occurred and the denominator is the total number of trials).

*I know how to:*

Problem and prediction	<ul style="list-style-type: none"> <li>• recognise a claim or investigative question that involves the likelihood or chance of an event happening</li> </ul>
Plan	<ul style="list-style-type: none"> <li>• list all the possible outcomes of simple experiments involving equally likely outcomes</li> <li>• conduct repeated trials in an experiment</li> </ul>
Data	<ul style="list-style-type: none"> <li>• gather and sort data, using lists, tables, and digital tools as appropriate</li> </ul>
Analysis	<ul style="list-style-type: none"> <li>• use frequency and visual displays to compare outcomes and estimate their likelihoods</li> <li>• consider what others say about probability in simple situations, referring to evidence</li> </ul>
Conclusion	<ul style="list-style-type: none"> <li>• identify which outcomes are always, the most, the least, and never likely to happen, by looking at the end results of repeated trials</li> <li>• use and explain the results of a game or situation to identify which outcomes are more likely to occur in future.</li> </ul>

#### Critical thinking in probability

*I know how to:*

- agree or disagree with others' conclusions and provide reasons why.

# Progress outcome, typically by the end of **year 6**

## Do

### Te whakatauirā me te tūhura | Modelling and investigating

*In my learning in mathematics and statistics, I can:*

- ask a question that can be investigated using mathematics or statistics
- understand and explain in my own words what a question means mathematically or statistically
- find entry points for addressing a question using mathematics or statistics, identifying relevant prior knowledge, givens, assumptions, constraints, and relationships
- formulate a mathematical or statistical solution pathway and carry it out
- monitor and evaluate progress, adjusting pathways if needed
- make sense of mathematical or statistical outcomes or conclusions in light of a given situation and context.

### Te whakaahua | Representing

*In my learning in mathematics and statistics, I can:*

- use representations to compare, explore, simplify, illustrate, prove, justify, and find patterns, variations, and trends
- use representations to learn new ideas
- use representations to explain ideas to others
- use representations to investigate conjectures and support arguments
- select physical, virtual, graphical, or diagrammatic representations appropriate for what I am working on
- use visualisation to mentally represent and manipulate things and ideas.

### Te honohono | Connecting

*In my learning in mathematics and statistics, I can:*

- recognise and explain connections between ideas and between different representations of ideas
- connect new ideas to things I already know
- recognise and explore mathematical and statistical ideas in a range of contexts, including matters that are important in my community
- use my mathematics and statistics learning to connect to ideas in other learning areas and to cultural, linguistic, and historical contexts.

### Te whakatau whānui | Generalising

*In my learning in mathematics and statistics, I can:*

- recognise, explore, conjecture, and draw conclusions about mathematical or statistical patterns
- use mathematical or statistical relationships that I know to propose new relationships and to identify similarities, differences, and new connections across my learning
- look for patterns and regularities that might be applied in another situation or always be true as I work with mathematical and statistical ideas
- use mathematical and statistical symbols to express generalisations.

### Te whakaaro arorau | Reasoning

*In my learning in mathematics and statistics, I can:*

- build statements and explanations inductively from observations or data, considering the context
- build statements and explanations deductively from mathematical and statistical knowledge, definitions, and rules
- make conjectures, using reasoning and counter examples to decide if they are true or not
- critically reflect on statements based on mathematical and statistical ideas, distinguishing correct logic from flawed logic and asking questions to clarify and understand
- use evidence, reasoning, and proofs to explain why I agree or disagree with statements or why something will always be true
- develop collective understandings with others by sharing ideas and comparing, contrasting, and building on them.

### Te reo matatini o te pāngarau | Communicating

*In my learning in mathematics and statistics, I can:*

- use mathematical and statistical language to describe my thinking
- record, share, and explain my thinking
- communicate using representations that are appropriate for the idea I am talking about
- choose an effective format for communicating an idea or argument
- answer questions, engage in discussion, and respond to others' ideas in constructive ways
- present concise and coherent explanations and arguments for an idea, solution, or process.

## Learning experiences

In the digital version of the refreshed New Zealand Curriculum, there will be classroom resources and kaiako support linked to each progress outcome.

# Progress outcome, typically by the end of **year 8**

*Te mōhio ki tōku tūrangawaewae me te kōkiri kaupapa | Knowing I belong and advocating for self and others*

## Understand

*Through building knowledge about contexts and drawing on mathematical and statistical practices, I am deepening my understanding that:*

- Mātauranga Māori and mathematics and statistics help make sense of the world.
- Mathematics and statistics have a continuous, evolving human history.
- Mathematics and statistics are elegant, explorative, creative, and powerful.
- Mathematics and statistics help us in our everyday lives and decisions and are key to many areas of knowledge and practice.
- Mathematics and statistics reward persistence and positivity.

## Know

### Mātauranga tau | Number

#### Whole numbers

*I know:*

- There are real-life situations described by quantities less than zero, such as the temperature or being in debt
- Positive and negative numbers can be placed on a number line
- Positive and negative numbers are opposites; joining a pair of opposites results in zero
- Negative numbers can be added and subtracted; to do this, I need to extend my understanding of joining (adding) and taking away (subtracting) to amounts less than zero
- Negative numbers can be added and subtracted with positive numbers; this requires reasoning about the impact of joining and taking away, as well as separating amounts that are above and below zero
- A square root undoes squaring a number
- In equations where there is more than one operation, there are rules for the order in which to apply the operations.

*I know how to:*

- add and subtract integers
- find square roots of numbers that are the squares of whole numbers (perfect squares)
- use the order of operations to solve problems
- recognise when a mathematical situation involves integers and figure out the mathematical relationships in the situation.

#### Rational numbers

*I know:*

- Decimals (tenths, hundredths, thousandths) result from continuing the place-value system using negative powers of ten
- Whole numbers and integers can be written as fractions
- Every fraction can be represented by an infinite set of equivalent fractions that occupy the same point on the number line

- There is no least or greatest fraction on the number line
- There is always another fraction between any two fractions on the number line
- A fraction can be a quotient, showing the result of one number being divided by another
- Fractions can be used to describe the likelihood of something happening
- The effect of adding or subtracting with decimals and fractions is the same as with whole numbers; the effect of multiplying with decimals smaller than 1 is different to multiplying with whole numbers
- When a number cannot be divided exactly by a divisor, the result includes a remainder; remainders can be interpreted as whole numbers, fractions, or decimals, depending on the context
- The product of two positive fractions or decimals each less than one is less than either of them
- To add and subtract, fractions with unlike denominators are expressed as equivalent fractions with like denominators
- Multiplying two fractions gives a fraction of a fraction; simplifying fractions and using models can help with this
- The commutative and associative properties of addition and multiplication work the same with fractions as with whole numbers
- The additive identity (0) and multiplicative identity (1) are the same with fractions and decimals as with whole numbers
- Benchmark fractions can be used to estimate calculations involving fractions and decimals.

*I know how to:*

- recognise, read, write, represent, compare, convert, and order fractions, decimals, and percentages
- add and subtract fractions and decimals
- divide by whole-number divisors
- multiply fractions and decimals
- convert fractions, decimals, and percentages and apply this when solving problems
- estimate the outcome of adding, subtracting, and multiplying familiar fractions and decimals by using an understanding of their structure and common benchmarks.

## Know

### Taurangi | Algebra

#### Patterns

##### *I know:*

- Functions are relationships or rules that uniquely associate members of one set with members of another set
- Linear patterns have rules that can be explicitly generalised as a function in which the two sets are the ordinal positions and the elements
- Linear relationships are represented by or form straight lines on XY-graphs
- When describing a rule, function, or generalisation, we can use letters as symbols to stand for any number (a variable)
- Some relationships in number patterns are not linear and do not correspond to straight lines on XY-graphs
- A function can be described as an algorithm that can be used to generate a pattern.

##### *I know how to:*

- form a generalisation about a linear pattern using variables, and use it to make predictions
- represent linear patterns using tables and XY-graphs
- use recursive methods (looking at the previous element) to predict further elements of a sequence in which the relationship is non-linear
- create and use algorithms in digital tools to generate patterns using integers
- identify and describe the properties of prime, composite (non-prime), and square numbers
- identify the divisibility rules for 2, 3, 5, 9, and 10.

#### Equations, expressions, and number properties

##### *I know:*

- Inequalities can also include equals (greater than or equal, and less than or equal) to show a relationship, and I can use this relationship to solve problems
- Operations have properties that can be generalised using variables
- A linear relationship can be shown by an XY-graph, a table, an equation, or in words
- When I do not know the number a variable stands for in an equation, I can work it out by thinking about what the equation says and using the relationships shown by the symbols in the equation
- I can use the inverse property to help work out missing numbers in equations.

##### *I know how to:*

- connect at least two different forms of linear relationships, such as an XY-graph of a line and the corresponding equation
- solve informally problems based on relationships and explain my reasoning
- find and justify a word formula to represent a practical situation
- use words and symbols to describe and apply all properties of operations as they apply to whole numbers (commutativity, distributivity, associativity, inverse, and identity)
- solve linear equations of the form  $a = b + cx$  by trial and improvement and by applying inverse operations.

### Inenga | Measurement

##### *I know:*

- When two line segments meet they form an angle, which can be thought of as a rotation of one of the line segments
- When two lines meet at right angles, they are perpendicular
- When I multiply lengths or divide an area or volume by a length, the result has a different unit (a derived unit)
- Shapes can be made up of other shapes or make up other shapes; this can help us find perimeters, areas, and volumes
- In the metric system, there are base measurements (metre, gram, litre) with prefixes added to show the size of units
- To make metric measurements easy to understand, I can either use decimals or change the unit so I am using whole numbers
- Measurements can involve wholes and parts or comparing parts to a measurement unit; so fractions and decimals are important in measurement.

##### *I know how to:*

- estimate and then measure length, area, volume, capacity, mass, temperature, data storage, time, and angle, using appropriate metric units
- read analogue and digital measurement tools, round appropriately, and interpret scales accurately
- visualise the amount of turn in angles and measure them using degrees
- find the perimeter and area of parallelograms, triangles, and shapes composed of quadrilaterals
- find the volume of triangular prisms and shapes composed of rectangular prisms
- use decimals to express parts of wholes in measurements
- convert between metric units, using whole numbers
- read, interpret, and use timetables and charts that present measurement information
- make connections between smaller and larger units of time, expressing parts of a unit of time using a smaller unit or fractions of the unit.

## Know

### Mokowā | Space

#### *I know:*

- Classes of two- and three-dimensional shapes can be identified by their geometric properties – in particular, polygons by their number of sides, three-dimensional shapes by the nature of their faces and surfaces, and two-dimensional closed curves by their symmetries
- Classes can be disjoint, such as scalene and isosceles triangles, and prisms and pyramids; or they can be sub-classes that are included within other classes, such as squares within rectangles
- Three-dimensional shapes can be modelled by a variety of two-dimensional representations, and vice versa
- Prisms are solids with a fixed cross-section; they are classified by their cross section
- The invariant properties of two- and three-dimensional shapes do not change under different transformations
- Position, direction, and pathways can be operated on using coordinate systems and maps
- Direction is plotted accurately by a clockwise angle measured from north; I need to understand the relationship between angle measures and compass points
- Distances on maps and grids are shown by a scale that can be written in three different ways: ratio, measurement, or a drawing
- Dynamic geometry software can be used to create and manipulate two-dimensional shapes.

### Shape

#### *I know how to:*

- identify the features and properties of triangles, quadrilaterals, and other polygons
- classify types of two-dimensional shapes (e.g., triangles) based on their features and properties
- identify angle properties of straight lines, angles at point, and vertically opposite angles
- find unknown angles using angle properties
- design an algorithm to sort and classify two- and three-dimensional shapes according to their properties, and identify and reason about the classifications
- visualise missing parts from composite shapes
- visualise three-dimensional shapes from drawings and diagrams
- visualise what prisms would look like when they are opened out to make nets
- create nets and construct prisms
- create isometric drawings of objects from different perspectives.

### Position and orientation

#### *I know how to:*

- mentally manipulate three-dimensional shapes, and draw or make the result of the manipulation
- recognise and use combinations of transformations to create tessellations and other geometric patterns
- resize two-dimensional shapes by a positive whole number or a unit fraction
- demonstrate that some properties of two-dimensional shapes do not change under different transformations
- use multi-steps to reflect, rotate, translate, or resize two-dimensional shapes, and represent the outcomes of the transformations using drawings, models, and digital tools
- use scale and compass points to interpret and describe location and direction
- read maps and interpret pictures, photographs, and diagrams spatially, finding pathways and directions, recognising different perspectives, and identifying where things are in relation to each other.

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## Know

### Tauanga | Statistics

**I know:**

- New variables are created by combining and modifying existing data
- Data visualisations show patterns, trends, and variations
- A relationship investigation looks for a relationship between two variables
- In an investigation or experiment, the groups of interest are those from whom the data will be collected
- Conjectures have an element of uncertainty and statements should reflect this
- A distribution is all the possible values of a variable.

### Data-based investigations

Ākonga use the statistical enquiry cycle (PPDAC) to conduct data-based investigations.

**I know how to:**

Problem	<ul style="list-style-type: none"> <li>• pose summary, comparison, and relationship investigative questions about local rohe or community matters</li> </ul>
Plan	<ul style="list-style-type: none"> <li>• make a plan to collect data for observational studies</li> <li>• determine the groups of interest</li> <li>• determine the variables of interest and identify reliable measurements for them</li> <li>• pose survey and data-collection questions</li> <li>• decide when and where to collect the data</li> <li>• consider whether the data collection may affect the people from whom it is being collected</li> </ul>
Data	<ul style="list-style-type: none"> <li>• collect and systematically record data to support answering investigative questions</li> <li>• set up spreadsheets to store data</li> <li>• export data into statistical-analysis tools</li> <li>• source and use existing multivariate datasets, including data dictionaries, to find out about variables and data-collection methods</li> <li>• check data collected for errors</li> </ul>
Analysis	<ul style="list-style-type: none"> <li>• use statistical-analysis tools to make data visualisations</li> <li>• create data visualisations for summary, comparison, relationship, and time-series situations using categorical, numerical, and continuous variables</li> <li>• describe features, patterns, or trends of the distributions of variables, including the median and range</li> <li>• use a variety of data visualisations to show local and global information about variables</li> <li>• refer to the context in statements about data visualisations, including variables of interest, groups of interest, counts or proportions for categorical variables, and values and units for numerical variables</li> <li>• use my findings to make conjectures about whether related groups of interest would have similar or different findings from mine</li> </ul>

Conclusion

- answer investigative questions using evidence from the analysis, connecting the evidence to the context
- communicate findings in context
- reflect on my conjectures and explore possible explanations.

### Critical thinking in statistics

**I know how to:**

- examine the data-collection methods and findings of others' statistical investigations to see if their claims are believable and reasonable
- critically consider data visualisations.

## Know

### Tūponotanga | Probability

**I know:**

- Results of past trials in probability experiments do not impact on future events
- There is variation between expected outcomes from models and experimental outcomes from trials
- If all possible outcomes in a chance situation are equally likely, the probability of an event happening is a fraction in which the numerator is the number of ways the event can happen and the denominator is the total number of possible outcomes.

**I know how to:**

Problem and prediction	<ul style="list-style-type: none"> <li>• recognise claims or misconceptions and pose investigative questions for single-stage events involving equally likely outcomes</li> <li>• consider how many trials should be conducted and what might happen if the number of trials is increased</li> </ul>
Plan	<ul style="list-style-type: none"> <li>• systematically list all possible outcomes of single-stage events with equally likely outcomes, using a table or tree diagram and assigning probabilities</li> <li>• conduct repeated single-stage chance experiments and run simulations with a large number of trials, using digital tools</li> </ul>
Data	<ul style="list-style-type: none"> <li>• gather and sort data systematically, using lists, tables, and digital tools</li> </ul>
Analysis	<ul style="list-style-type: none"> <li>• compare predictions about outcomes with observed results, and explain the differences</li> <li>• compare distributions from trialling with the expectations obtained from models</li> </ul>
Conclusion	<ul style="list-style-type: none"> <li>• explain and reason similarities and differences in results.</li> </ul>

### Critical thinking in probability

**I know how to:**

- question a conclusion by interrogating the way in which an investigation was conducted
- agree or disagree with others' conclusions and ask questions about their plan and analysis.

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# Progress outcome, typically by the end of **year 8**

## Do

### Te whakatauirā me te tūhura | Modelling and investigating

*In my learning in mathematics and statistics, I can:*

- ask a question that can be investigated using mathematics or statistics
- understand and explain in my own words what a question means mathematically or statistically
- find entry points for addressing a question using mathematics or statistics, identifying relevant prior knowledge, givens, assumptions, constraints, and relationships
- formulate a mathematical or statistical solution pathway and carry it out
- monitor and evaluate progress, adjusting pathways if needed
- make sense of mathematical or statistical outcomes or conclusions in light of a given situation and context.

### Te whakaahua | Representing

*In my learning in mathematics and statistics, I can:*

- use representations to compare, explore, simplify, illustrate, prove, justify, and find patterns, variations, and trends
- use representations to learn new ideas
- use representations to explain ideas to others
- use representations to investigate conjectures and support arguments
- select physical, virtual, graphical, or diagrammatic representations appropriate for what I am working on
- use visualisation to mentally represent and manipulate things and ideas.

### Te honohono | Connecting

*In my learning in mathematics and statistics, I can:*

- recognise and explain connections between ideas and between different representations of ideas
- connect new ideas to things I already know
- recognise and explore mathematical and statistical ideas in a range of contexts, including matters that are important in my community
- use my mathematics and statistics learning to connect to ideas in other learning areas and to cultural, linguistic, and historical contexts.

### Te whakatau whānui | Generalising

*In my learning in mathematics and statistics, I can:*

- recognise, explore, conjecture, and draw conclusions about mathematical or statistical patterns
- use mathematical or statistical relationships that I know to propose new relationships and to identify similarities, differences, and new connections across my learning
- look for patterns and regularities that might be applied in another situation or always be true as I work with mathematical and statistical ideas
- use mathematical and statistical symbols to express generalisations.

### Te whakaaro arorau | Reasoning

*In my learning in mathematics and statistics, I can:*

- build statements and explanations inductively from observations or data, considering the context
- build statements and explanations deductively from mathematical and statistical knowledge, definitions, and rules
- make conjectures, using reasoning and counter examples to decide if they are true or not
- critically reflect on statements based on mathematical and statistical ideas, distinguishing correct logic from flawed logic and asking questions to clarify and understand
- use evidence, reasoning, and proofs to explain why I agree or disagree with statements or why something will always be true
- develop collective understandings with others by sharing ideas and comparing, contrasting, and building on them.

### Te reo matatini o te pāngarau | Communicating

*In my learning in mathematics and statistics, I can:*

- use mathematical and statistical language to describe my thinking
- record, share, and explain my thinking
- communicate using representations that are appropriate for the idea I am talking about
- choose an effective format for communicating an idea or argument
- answer questions, engage in discussion, and respond to others' ideas in constructive ways
- present concise and coherent explanations and arguments for an idea, solution, or process.

## Learning experiences

In the digital version of the refreshed New Zealand Curriculum, there will be classroom resources and kaiako support linked to each progress outcome.

# Progress outcome, typically by the end of **year 10**

*Te whai ahunga, te manaaki i ētahi atu me te manawaroa | Having a purpose and being empathetic and resilient*

## Understand

*Through building knowledge about contexts and drawing on mathematical and statistical practices, I am deepening my understanding that:*

- Mātauranga Māori and mathematics and statistics help make sense of the world.
- Mathematics and statistics have a continuous, evolving human history.
- Mathematics and statistics are elegant, explorative, creative, and powerful.
- Mathematics and statistics help us in our everyday lives and decisions and are key to many areas of knowledge and practice.
- Mathematics and statistics reward persistence and positivity.

## Know

### Mātauranga tau | Number

#### Rational numbers

*I know:*

- There are an infinite number of fractions between any two fractions on the number line
- When I multiply numbers and one of them is less than 1, the result will be smaller than the other number
- When I divide numbers and the divisor is less than 1, the result will be bigger than the dividend
- When I multiply a fraction by its reciprocal, I get 1
- I can rewrite division of one number by another as a fraction
- When I multiply a fraction by an equivalent form of 1, the result is an equivalent fraction
- A rate compares two different quantities that have different units of measure; the resulting unit often uses 'per'
- A ratio is a comparison of two like quantities.

*I know how to:*

- compare any two rational numbers
- estimate the outcome of operations with fractions and decimals by using an understanding of their structure and benchmarks
- make sense of the process of multiplying and dividing with parts of a whole
- multiply and divide fractions
- multiply and divide decimals
- find fractions, decimals, and percentages of any number
- explore concepts of change using rates
- explore the use of ratios in everyday contexts.

#### Real numbers

*I know:*

- Multiplying can be thought of as repeated addition, so multiplying negative numbers can start with adding them and then generalising
- Division can be thought of as repeated subtraction, so dividing negative numbers can start with subtracting them and then generalising
- Rational numbers are ratios of whole numbers
- Decimals can be terminating, repeating, or non-repeating and infinite (irrational numbers)
- There are an infinite number of numbers between any two numbers
- Some irrational numbers such as  $\sqrt{2}$  and  $\pi$  are represented by special symbols
- Real numbers consist of positive and negative whole numbers, rational numbers, and irrational numbers.

*I know how to:*

- compare any two numbers
- multiply and divide positive and negative numbers
- round numbers appropriately considering the context
- express solutions involving irrational numbers in either rounded or exact form.

## Know

### Taurangi | Algebra

#### Patterns

*I know:*

- Functions can be expressed as an algebraic expression, an XY-graph, a table, or in words
- Patterns have rules that can be explicitly generalised as a function.

*I know how to:*

- find the relationship between a position and its corresponding element in a pattern, including linear and simple quadratic patterns
- find the other forms of a function of a pattern, when it is expressed in words, as an equation, in a table, or as a graph
- create and use algorithms in digital tools to generate patterns.

#### Equations, expressions, and number properties

*I know:*

- The commutative and associative properties of addition and multiplication work with all real numbers
- The additive identity (0) and multiplicative identity (1) are the same for all real numbers
- The distributive property works with all real numbers
- There are different forms of equations for expressing linear relationships
- Graphs may contain breaks; this can be useful for modelling practical situations
- Substitution means putting numbers or different variables in place of variables
- The exponent of a number says how many times the base (the number) is multiplied by itself; exponents are also called 'powers'
- Like terms are terms whose variables and their corresponding exponents are the same
- Factorising is finding the numbers or terms that multiply together to get an expression.

*I know how to:*

- write equations to explore practical situations
- graph linear equations and interpret the gradient, x-intercept, and y-intercept in relation to the equation or the practical situation represented
- sketch and interpret XY-graphs to represent everyday dynamic situations
- use a formula, rule, or equation and solve for unknowns, evaluate by substitution, and check solutions
- operate on numbers with whole-number exponents and from this generalise the rule for multiplying or dividing numbers with exponents and the same base
- apply the distributive property to expand, rearrange, and simplify linear expressions
- apply an understanding of factors to determine the common factors of expressions
- combine like terms in algebraic expressions
- simplify algebraic expressions using the order of operations
- solve linear equations
- solve linear inequalities and graph solutions on a number line.

### Inenga | Measurement

*I know:*

- The number of significant figures in a measurement is the number of digits that contribute to the degree of accuracy of the measurement
- Features of circles and polygons describe what they are and how they are constructed
- Features of prisms describe what they are and how they are formed
- Resizing a shape changes its perimeter, area, and volume
- In right-angled triangles, there is a fixed relationship between side lengths given by Pythagoras' theorem
- Decimal measures are used for very small durations (milliseconds); the rest of time measurement uses a different system (based on 12 and 60).

*I know how to:*

- estimate and then measure length, area, volume, capacity, mass, temperature, time, and angle, selecting appropriate tools and units
- read analogue and digital measurement tools, round appropriately, and interpret scales accurately using significant figures
- convert between metric units, using decimals
- determine the perimeter, area, and volume of shapes when a length has been resized by multiplying or dividing by a whole number
- reason about the perimeter and area of polygons and circles, using their properties
- use generalisations to find the perimeter and area of polygons and circles
- find the surface area of prisms, including cylinders, by visualising the area and how it could be 'covered'
- find the volume or capacity of prisms, including cylinders, by visualising the volume or capacity and how it could be filled
- use and apply Pythagoras' theorem to find the side lengths of right-angled triangles
- reason about duration using different units of time, fractions of units of time, and decimal fractions (milliseconds) where appropriate.

## Know

### Mokowā | Space

*I know:*

- Geometric properties of two-dimensional shapes, including internal and external angles of polygons and angles on intersecting and parallel lines, can be used to calculate lengths and angles
- A point has zero dimensions, a line has one dimension, a plane is two-dimensional, and a solid is three-dimensional; in mathematics there are more dimensions, but we cannot draw them because we live in a three-dimensional world
- In similar shapes, corresponding angles are equal and lengths of corresponding sides are proportional.

### Shape

*I know how to:*

- visualise how polygons fit together to form a model of a three-dimensional shape
- recognise similar two-dimensional shapes and their features
- create accurate nets for platonic solids, cuboids, right-angled prisms, and pyramids
- connect three-dimensional shapes with two-dimensional drawings, including plan views, isometric projections, and nets
- identify in diagrams and two-dimensional shapes identical angles formed by a transversal crossing parallel lines
- generalise and use angle properties of parallel lines and a transversal to reason about unknown angles
- generalise and use interior and exterior angle properties of polygons to reason about unknown angles
- decide which polygons can fit together to form a model of a three-dimensional shape, and why
- use the properties of similarity in two-dimensional shapes, including right-angled triangles, to find unknown lengths.

### Position and orientation

*I know how to:*

- fit two-dimensional shapes together, because the interior angles of each shape at their vertex add to 360
- justify and reason how similar two-dimensional shapes have corresponding sides that are in proportion
- interpret and describe location on a map by calculating distances using map scales and directions using one or more compass bearings
- define and accurately describe the transformation a two-dimensional shape has undergone, and identify the invariant properties
- describe transformations of a set of points using coordinates in the XY-plane, translations and reflections on an axis, and rotations about a given point by 90 or 180 degrees.

### Tauanga | Statistics

*I know:*

- It is not always possible to get data about everything, or from everyone (the entire population); there are ways of addressing this
- In experiments, the experimental units are the persons or objects that are the subject of the experiment
- Data is not always accurate and may need 'cleaning'
- Features of same-size sample distributions of a variable from a population vary from sample to sample
- Uncertainty should be taken into account when making claims.

### Data-based investigations

Ākonga use the statistical enquiry cycle (PPDAC) to conduct data-based investigations.

*I know how to:*

Problem	<ul style="list-style-type: none"> <li>• pose summary and comparison investigative questions about populations</li> <li>• pose time-series and relationship investigative questions</li> <li>• pose investigative questions that can be answered through experiments</li> </ul>
Plan	<ul style="list-style-type: none"> <li>• make a plan to collect or source data for observational studies and to collect data for experiments</li> <li>• decide whether to take a sample from the population of interest or use the group of interest, depending on the investigative question</li> <li>• determine the experimental units when undertaking an experiment</li> <li>• determine the variables of interest needed to answer investigative questions</li> <li>• identify valid and reliable measurements for variables when posing survey and data-collection questions</li> <li>• decide when and where to collect data</li> <li>• examine data-collection plans from an ethical viewpoint, considering the variables and how the data will be collected, stored, used, and reported</li> </ul>
Data	<ul style="list-style-type: none"> <li>• collect, record, store, and import data into statistical-analysis tools to support answering investigative questions</li> <li>• clean the data by editing and, where appropriate, deleting data</li> <li>• create new variables by combining and modifying existing data</li> <li>• source, interrogate and use existing multivariate datasets</li> </ul>
Analysis	<ul style="list-style-type: none"> <li>• use statistical-analysis tools to make data visualisations</li> <li>• describe features, patterns, or trends within and between distributions of variables, in context</li> <li>• investigate features, patterns, and trends – including, where appropriate, clusters, gaps, outliers, measures of centre and spread, the shape of distributions, the context, and the position of data – and look for relevant reasons and possible explanations</li> </ul>

## Know

### Tauanga | Statistics *(continued)*

Conclusion	<ul style="list-style-type: none"> <li>answer investigative questions using evidence from the analysis</li> <li>link my statements to the context, reflect on my conjectures, and provide possible explanations for findings</li> <li>make claims about populations from samples, for summary and comparison situations</li> <li>communicate my findings with appropriate displays and in ways that non-specialist audiences will understand.</li> </ul>
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#### Critical thinking in statistics

***I know how to:***

- evaluate the findings and claims of others by interrogating all phases of the statistical enquiry cycle
- propose reasons why particular data visualisations were used to support others' claims.

### Tūponotanga | Probability

***I know:***

- Some variation between theoretical probabilities and estimates from experiments is normal
- A simulation is a way to model random events, generally using digital tools
- Elements of chance affect the certainty of results from surveys or experiments.

***I know how to:***

Problem and prediction	<ul style="list-style-type: none"> <li>recognise and pose investigative questions for two-stage, equally likely events</li> </ul>
Plan	<ul style="list-style-type: none"> <li>design and conduct repeated experiments and simulations for two-stage events</li> <li>systematically list the sample space (all possible outcomes) of two-stage events with equally likely outcomes, using a table and tree diagram and assigning probabilities</li> </ul>
Data	<ul style="list-style-type: none"> <li>gather and sort data systematically, using lists, tables, and digital tools as appropriate</li> </ul>
Analysis	<ul style="list-style-type: none"> <li>use digital tools to compare probabilities of one or two events, and describe observed results</li> <li>link an experiment to a theoretical model</li> <li>compare distributions from trialling with the expectations obtained from models</li> </ul>
Conclusion	<ul style="list-style-type: none"> <li>use data to back up statements I or others have made.</li> </ul>

#### Critical thinking in probability

***I know how to:***

- identify why claims might not be valid and suggest suitable improvements to them
- use findings to evaluate the reasonableness of others' claims
- explain whether an advertising claim is misleading and why probability might have been used in a particular way to arrive at the claim
- recognise potential issues in the match between the real world, in which additional factors might influence the chance of something occurring, and a theoretical model.

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# Progress outcome, typically by the end of **year 10**

## Do

### Te whakatauirā me te tūhura | Modelling and investigating

*In my learning in mathematics and statistics, I can:*

- ask a question that can be investigated using mathematics or statistics
- understand and explain in my own words what a question means mathematically or statistically
- find entry points for addressing a question using mathematics or statistics, identifying relevant prior knowledge, givens, assumptions, constraints, and relationships
- formulate a mathematical or statistical solution pathway and carry it out
- monitor and evaluate progress, adjusting pathways if needed
- make sense of mathematical or statistical outcomes or conclusions in light of a given situation and context.

### Te whakaahua | Representing

*In my learning in mathematics and statistics, I can:*

- use representations to compare, explore, simplify, illustrate, prove, justify, and find patterns, variations, and trends
- use representations to learn new ideas
- use representations to explain ideas to others
- use representations to investigate conjectures and support arguments
- select physical, virtual, graphical, or diagrammatic representations appropriate for what I am working on
- use visualisation to mentally represent and manipulate things and ideas.

### Te honohono | Connecting

*In my learning in mathematics and statistics, I can:*

- recognise and explain connections between ideas and between different representations of ideas
- connect new ideas to things I already know
- recognise and explore mathematical and statistical ideas in a range of contexts, including matters that are important in my community
- use my mathematics and statistics learning to connect to ideas in other learning areas and to cultural, linguistic, and historical contexts.

### Te whakatau whānui | Generalising

*In my learning in mathematics and statistics, I can:*

- recognise, explore, conjecture, and draw conclusions about mathematical or statistical patterns
- use mathematical or statistical relationships that I know to propose new relationships and to identify similarities, differences, and new connections across my learning
- look for patterns and regularities that might be applied in another situation or always be true as I work with mathematical and statistical ideas
- use mathematical and statistical symbols to express generalisations.

### Te whakaaro arorau | Reasoning

*In my learning in mathematics and statistics, I can:*

- build statements and explanations inductively from observations or data, considering the context
- build statements and explanations deductively from mathematical and statistical knowledge, definitions, and rules
- make conjectures, using reasoning and counter examples to decide if they are true or not
- critically reflect on statements based on mathematical and statistical ideas, distinguishing correct logic from flawed logic and asking questions to clarify and understand
- use evidence, reasoning, and proofs to explain why I agree or disagree with statements or why something will always be true
- develop collective understandings with others by sharing ideas and comparing, contrasting, and building on them.

### Te reo matatini o te pāngarau | Communicating

*In my learning in mathematics and statistics, I can:*

- use mathematical and statistical language to describe my thinking
- record, share, and explain my thinking
- communicate using representations that are appropriate for the idea I am talking about
- choose an effective format for communicating an idea or argument
- answer questions, engage in discussion, and respond to others' ideas in constructive ways
- present concise and coherent explanations and arguments for an idea, solution, or process.

## Learning experiences

In the digital version of the refreshed New Zealand Curriculum, there will be classroom resources and kaiako support linked to each progress outcome.

# Progress outcome, typically by the end of **year 13**

*Te whakaterere me te waihanga i ngā arawātea me ngā ahunga whakamua mō anamata*  
*Navigating a pathway and contributing to future possibilities*

## Understand

*Through building knowledge about contexts and drawing on mathematical and statistical practices, I am deepening my understanding that:*

- Mātauranga Māori and mathematics and statistics help make sense of the world.
- Mathematics and statistics have a continuous, evolving human history.
- Mathematics and statistics are elegant, explorative, creative, and powerful.
- Mathematics and statistics help us in our everyday lives and decisions and are key to many areas of knowledge and practice.
- Mathematics and statistics reward persistence and positivity.

## Know

### Mātauranga tau | Number

#### *I know:*

- Proportional reasoning involves thinking about relationships and comparing quantities
- Geometric sequences can be generated using recursion, and they can help model exponential growth and decay in discrete situations
- Reciprocal, square roots, cube roots, and other roots can be written using exponents
- A logarithm is the power to which a number must be raised, to yield another specific number
- Exponents and logarithms are inversely related
- The two commonly used bases for logarithms are 10 and e
- Sums or series can be represented using sigma notation
- The Counting Principle is used to count the total number of possible outcomes
- The square root of negative one ( $\sqrt{-1}$ ) is represented by the symbol  $i$ .

#### *I know how to:*

- use rounding and estimating as appropriate
- fluently operate with whole numbers, decimals, fractions, percentages, rates, and ratios
- evaluate and determine the reasonableness of numeric solutions
- use proportional reasoning to investigate situations involving percentages, rates, and ratios
- operate on very large and on very small numbers using scientific notation, in both written and digital forms
- add, subtract, multiply, divide, find powers, and find roots of fractions
- create and follow algorithms that find prime factorisation of positive integers, including greatest common factors and least common multiples
- operate on numbers with real number exponents by applying exponent rules, including evaluating integer and rational exponents

- evaluate, simplify, and operate on logarithmic expressions
- apply arithmetic sequences and their sums to increasing and decreasing linear patterns
- apply geometric sequences and their sums to geometric growth and decay (e.g., as in compound interest)
- apply and interpret interest compounded continuously
- describe continuous sets of numbers using interval notation and inequalities
- operate on irrational numbers, expressing them in exact forms as appropriate – for example, by using simplifying (e.g.,  $\sqrt{20} = 2\sqrt{5}$ ) or adding (e.g.,  $\frac{\pi}{3} + \frac{5\pi}{6} = \frac{7\pi}{6}$ )
- find square roots of negative numbers
- calculate the number of possible arrangements in a set, both when the order of the arrangements does matter (permutations) and when the order does not matter (combinations).

### Taurangi | Algebra

#### Equations, expressions, and formulae

#### *I know:*

- Formulae can appear in different, but equivalent, forms
- The distributive property is limited to expanding multiplication over addition; it does not hold for expressions with powers and square roots
- In a polynomial,  $x = a$  is only a root if  $(x-a)$  is a factor of the polynomial, and vice versa
- The zero-product property says that if two expressions multiply to be zero, then one or the other is zero; this property is useful for solving equations
- Logarithm laws provide the means to solve exponential equations.

## Know

### Taurangi | Algebra (continued)

#### *I know how to:*

- rearrange and use formulae
- expand and factorise polynomials
- determine the nature of the roots of a quadratic equation
- operate on algebraic fractions
- solve equations
- expand expressions, using the distributive property
- algebraically solve systems of two linear equations in two-dimensional space, and of three linear equations in three-dimensional space, and geometrically interpret the solutions.

### Functions

#### *I know:*

- Representing functions in different forms allows for deeper understanding and reveals alternative approaches
- Equivalent representations of quadratic functions can reveal different characteristics of the same relationship
- Functions can be combined to create new functions by operating on them, including through function composition
- Composition of a function and its inverse function results in the identity function
- Exponential functions are used to model growth and decay
- Logarithmic and exponential functions with the same base are inverse functions
- The derivative can be represented as a rate of change
- The chain rule helps differentiate composite functions
- The indefinite integral is the antiderivative; the derivative of the indefinite integral is the original function
- Initial-value problems include both a value of, and the derivative of, the original function
- Calculus can be used to work with continuous variables in statistics and probability.

#### *I know how to:*

- find the inverse function of a function, restricting the domain if required
- find limits of functions using equations and graphs
- use first principles to find the derivatives of simple functions
- differentiate polynomial, rational-power, sine, cosine, natural-logarithm, and exponential (with base  $e$ ) functions, including using the product, quotient, or chain rule
- integrate polynomial, rational-power, sine, cosine, reciprocal and exponential (with base  $e$ ) functions
- solve initial-value problems, including kinematic problems and exponential growth or decay.

### Inenga | Measurement

#### *I know:*

- Measurements involve variation and uncertainty
- Quantities calculated from two or more measures are derived measures (e.g., square metres and metres per second)
- A right-angled triangle has a fixed relationship between its side lengths and its angles, given by the trigonometric ratios
- In any triangle, the relationships between side lengths and angles are given by the trigonometric formulae involving sine and cosine
- The standard unit of angular measurement is a radian, which is geometrically related to the length of the radius of a unit circle
- A point on a unit circle at an angle of  $\theta$  with the positive x-axis (in standard position) is represented by the coordinates  $(\cos \theta, \sin \theta)$ ; through this relationship, I can visually and algebraically represent the periodic and symmetric nature of the sine and cosine functions
- Trigonometric ratios and functions can model contextual situations
- The area under a curve between two points can be calculated by finding the definite integral between the two points
- Derivatives and integrals describe kinematic problems in two- and three-dimensional space by examining displacement, velocity, and acceleration.

#### *I know how to:*

- estimate, accurately measure, and calculate quantities using appropriate tools and formulae
- estimate the area or volume of regular, irregular, and compound shapes
- recognise the complexity of the attribute being measured and use correct units with appropriate precision
- determine how length, area, or volume change when resizing similar shapes
- preserve units in calculations, including those involving derived measures and metric conversions
- find the side lengths, angle sizes, and area of any triangle when given appropriate information
- convert between radians and degrees in appropriate situations
- connect the sine and cosine functions to the unit circle
- use simple trigonometric identities such as  $\sin^2(\theta) + \cos^2(\theta) = 1$  to simplify calculations
- apply calculus to trigonometric functions
- approximate the area under a curve using rectangles or trapeziums, and improve the approximation.

## Know

### Mokowā | Space

#### *I know:*

- The rate of change for a linear function is the gradient
- Parameters are the non-variable letters used in some algebraic formulae (e.g.,  $m$  and  $b$  in  $y = mx + b$ ); when a parameter is changed in a formula, an aspect of the corresponding graph will change in response
- Function transformations change both the algebraic formula and the graph
- The solution of a system of equations can be interpreted geometrically as intersections of the corresponding graphs
- A tangent line is a line that just touches a curve at a point and matches the curve's gradient at that point
- At a single point, the derivative of a function equals the gradient of the tangent line
- The derivative may be represented as the gradient function
- Tangent lines are local approximations of a function; near a specific point, the tangent line and the function have approximately the same graph
- The gradient of the tangent line at a maxima and minima of a function is 0 or undefined
- Examining rates of change close to turning points helps to identify intervals where the function increases or decreases, and helps to identify the concavity of the function.

#### *I know how to:*

- approximate irregular shapes with triangles, rectangles, and circles
- find the equation for the line between two points, as well as the distance between them and the midpoint
- find equations for parallel and perpendicular lines, including horizontal and vertical lines
- graph linear, quadratic, polynomial, exponential, logarithmic, trigonometric, and absolute-value functions and interpret  $x$ -intercepts, the  $y$ -intercept, and key features in relation to the equation or the situation
- transform graphs of functions
- graph a function and its inverse function by restricting the domain if required
- graph the solutions of an inequality in two variables
- find and graph tangents of a function
- find a function that explains paired data (empirical models) and use this to make predictions
- describe curves and circles using parametric equations.

### Tauanga | Statistics

#### *I know:*

- Contexts are integral to all statistical investigations; this includes considering diverse perspectives
- Data can be extracted from a wide range of sources, including text, images, sounds, and experiences
- Decisions made to collect and generate data (including what questions are asked and how measures are defined) will affect the quality, diversity, and quantity of data, as well as the conclusions
- Data-based information can be used to influence decisions, behaviours, policies, and opinions; the consequences for affected communities must be considered
- Predictive modelling involves labelling variables to determine what may happen in a new occurrence (forecasting)
- Data can be used to develop algorithmic models; the predictions and recommendations generated by these models may be harmful
- In statistical inference, the data is sometimes observed in a way that we have no control over (sample-to-population inferences) and is sometimes from experiments that we design (experiment-to-causation inferences); the two situations are handled differently
- Simulation-based methods such as randomisation tests can assess the strength of evidence
- Intervals can communicate the uncertainty associated with estimates or predictions
- Proportions and counts of categorical variables need to be considered along with the groups of interest
- Findings are tentative and subject to revision while more evidence and insights accumulate.

#### *I know how to:*

- create, manipulate, and restructure data from a variety of sources using non-automated and computational methods
- use ethical and responsible data practices when designing and evaluating studies involving questionnaires and experiments
- evaluate a wide range of data-based information from sources including the media
- recognise when statistical claims are made and critique the statistical information behind the claim
- identify likely and unlikely outcomes within distributions when making predictions, and explore the impact of conditioning on other variables
- evaluate poll-based reports by interpreting the informal margin of error for proportions
- explore data by merging data sources, developing new variables, and creating a wide range of data visualisations
- make informal sample-to-population inferences by visually comparing medians and proportions
- make formal sample-to-population inferences by using a resampling method to construct confidence intervals for means, medians, or proportions
- make experiment-to-causation inferences based on a single proportion-based experiment and assess the strength of evidence with a simulation-based randomisation test
- make experiment-to-causation inferences involving a comparison of two independent groups by considering the study design and by using a simulation-based randomisation test to assess the strength of evidence
- select, use, and evaluate appropriate statistical models to make predictions (including linear-regression, additive-time-series, and classification models).

## Know

### Tūponotanga | Probability

*I know:*

- Probability models can be derived from gathered (empirical) data or from theories
- Expected values can be calculated for discrete random variables
- Uniform, binomial, Poisson, and normal probability distributions have key features
- Generating data from a probability model through simulation can demonstrate what outcomes are likely or unlikely under certain conditions, as well as the variability of the proportions for the outcomes.

*I know how to:*

- use representations, such as tree diagrams and two-way tables, to determine probabilities of events
- determine probabilities of independent, combined, and conditional events using probability-distribution models
- calculate and interpret expected values for simple models, and, when necessary, apply expected-value rules
- estimate probability using probability-distribution models selected by recognising the underlying structure and conditions of the data-generating process
- explain effects caused by changing the parameters of probability distributions
- conduct large-scale, technology-supported simulations to model probabilities
- calculate, interpret, and evaluate risk situations (e.g., relative risk, increased or decreased risk, and recurrence intervals)
- use informal tests to determine how well a probability-distribution model fits a set of observations (goodness-of-fit).

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## Do

### Te whakatauirā me te tūhura | Modelling and investigating

*In my learning in mathematics and statistics, I can:*

- ask a question that can be investigated using mathematics or statistics
- understand and explain in my own words what a question means mathematically or statistically
- find entry points for addressing a question using mathematics or statistics, identifying relevant prior knowledge, givens, assumptions, constraints, and relationships
- formulate a mathematical or statistical solution pathway and carry it out
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### Te whakaahua | Representing

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- use mathematical and statistical symbols to express generalisations.

### Te whakaaro arorau | Reasoning

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- build statements and explanations deductively from mathematical and statistical knowledge, definitions, and rules
- make conjectures, using reasoning and counter examples to decide if they are true or not
- critically reflect on statements based on mathematical and statistical ideas, distinguishing correct logic from flawed logic and asking questions to clarify and understand
- use evidence, reasoning, and proofs to explain why I agree or disagree with statements or why something will always be true
- develop collective understandings with others by sharing ideas and comparing, contrasting, and building on them.

### Te reo matatini o te pāngarau | Communicating

*In my learning in mathematics and statistics, I can:*

- use mathematical and statistical language to describe my thinking
- record, share, and explain my thinking
- communicate using representations that are appropriate for the idea I am talking about
- choose an effective format for communicating an idea or argument
- answer questions, engage in discussion, and respond to others' ideas in constructive ways
- present concise and coherent explanations and arguments for an idea, solution, or process.

At years 11–13, learning in mathematics and statistics becomes increasingly specialised. In addition, all ākonga have opportunities to further develop and use what they have learnt in years 1–10 in a range of approved NCEA subjects.



**Te Poutāhū**  
Curriculum Centre

# Mathematics and statistics

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THE NEW ZEALAND CURRICULUM REFRESH

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**Te Tāhuhu o  
te Mātauranga**  
Ministry of Education

**Te Kāwanatanga  
o Aotearoa**  
New Zealand Government

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# Purpose statement for mathematics and statistics

## **Ānō me he whare pūngāwerewere.** *Behold, it is like the web of a spider.*

This whakataukī celebrates intricacy, complexity, interconnectedness, and strength. The learning area of mathematics and statistics weaves together the effort and creativity of many cultures that over time have used mathematical and statistical ideas to understand their world.

Mathematics and statistics enables ākonga to appreciate and draw on the power of abstraction and symbolic representation to investigate, interpret, and explain patterns and relationships in quantity, space, time, data, and uncertainty. Like mathematics and statistics, mātauranga Māori is a body of knowledge with a history and a future. When we afford mana ōrite to mātauranga mathematics and statistics and mātauranga Māori while retaining their distinctiveness, ākonga can draw from both in ways that are beneficial to both spheres of knowledge.

The learning area has been designed to support the vision of Mātairiti and reflects the four kinds of value each learning area contributes: personal value, participatory value, pathways value, and planetary value. Collectively, these express the richness and value of mathematics and statistics learning for ākonga. Ākonga discover inherent personal enjoyment and satisfaction in persistence, solving problems, identifying patterns, and seeing the beauty in mathematics and statistics. They come to appreciate the everyday use of mathematical and statistical tools in, for example, personal finance, music and dance, estimation, and measurement. They recognise how their culture is included and valued in the learning area.

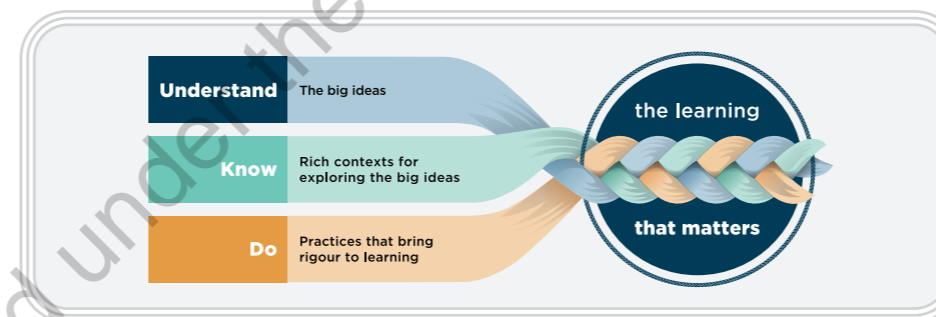
Ākonga participate as they take part in discussions with their peers about their mathematical and statistical thinking and the thinking of others. They discuss and take action on important social matters such as the ethical gathering, interpretation, and communication of data, and challenging misinformation and disinformation. They also engage with diverse cultural perspectives, including te ao Māori and Pacific world-views, on being numerate in Aotearoa New Zealand.

Through the learning area, ākonga discover pathways into STEAM-integrated careers and opportunities across a wide range of industry sectors. They build a base of knowledge that supports Māori and Pacific aspirations and priorities and helps them function in our increasingly technology- and information-rich world of work.

Ākonga also come to understand the value of mathematical and statistical modelling as a lens for resolving collective global challenges – for example, in adapting to and mitigating climate change and in helping to build an equitable, sustainable future for all.

Learning in mathematics and statistics builds both literacy and numeracy. Mathematics and statistics contribute to ākonga literacy by developing their skills in oral and written communication, meaning-making, and the use of specific vocabulary and symbols. Statistics and probability, in particular, support the understanding of tables, graphs, and diagrams as well as critical thinking about the quality of data and stories told about it.

As this whakataukī tells us, connections between different concepts, knowledge, and practices are central to mathematics and statistics. Kaiako weave together the elements of Understand, Know, and Do to ensure ākonga learn mathematics and statistics as a connected body of knowledge.



## **Important considerations for teaching mathematics and statistics**

The most effective teaching of mathematics and statistics follows a strengths-based approach that creates opportunities for all students to learn and progress. Such an approach recognises that all ākonga are culturally located and includes parity for mātauranga Māori.

It is important for teaching to be ambitious within and potentially beyond each phase of learning. When designing a mathematics and statistics programme, kaiako need to plan for providing ākonga with multiple opportunities to progress.

Learning happens best when mathematics and statistics are taught daily, using purposeful tasks related to both mathematical contexts and wider contexts relevant to the communities, cultures, interests, and aspirations of ākonga.

When planning how to support progress, kaiako can ask: What opportunities do ākonga have to:

- learn new mathematics and statistics concepts and practices?
- use mathematics and statistics to investigate relevant tasks?
- communicate and critique mathematical findings and understandings?
- understand the interrelated nature of skills and concepts in mathematics and statistics?
- practise the mathematics and statistics that they have learned?

When planning tasks, kaiako can ask:

- What are the cultural contexts that will resonate with my ākonga?
- How can I support ākonga to engage with a context's whakapapa, tikanga, and significance while honouring and maintaining the integrity of both the mathematics and the context?

As they prepare, kaiako can work through the tasks themselves and ask:

- How can I help ākonga find the joy in this learning?
- How can I help build the resilience of ākonga?
- How can I value and reward persistence?
- How can I help ākonga see the broad relevance of this work to their lives, including purposeful contexts, mathematical skills, social skills, knowledge, cognitive development, and cultural competence?

## Understand

### Big ideas

*Kotahi te kōhao o te ngira e kuhuna ai te miro mā, te miro pango, te miro whero. (Pōtatau Te Wherowhero)*

*There is but a single eye of the needle through which white, black, and red threads must pass together, yet each thread keeps its own colour and integrity while adding its strength and beauty to the others.*

**The interface between mātauranga Māori and mātauranga mathematics and statistics offers opportunities for insights that uphold the integrity of each knowledge system.**

Mātauranga Māori and mathematics and statistics are different systems for viewing, understanding, and organising the world and for guiding how we operate within it. Mātauranga Māori makes meaningful and distinctive contributions to mathematical inquiry and knowledge in Aotearoa New Zealand, just as mathematical and statistical insights contribute to mātauranga Māori. When considering concepts, processes, and artifacts from te ao Māori, we maintain their integrity by exploring the mātauranga Māori associated with them before formulating mathematical and statistical hypotheses about them.

*Nō ngā tūpuna, tuku iho, tuku iho.*

*The human ideas that have been passed down from generation to generation can help us develop our thinking today.*

**Mathematics and statistics have a continuous, evolving human history.**

Mathematics and statistics have been constructed over thousands of years across the globe as we have grappled with notions of quantity, numerical representation, measurement, dimension, and pattern. They continue to be constructed from ideas drawn from many cultures. In Aotearoa New Zealand, our location in Te Moana-nui-a-Kiwa – with its multiple cultures, artifacts, and knowledges – contributes to mathematics and statistics.

*Whiria te kaha tūātinitini, whiria te kaha tūāmanomano.*

*Together we can use our strengths to achieve more. All learning contributes specific threads that we can use to weave a rope strong enough to get us where we want to go, do what we want to do, and be what we want to be.*

**The world is full of patterns and structures that we use mathematics and statistics to understand.**

Mathematics and statistics enable us to notice, explore, and describe similarities, regularities and irregularities, and trends in the natural, mathematical, technological, and social worlds. They provide tools and ways of working that can reveal patterns and structures useful for decision making, understanding and predicting phenomena, and creating new insights.

## Know

### Contexts

#### Mātauranga tau | Number

Cultures use *Number* to represent, describe, and compare quantities. We operate on these quantities, and use them to estimate, calculate, reason, and justify.

#### Taurangi | Algebra

*Algebra* focuses on making and using generalisations to reason mathematically, and on identifying patterns and underlying mathematical relationships. These generalisations, patterns, and relationships can be represented and communicated using diagrams, graphs, and symbols (including variables).

#### Ine | Measurement

*Measurement* provides the tools and concepts for quantifying phenomena in the world by estimating, measuring accurately, and using appropriate units, including those from Māori, Pacific, and metric systems of measurement. Many cultures use both standard and non-standard units to measure tangible and intangible quantities.

#### Mokowā | Space

*Space* focuses on visualising, representing, and reasoning about the shape, position, orientation, and transformation of objects. It takes account of tools and techniques from the natural world used by many cultures.

#### Tauanga | Statistics

*Statistics* focuses on tools, concepts, and systematic process for interpreting situations, using data and its context to understand uncertainty and make predictions. Every piece of data is a taonga to be kept safe and treated ethically and respectfully.

#### Tūponotanga | Probability

*Probability* focuses on tools and concepts for quantifying chance, dealing with expectation, and using evidence to identify how likely events are to occur. Probabilistic thinking is evident throughout tikanga and mātauranga Māori.

## Do

### Practices

#### Te tūhura pūāhua | Investigating situations

When we investigate situations using mathematics and statistics, we describe and explore them to build our understanding of them. Māori, Pacific, and other world-views may offer us different ways of understanding them. When investigating, we need to decide which approaches, concepts, and tools to use and how to use them. We often begin with a question or focus of interest and proceed in systematic but flexible ways, using mathematical and statistical concepts to make sense of solutions, stories about data, and conclusions in context. We conclude by evaluating the investigation, which involves reflecting on the solutions and outcomes and our approaches and choices to determine whether they were reasonable, made sense in context, and could be improved on in future investigations.

#### Te whakaata pūāhua | Representing situations

When we represent situations mathematically and statistically, we use words or symbols and mental, oral, physical, virtual, graphical, or diagrammatic ways to show concepts and findings. We can use representations to compare, explore, simplify, illustrate, prove, and justify as well as to look for patterns, variations, and trends. They can draw from mātauranga Māori, from Pacific cultures, and from diverse places and periods of history. Representing a situation in multiple ways enables a deeper and more flexible understanding of the situation. It also allows us to communicate with different audiences.

## Understand

### Big ideas

*Kei hopu tōu ringa ki te aka tāepa, engari kia mau ki te aka matua.*

*Do not catch hold of the loose vine but lay hold of the main vine. You can use the strength of the aka matua (main vine) for the sure footing you will need to reach for new ideas and to climb to new heights.*

#### **The world is characterised by change and variation that we use mathematics and statistics to understand.**

The world embodies a multitude of temporary and permanent relationships in which change and variation occur. Some relationships are linear; others are exponential. Mathematics and statistics enable us to systematically describe and analyse different types of change and variation, and to generate insights and make predictions about them.

*Ko te pae tawhiti whāia kia tata, ko te pae tata whakamaua kia tina.*

*Seek to bring distant horizons closer and cherish those that you have attained. There will always be pae tawhiti, the 'not yet', but we can move ever closer to that for which we strive.*

#### **Mathematical and statistical logic and reasoning enable us to identify and explain relationships and to justify conclusions.**

Reasoning from observation (induction) and reasoning from theory (deduction) allow us to explore situations using mathematics and statistics. Mathematical and statistical logic and reasoning differentiate what is probable from what is possible and allow us to draw reliable conclusions about what is reasonable or not.

## Know

### Contexts

## Do

### Practices

#### **Te tūhono pūāhua | Connecting situations**

When we connect situations using mathematics and statistics, we recognise and make links by noticing similarities and differences. Connecting helps us to understand the relationships between concepts, facts, and procedures in mathematics and statistics. This is important because number, algebra, space, measurement, statistics, and probability form a web of interconnected ideas and approaches that can be easier to remember and understand if the connections between them are clear. Connecting also involves linking mathematics and statistics to other learning areas, to mātauranga Māori, and to a range of contexts, including cultural, linguistic, and historical contexts.

#### **Te whakatauhānui i ngā kitenga | Generalising findings**

When we generalise mathematical and statistical findings, we move from specific examples to general principles. We use the patterns, regularities, and structures that we observe to make conjectures that might apply more generally. Further investigation can test and refine these conjectures and determine if they apply in all cases. In statistics, we generalise by using trends and variation in data to make inferences and predictions and to articulate and evaluate claims about similar situations.

#### **Te whakamārama me te parahau i ngā kitenga | Explaining and justifying findings**

When we explain and justify, we use mathematical and statistical ways of communicating and reasoning to share our ideas and to respond to the ideas, reasoning, and inferences of others. Explaining is how we communicate our inferences and predictions, build arguments, and unpack stories from data. Justifying involves describing why decisions and findings are reasonable, taking into account limitations arising from assumptions and choices and the evidence on which findings are based.

# Progress outcome by the end of **year 3** (Foundation)

## Understand

I am building knowledge about number, algebra, measurement, space, statistics, and probability, and drawing on the practices of mathematics and statistics.

*Through this, I am deepening my understanding that:*

**Kotahi te kōhao o te ngira e kuhuna ai te miro mā, te miro pango, te miro where.**

The interface between mātauranga Māori and mātauranga mathematics and statistics offers opportunities for insights that uphold the integrity of each knowledge system.

**Nō ngā tūpuna, tuku iho, tuku iho.**

Mathematics and statistics have a continuous, evolving human history.

**Whiria te kaha tūātinini, whiria te kaha tūāmanomano.**

The world is full of patterns and structures that we use mathematics and statistics to understand.

## Know

*I know that:*

### Mātauranga tau | Number

In base 10, there are ten digit symbols and their value is defined by their position within a number. Digits in any column are worth ten times as much as those in the column to the right.

Te reo Māori and other Pacific languages explicitly describe the logic of the base 10 numbering system.

Numbers can be composed and decomposed in different ways by using patterns.

Multiplication and division involve recognising and working with groups, the number of groups, and the total.

Fractions show parts of a whole in a region, a measurement, or a set of objects. The same amount (e.g., a half or a quarter) can be shown by equivalent fractions.

### Taurangi | Algebra

The commutative property applies to addition (e.g.,  $2 + 5 = 5 + 2$ ) and multiplication (e.g.,  $5 \times 2 = 2 \times 5$ ).

The additive identity is 0 (e.g.,  $4 + 0 = 4$  and  $5 - 0 = 5$ ), and the multiplicative identity is 1 (e.g.,  $5 \times 1 = 5$  and  $4 \div 1 = 4$ ).

The equal sign is relational; it shows that the two sides of an equation are the same.

Patterns are made of numeric or spatial elements in a sequence governed by a rule.

Identifying the rule of a pattern involves working out the unit of repeat.

An algorithm is a sequence of rules that can be followed.

### Ine | Measurement

Measuring starts at the beginning of the object being measured. The size of the measurement unit must remain the same. Measurement units are repeated with no gaps or overlaps. The measurement is the total number of units used.

Length around the outside of a two-dimensional shape gives perimeter, covering a surface gives area, filling a three-dimensional shape gives capacity or volume.

*I know how to:*

- recognise, read, write, and order whole numbers up to 10,000
- group, partition, and recombine whole numbers up to 1,000
- add and subtract two- and three-digit numbers
- multiply two single-digit numbers or multiply a single-digit and a two-digit number
- divide whole numbers with a single-digit divisor and no remainders
- recognise, read, write, represent, and order halves, thirds, quarters, fifths, sixths, and eighths
- find a unit fraction of a whole (e.g., a region, measurement, or set of objects), and add unit fractions with like denominators.

*I know how to:*

- recall addition facts to 20 and their corresponding subtraction facts
- recall multiplication and corresponding division facts for twos, fives, and tens
- solve true and false number sentences and open number sentences
- use the additive and multiplicative identities and commutative property
- find another element of a pattern, given part of it
- describe a rule that explains how a pattern works
- follow, and create patterns from, rules or simple algorithms.

*I know how to:*

- estimate and then reliably measure length, area, volume, capacity, and mass, using standard metric units
- use rulers, scales, square grids, and cubes to measure
- tell the time to hours, half hours, and quarter past or quarter to the hour, using language and a range of cultural tools, including analogue and digital clocks
- find out how far something has been turned, using half and quarter turns as benchmarks.

## Do

### Te tūhura pūāhua | Investigating situations

*I can:*

- work with others to pose a question for investigation
- find entry points for addressing a question, identifying relationships and relevant prior experience and knowledge
- work with others to plan an investigation pathway and follow it
- describe progress on the investigation pathway
- work with others to make sense of outcomes or conclusions in light of a given situation and context.

### Te whakaata pūāhua | Representing situations

*I can:*

- use representations to explore, find, and illustrate patterns
- use representations to learn new ideas and explain ideas to others
- select or create appropriate mental, oral, physical, or virtual representations
- use visualisation to mentally represent and manipulate groups and shapes.

### Te tūhono pūāhua | Connecting situations

*I can:*

- suggest connections between ideas and approaches
- suggest connections between different representations
- connect new ideas to things I already know
- make connections with ideas in other learning areas and in familiar local contexts.

## Understand

*Kei hopu tōu ringa ki te aka tāepa, engari kia mau ki te aka matua.*

The world is characterised by change and variation that we use mathematics and statistics to understand.

*Ko te pae tawhiti whāia kia tata, ko te pae tata whakamaua kia tina.*

Mathematical and statistical logic and reasoning enable us to identify and explain relationships and to justify conclusions.

## Know

### Mokowā | Space

Patterns and regularities in shapes can be used to compare, classify, and predict.

Two-dimensional shapes can be composed or decomposed to form new shapes and can have symmetry.

Shapes and objects can flip (reflection), turn (rotation), and slide (translation) and be used to create patterns.

Objects can be rotated in space and may appear different from other perspectives.

Maps are two-dimensional representations of places in the world. They use symbols to show locations and landmarks.

### Tauanga | Statistics

Data is information about the world and comes in many forms.

People and the environment are not data, but data can tell us things about people, their lives, and their environment.

Summary investigative questions and the statistical enquiry cycle (PPDAC – Problem, Plan, Data, Analysis, Conclusion) are used to investigate a group.

Data visualisations are representations of all available values of one or more variables that reveal relationships or tell a story.

### Tūponotanga | Probability

A chance-based situation has a set of possible outcomes that can be arranged into events. The probability of an event is the chance of it occurring.

The statistical enquiry cycle (PPDAC) can be used for chance-based investigations for predicting outcomes of everyday situations and activities and whether they are certain, likely, possible, unlikely, or impossible.

#### *I know how to:*

- visualise, identify, compare, and classify two- and three-dimensional shapes
- compose and decompose two-dimensional shapes using the properties of shapes, such as lines of symmetry
- predict and justify what will happen to two-dimensional shapes if you rotate, reflect, or translate them
- use pepeha to describe location by referring to environmental features
- draw simple maps of familiar places to provide directions
- interpret simple maps to locate objects and pathways.

#### *I know how to:*

- explore summary investigative questions about everyday situations, using categorical data and discrete numerical (whole-number) data
- use survey and data-collection questions
- collect, record, and sort data or use secondary data sources
- create and make statements about data visualisations
- answer an investigative question by choosing statements from findings
- identify relevant features in others' data visualisations.

#### *I know how to:*

- explore chance-based investigative questions about games and everyday situations in my life
- collect and record data to answer chance-based investigative questions
- create and describe data visualisations for the frequencies of outcomes in chance-based situations
- explain and question statements about chance-based situations, with reference to data.

## Do

### Te whakatauwhānui i ngā kitenga | Generalising findings

#### *I can:*

- recognise and explore patterns and make conjectures and draw conclusions about them
- identify relationships including similarities, differences, and new connections
- look for patterns and regularities that might be applied in another situation or always be true
- make and test conjectures, using reasoning and counterexamples to decide if they are true or not
- use words and pictures to express generalisations.

### Te whakamārama me te parahau i ngā kitenga | Explaining and justifying findings

#### *I can:*

- make statements and give explanations about what I notice and wonder
- make statements and give explanations deductively based on prior knowledge
- ask questions to clarify and understand others' thinking
- use evidence and reasoning to explain why I agree or disagree with statements
- develop collective understandings by sharing and building on ideas with others
- present basic explanations and arguments for an idea, solution, or process.

# Mathematics and statistics **progress steps**

Progress steps alert kaiako to aspects of learning that are essential and time-sensitive as ākongā work towards the progress outcome for this phase. They support kaiako to notice, recognise, and respond to ākongā learning in a timely fashion through the provision of multiple opportunities for learning and practice. These opportunities will be meaningful for ākongā if they connect with their home languages and value the ways they make meaning, reason, and communicate (including through the use of augmentative and alternative communication or assistive technology).

Subitising
Number structure:
Operations: Addition and subtraction
Operations: Multiplication and division
Rational numbers
Equality
Patterns
Measurement
Classification
Spatial reasoning
Variability

## Mathematics and statistics progress step **during the first six months**

*Building on the foundation from early learning, ākongā can:*

- recognise instantly the total number of objects in a group of up to six

- join and separate groups of up to a total of 10 objects, and find the result by grouping and counting

- copy, continue, create, and describe a repeating pattern with two elements

- compare directly two objects by an attribute (e.g., length, weight, capacity)

- sort shapes and objects by one feature (e.g., colour, shape), identifying the feature chosen

- compose by trial and error an outlined target shape using smaller shapes, and decompose a shape into smaller shapes
- follow instructions to move to a familiar location or locate an object.

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## Mathematics and statistics progress step **during the first year**

*Building on their progress from previous months, ākonga can:*

- recognise instantly the total number of objects in two patterns, each of up to five objects
- partition and recombine sets of up to 10 in different ways
- recognise and represent in different ways, including in te reo Māori, the tens and one structure of teen numbers (11-19)
- join and separate groups of up to a total of 20 objects, and find the difference between groups by grouping and counting
- multiply and divide by making equal groups and using grouping or counting
- recognise, and represent in different ways, halves and quarters of sets and regions
- copy, continue, create, and describe a repeating pattern with three elements, and identify missing elements in a pattern
- compare the length, weight, volume, and capacity of objects indirectly (i.e., by comparing each of them with another object)
- sort and re-sort shapes and objects by features, identifying the feature chosen
- visualise and anticipate which smaller shapes might compose a target shape, and then check by making the shape
- follow and give instructions to move to a familiar location or locate an object.

## Mathematics and statistics progress step **during the second year**

*Building on their progress from the previous year, ākonga can:*

- partition a pattern of up to 10 objects, instantly recognise the number of objects in each part, and confirm the total number in the pattern using the parts
- group, partition, and recombine whole numbers up to 100
- add and subtract numbers up to 100 by grouping and using number patterns
- multiply and divide by grouping and using number patterns
- recognise the relationships between related fractions (e.g., one half is the same as two quarters)
- find a half, quarter, or a third of a set by recognising groups and patterns rather than sharing by ones
- show that in an equation, both sides of the equal sign represent the same quantity
- use both the unit of repeat and the ordinal position (e.g. first, second, and third) of a repeating pattern to predict further elements
- use a standard informal unit repeatedly to measure the length, weight, volume, or capacity of an object
- visualise and anticipate which smaller shapes might compose or decompose a target shape, and then check by making the shape
- follow and give movement instructions that involve familiar reference points, direction, distances (number of steps), and half and quarter turns
- identify possible outcomes and notice variations in outcomes for familiar activities and situations involving chance.

As ākonga move through school and progress through each phase of learning, the emphasis of learning in numeracy shifts from developing foundational skills to the specific demands of the different learning areas and to contexts outside the classroom. For this reason, from year 4 onwards, learning in numeracy is described in the progress outcomes for each learning area.

# Progress outcome by the end of year 6

## Understand

I am building knowledge about number, algebra, measurement, space, statistics, and probability, and drawing on the practices of mathematics and statistics.

*Through this, I am deepening my understanding that:*

**Kotahi te kōhao o te ngira e kuhuna ai te miro mā, te miro pango, te miro wheno.**

The interface between mātauranga Māori and mātauranga mathematics and statistics offers opportunities for insights that uphold the integrity of each knowledge system.

**Nō ngā tūpuna, tuku iho, tuku iho.**

Mathematics and statistics have a continuous, evolving human history.

**Whiria te kaha tūātinini, whiria te kaha tūāmanomano.**

The world is full of patterns and structures that we use mathematics and statistics to understand.

## Know

*I know that:*

### Mātauranga tau | Number

In our number system each place value is a power of 10, and this continues infinitely.

Multiplication and division problems can involve equal groups, rates, comparisons, combinations, part-whole relationships, areas, and volumes.

Fractions are numbers and can describe a measure, a proportional relationship, or an action on another number.

Fractions express ways of sharing that may be different from those in tikanga and mātauranga Māori.

Decimals are a set of fractions that have powers of 10 as their denominators (e.g.,  $\frac{7}{10}$  or  $\frac{7}{100}$ ) and that can be written as numbers using a decimal point (e.g., 0.7 or 0.07).

A percentage is the number of 100ths of a whole (e.g.,  $\frac{7}{100}$  is 7%).

*I know how to:*

- recognise, read, write, order, partition, recombine, and represent whole numbers up to 1,000,000
- add and subtract whole numbers and decimal numbers to two places
- multiply two- and three-digit whole numbers
- divide whole numbers by one- or two-digit divisors
- find factors of numbers up to 100
- recognise, read, write, represent, compare, order, and convert between fractions, decimals (to three places), and percentages
- find equivalent fractions for halves, thirds, quarters, sixths, and eighths, and represent fractions in their simplest form
- find a fraction or a percentage of a whole number.

### Taurangi | Algebra

The associative property applies to addition and multiplication (e.g.,  $3 \times (2 \times 7) = (3 \times 2) \times 7$ ).

The distributive property applies to multiplication over addition and subtraction (e.g.,  $3 \times (10 + 7) = (3 \times 10) + (3 \times 7)$ ).

The equal (=) and inequality (<, >) signs show relationships.

In a pattern, the relationship between the ordinal position (e.g., first, second, third) and the corresponding element is useful for finding the pattern rule.

Tables and XY-graphs provide a way of organising the positions and elements of a pattern to reveal relationships or rules.

An algorithm is a set of instructions for solving a problem.

*I know how to:*

- recall multiplication facts to  $10 \times 10$  and corresponding division facts
- use the distributive, commutative, and associative properties
- solve open number sentences and true and false number sentences involving equality or inequality
- use tables, XY-graphs, and diagrams to find relationships between elements of growing patterns
- develop a rule in words about a linear pattern
- use a rule to make predictions
- create and use algorithms for making decisions that involve clear choices.

### Ine | Measurement

Mātauranga Māori draws on knowledge of te taiao and has meaningful ways of measuring things (e.g., Maramataka).

The metric measurement system is based on powers of ten.

Measurements can contain units and parts of units, and need the unit recorded with the amount (e.g., 1.3 km).

Angles are a measure of turn and can be measured in degrees.

*I know how to:*

- read measurement tools and interpret scales accurately
- convert between units of time and solve duration-of-time problems
- find the perimeter and area of rectangles and the volume of cuboids
- describe an angle using the benchmarks 90 degrees, 180 degrees, and 360 degrees.

## Do

### Te tūhura pūāhua | Investigating situations

*I can:*

- pose a question for investigation
- find entry points for addressing a question, identifying relevant prior knowledge, givens, and relationships
- plan an investigation pathway and follow it step by step
- monitor and evaluate progress, adjusting the investigation pathway if necessary
- make sense of outcomes or conclusions in light of a given situation and context.

### Te whakaata pūāhua | Representing situations

*I can:*

- use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns and variations
- use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments
- select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations
- use visualisation to mentally represent and manipulate objects and ideas.

### Te tūhono pūāhua | Connecting situations

*I can:*

- suggest connections between ideas and approaches
- suggest connections between different representations
- connect new ideas to things I already know
- make connections with ideas in other learning areas and in familiar cultural, linguistic, and historical contexts.

## Understand

*Kei hopu tōu ringa ki te aka tāepa, engari kia mau ki te aka matua.*

The world is characterised by change and variation that we use mathematics and statistics to understand.

*Ko te pae tawhiti whāia kia tata, ko te pae tata whakamaua kia tina.*

Mathematical and statistical logic and reasoning enable us to identify and explain relationships and to justify conclusions.

## Know

### Mokowā | Space

Two- and three-dimensional shapes have consistent properties that can be used to define, compare, classify, predict, and identify relationships between them.

Shapes can be rotated, reflected, translated, and resized.

Viewing objects from different angles gives different perspectives.

Mātauranga Māori often identifies location in the natural world as a form of mapping (e.g., for travelling).

Position can be described using known environmental features and signs from te taiao.

Maps use grid references or coordinates to specify places, scales to show distances, and connections to show pathways.

#### *I know how to:*

- classify two-dimensional shapes and prisms using their spatial properties to justify my classifications
- perform and describe rotations, reflections, translations, enlargements, and reductions on two-dimensional shapes and simple geometric patterns
- visualise and represent three-dimensional shapes from different viewpoints
- visualise and draw nets for rectangular prisms
- use grid references, simple scales, the language of direction (compass points), distance (in m, km), and turn (in degrees) to locate and describe positions and pathways.

### Tauanga | Statistics

Data about people and te taiao can have negative impacts so must be collected, used, and stored carefully.

The statistical enquiry cycle (PPDAC – Problem, Plan, Data, Analysis, Conclusion) can be used in summary, comparison, and time-series investigations (e.g., about school related matters).

Different data visualisations for the same data can lead to different insights.

#### *I know how to:*

- pose investigative questions about school contexts for summary, comparison, and time series situations, and make predictions or assertions about what I expect to find
- plan how to collect primary data or to use provided secondary data
- use and describe a variety of data visualisations, identifying features, patterns, and trends in context and making connections to the group of interest
- interrogate others' survey or data-collection questions, and identify and explain features and errors in others' data visualisations and statements about data.

### Tūponotanga | Probability

The statistical enquiry cycle (PPDAC) can be used for chance-based investigations.

Probabilities and the language of probability are associated with values between 0 or 0% (impossible) and 1 or 100% (certain).

A probability experiment involves repeated trials. Results may vary in trials. The experimental probability of an event is the number of times the event occurs divided by the total number of trials.

#### *I know how to:*

- pose investigative questions for a chance-based situation with equally likely outcomes, listing all possible outcomes for the situation
- plan, conduct, and record data for a probability experiment
- create and describe data visualisations for the distribution of observed outcomes from a probability experiment, using them to answer the investigative question
- compare my findings with those of others when undertaking probability experiments.

## Do

### Te whakatauhānui i ngā kitenga | Generalising findings

#### *I can:*

- recognise and explore patterns and make conjectures and draw conclusions about them
- identify relationships including similarities, differences, and new connections
- look for patterns and regularities that might be applied in another situation or always be true
- make and test conjectures, using reasoning and counterexamples to decide if they are true or not
- use appropriate symbols to express generalisations.

### Te whakamārama me te parahau i ngā kitenga | Explaining and justifying findings

#### *I can:*

- make statements and give explanations inductively based on observations or data
- make statements and give explanations deductively based on knowledge, definitions, and rules
- critically reflect on others' thinking, evaluating their logic and asking questions to clarify and understand
- use evidence, reasoning, and proofs to explain why I agree or disagree with statements
- develop collective understandings by sharing, comparing, contrasting, critiquing, and building on ideas with others
- present reasoned explanations and arguments for an idea, solution, or process.

# Mathematics and statistics **progress steps**

Progress steps alert kaiako to aspects of learning that are essential and time-sensitive as ākonga work towards the progress outcome for this phase. They support kaiako to notice, recognise, and respond to ākonga learning in a timely fashion through the provision of multiple opportunities for learning and practice. These opportunities will be meaningful for ākonga if they connect with their home languages and value the ways they make meaning, reason, and communicate (including through the use of augmentative and alternative communication or assistive technology).

**Number structure:**

**Operations: Addition and subtraction**

**Operations: Multiplication and division**

**Rational numbers**

**Equality**

**Spatial reasoning**

**Variability**

## Mathematics and statistics progress step **during year 4**

*Building on their progress from the previous year, ākonga can:*

- recognise, read, write, order, partition, recombine, and represent whole numbers up to 10,000
- use their recalled addition and subtraction basic facts to solve problems
- add and subtract two- and three-digit numbers reliably and efficiently
- add and subtract using the commutative property
- use the relationship between multiplication and division to divide
- recall multiplication and corresponding division facts for threes and fours
- represent common fractions, including those greater than 1, on a number line
- solve addition and subtraction open number sentences using the relationship between the two sides of the equal sign
- identify which shape is a reflection, rotation, or translation of a given shape

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## Mathematics and statistics progress step **during year 5**

*Building on their progress from the previous year, ākonga can:*

- recognise, read, write, order, partition, recombine, and represent whole numbers up to 100,000
- add and subtract reliably and efficiently
- multiply two-digit numbers using the distributive property
- multiply reliably and efficiently
- recall multiplication and corresponding division facts for sixes, eights, and nines
- compare fractions with a benchmark fraction and put them in order
- convert between benchmark fractions, decimals, and percentages (e.g.,  $\frac{1}{2} = 0.5 = 50\%$ )
- represent decimals, fractions, and percentages using both discrete and continuous models
- solve open number sentences involving all operations using the relationship between the two sides of the equal sign
- visualise and draw nets for a cube
- recognise the need for relevant and usable data to answer investigative questions
- suggest reasons why data may vary in a familiar context.

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# Progress outcome by the end of year 8

## Understand

I am building knowledge about number, algebra, measurement, space, statistics, and probability, and drawing on the practices of mathematics and statistics.

*Through this, I am deepening my understanding that:*

**Kotahi te kōhao o te ngira e kuhuna ai te miro mā, te miro pango, te miro wheno.**

The interface between mātauranga Māori and mātauranga mathematics and statistics offers opportunities for insights that uphold the integrity of each knowledge system.

**Nō ngā tūpuna, tuku iho, tuku iho.**

Mathematics and statistics have a continuous, evolving human history.

**Whiria te kaha tūātinini, whiria te kaha tūāmanomano.**

The world is full of patterns and structures that we use mathematics and statistics to understand.

## Know

*I know that:*

### Mātauranga tau | Number

Decimals continue the place-value system using negative powers of ten.

Multiplying a positive number by a number less than 1 results in an answer smaller than the original number.

Division can result in a remainder expressed as a whole number, fraction, or decimal.

On a number line, fractions and decimals occur between whole numbers, and negative numbers go to the left of 0.

Positive and negative numbers can be added and subtracted.

### Taurangi | Algebra

The inverse property applies to addition (e.g.,  $3 + -3 = 0$ ) and multiplication (e.g.,  $3 \times \frac{1}{3} = 1$ ).

Operations to both sides preserve the balance of an equation.

The commutative, associative, distributive, and identity properties work the same for all numbers.

A variable can be used to stand for any number.

Functions are relationships or rules that uniquely associate members of one set with members of another set.

Linear patterns and functions have a constant rate of change. They can be represented by ordered pairs, tables, XY-graphs, and a rule (equation).

Algorithms help solve problems in a systematic way. Their instructions are created, tested, and revised.

### Ine | Measurement

In the metric system, there are base measurements with prefixes added to show the size of units.

Metric measurements can be converted from fractions to whole numbers, and vice versa, by changing units.

Shapes can be decomposed or recomposed to help us find perimeters, areas, and volumes.

When two line segments meet they form an angle, which can be thought of as a rotation of one of the line segments.

*I know how to:*

- represent whole and decimal numbers using powers of ten
- divide whole numbers, reliably and efficiently
- recognise, read, write, represent, compare, order, and convert between fractions, decimals, and percentages
- add and subtract decimals to three places
- add and subtract fractions with the same denominator
- multiply fractions and decimals by whole numbers
- represent fractions in their simplest form
- add and subtract integers.

*I know how to:*

- identify and describe the properties of prime, composite, and square numbers and the divisibility rules for 2, 3, 5, 9, and 10
- use words and symbols to describe and represent the properties of operations (commutative, distributive, associative, inverse, and identity)
- solve linear equations by trial-and-improvement and by applying inverse operations
- use variables to represent a rule about a linear pattern, and use the rule to make predictions
- represent and connect linear functions using tables, equations, and XY-graphs
- create and use algorithms to identify, interpret, and explain patterns.

*I know how to:*

- estimate and then measure length, area, volume, capacity, mass, temperature, data storage, time, and angle, using appropriate metric units
- convert between measurement units
- read analogue and digital measurement tools, round appropriately, and interpret scales accurately
- find the perimeter and area of parallelograms and shapes composed of rectangles
- read, interpret, and use timetables and charts that present measurement information.

## Do

### Te tūhura pūāhua | Investigating situations

*I can:*

- pose a question for investigation
- find entry points for addressing a question, identifying relevant prior knowledge, givens, assumptions, and relationships
- plan an investigation pathway and follow it in an organised way
- monitor and evaluate progress, adjusting the investigation pathway if necessary
- make sense of outcomes or conclusions in light of a given situation and context.

### Te whakaata pūāhua | Representing situations

*I can:*

- use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns, variations, and trends
- use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments
- select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations
- use visualisation to mentally represent and manipulate relationships, objects, and ideas.

### Te tūhono pūāhua | Connecting situations

*I can:*

- suggest connections between ideas and approaches
- suggest connections between different representations
- connect new ideas to things I already know
- make connections to ideas in other learning areas and in diverse cultural, linguistic, and historical contexts.

## Understand

*Kei hopu tōu ringa ki te aka tāepa, engari kia mau ki te aka matua.*

The world is characterised by change and variation that we use mathematics and statistics to understand.

*Ko te pae tawhiti whāia kia tata, ko te pae tata whakamaua kia tina.*

Mathematical and statistical logic and reasoning enable us to identify and explain relationships and to justify conclusions.

## Know

### Mokowā | Space

Spatial properties of simple polygons and polyhedra can also apply to more complex two- and three-dimensional shapes.

Three-dimensional shapes can be represented by two-dimensional images.

The invariant properties of two- and three-dimensional shapes do not change under different transformations.

Position, direction, and pathways can be described using te taiao, as in Māori and Pacific systems of knowledge, or using scale, compass points, and environmental features.

Coordinate systems and maps can express position, direction, and pathways.

#### *I know how to:*

- classify shapes based on their properties
- visualise and draw nets for prisms that have a fixed cross section
- use plan-view drawings to visualise and construct three-dimensional shapes
- find unknown angles and identify angle properties of intersecting lines
- make combinations of transformations that use the invariant properties of shapes
- use scale, compass points, and coordinate systems to interpret and describe distance, location, and direction.

### Tauanga | Statistics

Datasets have a whakapapa. How and why data about people and te taiao is collected, interpreted, and stored needs to benefit, include, and protect them, and must not harm them. People need to know who they are giving data to and why before they agree to contribute to a dataset.

The statistical enquiry cycle (PPDAC – Problem, Plan, Data, Analysis, Conclusion) can be used to conduct data-based investigations about the wider community.

Data visualisations show patterns, trends, and variations. Alternative visualisations of the same data can lead to different insights and communicate different information.

#### *I know how to:*

- pose investigative questions about local rohe and community matters and make predictions or assertions about what I expect to find
- determine the variables needed to answer investigative questions, and plan how to collect data for each variable
- collect data from a group (when all of the group can be surveyed), or source and use data collected by others
- communicate findings in context
- examine the data-collection methods, data visualisations, and findings of others' statistical investigations to see if their claims are believable and reasonable.

### Tūponotanga | Probability

The statistical enquiry cycle (PPDAC) can be used for chance-based investigations, using sampling with replacement.

In a probability experiment with independent trials, results from one trial do not affect results from other trials.

Estimated probabilities and theoretical model probabilities will differ.

If all possible outcomes in a chance-based situation are equally likely, the probability of an event =

$$\frac{\text{the number of ways the event can happen}}{\text{the total number of possible outcomes}}$$

#### *I know how to:*

- recognise claims or misconceptions in relation to chance-based situations
- pose investigative questions for chance-based situations, including those with not equally likely outcomes
- plan, conduct, and systematically record data from probability experiments
- use data visualisations to describe the distribution of observed outcomes from probability experiments and possible outcomes for theoretical probability models
- agree or disagree with others' conclusions by interrogating their chance-based investigations.

## Do

### Te whakatauwānui i ngā kitenga | Generalising findings

#### *I can:*

- recognise and explore patterns and make conjectures and draw conclusions about them
- identify relationships including similarities, differences, and new connections
- look for patterns and regularities that can be applied in another situation or are always true
- make and test conjectures, using reasoning and counterexamples to decide if they are true or not
- use appropriate symbols to express generalisations.

### Te whakamārama me te parahau i ngā kitenga | Explaining and justifying findings

#### *I can:*

- make statements and give explanations inductively based on observations or data
- make statements and give explanations deductively based on knowledge, definitions, and rules
- critically reflect on others' thinking, distinguishing between correct and flawed logic and asking questions to clarify and understand
- use evidence, reasoning, and proofs to explain why I agree or disagree with statements
- develop collective understandings by sharing, comparing, contrasting, critiquing, and building on ideas with others
- present reasoned, coherent explanations and arguments for an idea, solution, or process.

# Progress outcome by the end of year 10

## Understand

I am building knowledge about number, algebra, measurement, space, statistics, and probability, and drawing on the practices of mathematics and statistics.

*Through this, I am deepening my understanding that:*

**Kotahi te kōhao o te ngira e kuhuna ai te miro mā, te miro pango, te miro wheno.**

The interface between mātauranga Māori and mātauranga mathematics and statistics offers opportunities for insights that uphold the integrity of each knowledge system.

**Nō ngā tūpuna, tuku iho, tuku iho.**

Mathematics and statistics have a continuous, evolving human history.

**Whiria te kaha tūātinini, whiria te kaha tūāmanomano.**

The world is full of patterns and structures that we use mathematics and statistics to understand.

## Know

*I know that:*

### Mātauranga tau | Number

Decimals can be terminating, repeating and infinite, or non-repeating and infinite (irrational numbers).

There are an infinite number of fractions between any two numbers on the number line.

Multiplying a fraction by an equivalent form of 1 (e.g.,  $\frac{3}{3}$ ) results in an equivalent fraction.

Dividing by a divisor less than 1 gives a result bigger than the dividend.

A rate compares two quantities that have different units of measure. A ratio is a comparison of two like quantities.

### Taurangi | Algebra

The properties of operations (commutative, distributive, associative, inverse, and identity) apply to numbers and variables.

There is an order of operations when using numbers and variables.

Functions can be expressed as algebraic expressions, XY-graphs, tables, or in words.

There are many different, equivalent equations for expressing a linear function.

Algorithms can be efficient or inefficient. More efficient algorithms have fewer steps.

### Ine | Measurement

Decimal measures are used for very small durations (milliseconds); the rest of time measurement uses a different system (based on 12 and 60).

The defining characteristics of prisms are used to describe what they are and how they are formed.

Resizing a shape changes its perimeter, area, and volume.

*I know how to:*

- add and subtract fractions with unlike denominators by using equivalent fractions
- multiply and divide two fractions or two decimals
- find fractions or percentages of a number
- compare fractions that arise from division-as-sharing (e.g., sharing 2 among 5 vs sharing 3 among 7)
- simplify fractions using the highest common factor
- use rates to model and represent change
- use and apply ratios to model everyday situations.

*I know how to:*

- operate on numbers with whole-number exponents and generalise the rule for multiplying or dividing numbers with exponents and the same base
- express functions arising from linear and simple quadratic patterns
- graph linear functions and interpret the gradient, x-intercept, and y-intercept in relation to the function or the practical situation represented
- substitute into, rearrange, and simplify expressions, combining like terms as needed
- create or use a formula, rule, equation, or inequality, solve for unknowns, and evaluate by substitution
- identify how sequence, selection, and iteration are used in algorithms for generating patterns.

*I know how to:*

- calculate and represent accurately measurements using significant figures
- derive and use the formula for the perimeter and area of polygons and circles
- find the surface area and volume or capacity of prisms and cylinders
- scale a shape by a factor and then derive the scale factor for the new shape's area or volume.

## Do

### Te tūhura pūāhua | Investigating situations

*I can:*

- pose a question for investigation
- find entry points for addressing a question, identifying relevant prior knowledge, givens, assumptions, constraints, and relationships
- plan an investigation pathway and follow it in a systematic and organised way
- monitor and evaluate progress, adjusting the investigation pathway if needed
- make sense of outcomes or conclusions in light of a given situation and context.

### Te whakaata pūāhua | Representing situations

*I can:*

- use representations to find, compare, explore, simplify, illustrate, prove, and justify, patterns, variations, and trends
- use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments
- select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations
- use visualisation to mentally represent and manipulate relationships, objects, and ideas.

### Te tūhono pūāhua | Connecting situations

*I can:*

- suggest connections between ideas and approaches
- suggest connections between different representations
- connect new ideas to things I already know
- make connections to ideas in other learning areas and in a range of cultural, linguistic, and historical contexts.

## Understand

*Kei hopu tōu ringa ki te aka tāepa, engari kia mau ki te aka matua.*

The world is characterised by change and variation that we use mathematics and statistics to understand.

*Ko te pae tawhiti whāia kia tata, ko te pae tata whakamaua kia tina.*

Mathematical and statistical logic and reasoning enable us to identify and explain relationships and to justify conclusions.

## Know

### Mokowā | Space

For all polygons, there is a generalisation for the sum of interior angles and the sum of exterior angles.

Angles between parallel lines and a transversal have known relationships (corresponding, alternate, or co-interior angles).

In similar shapes, corresponding angles are equal and lengths of corresponding sides are proportional.

In right-angled triangles, there is a fixed relationship between the lengths of the three sides given by Pythagoras' theorem.

A point has zero dimensions, a line has one dimension, a plane is two-dimensional, and a solid is three-dimensional. In mathematics, there can be more than three dimensions.

#### *I know how to:*

- reason about unknown angles in situations involving parallel lines and transversals and the interior and exterior angles of polygons
- use the properties of similarity in two-dimensional shapes, including right-angled triangles, to find unknown lengths
- use and apply Pythagoras' theorem to find the length of an unknown side in a right-angled triangle
- use invariant properties to transform a set of points in the XY-plane by translation, reflection about an axis, and rotation about a given point by a multiple of 90 degrees
- represent three-dimensional shapes with two-dimensional drawings and digital tools.

### Tauanga | Statistics

Māori have rangatiratanga over their data including inherent rights and interests in relation to the collection, use, storage, and ownership of it.

People have rights and obligations in relation to their own data and that of others. Different countries have different laws about data and privacy.

The statistical enquiry cycle (PPDAC) can be used to conduct data-based investigations that involve sampling from populations. When sampling from a population, the distribution for a variable varies from sample to sample.

New variables can be created by combining and modifying existing variables.

#### *I know how to:*

- pose summary and comparison investigative questions about populations, and investigative questions for time-series and relationship data and for experiments
- plan to collect data for observational studies and experiments, including selecting valid and reliable measurements for variables or sourcing existing datasets
- recategorise variables if needed, and use multiple representations to analyse and visualise data
- communicate findings using evidence from analysis, provide possible explanations for findings, and reflect on predictions or assertions
- critique the findings and claims of others by interrogating all phases of the statistical enquiry cycle.

### Tūponotanga | Probability

The statistical enquiry cycle (PPDAC) can be used to conduct chance-based investigations involving simulations.

Theoretical model probabilities and experimental estimates of probabilities are approximations of the true probabilities, which are never known.

In two-stage chance-based situations, the stages can be dependent (leading to conditional probabilities) or independent.

#### *I know how to:*

- recognise and pose investigative questions, anticipate what will happen, and state my assumptions
- design probability experiments that use real data to create probability distributions for numerical variables, run simulations, and record data
- describe probability distributions including those involving simple, joint, and conditional probabilities
- identify why claims about chance-based situations might not be valid and consider improvements to how the claim was investigated.

## Do

### Te whakatauwānui i ngā kitenga | Generalising findings

#### *I can:*

- recognise and explore patterns and make conjectures and draw conclusions about them
- identify relationships including similarities, differences, and new connections
- look for patterns and regularities that can be applied in another situation or are always true
- make and test conjectures, using reasoning and counterexamples to decide if they are true or not
- use appropriate symbols to express generalisations.

### Te whakamārama me te parahau i ngā kitenga | Explaining and justifying findings

#### *I can:*

- make statements and give explanations inductively based on observations or data
- make statements and give explanations deductively based on knowledge, definitions, and rules
- critically reflect on others' thinking, distinguishing between correct and flawed logic and asking questions to clarify and understand
- use evidence, reasoning, and proofs to explain why I agree or disagree with statements
- develop collective understandings by sharing, comparing, contrasting, critiquing, and building on ideas with others
- present reasoned, coherent explanations and arguments for an idea, solution, or process.

# Progress outcome by the end of year 13

## Understand

I am building knowledge about number, algebra, measurement, space, statistics, and probability, and drawing on the practices of mathematics and statistics.

*Through this, I am deepening my understanding that:*

**Kotahi te kōhao o te ngira e kuhuna ai te miro mā, te miro pango, te miro whero.**

The interface between mātauranga Māori and mātauranga mathematics and statistics offers opportunities for insights that uphold the integrity of each knowledge system.

**Nō ngā tūpuna, tuku iho, tuku iho.**

Mathematics and statistics have a continuous, evolving human history.

**Whiria te kaha tūātinini, whiria te kaha tūāmanomano.**

The world is full of patterns and structures that we use mathematics and statistics to understand.

## Know

*I know that:*

### Mātauranga tau | Number

Geometric sequences can be generated using recursion, and they can help model exponential growth and decay.

The square root of negative one ( $\sqrt{-1}$ ) is represented by  $i$ .

Sums or series can be represented using sigma notation.

### Taurangi | Algebra

An equation can undergo a variety of transformations with predictable effects on the solution.

A formula can include multiple parameters and variables, which are represented in different ways depending on the context in which the formula is being used.

Functions can be combined to create new functions by operating on them, including through function composition.

The derivative of a function can be interpreted as its rate of change.

The indefinite integral is the antiderivative; the derivative of the indefinite integral is the original function.

### Ine | Measurement

A point on a unit circle at an angle of  $\theta$  with the positive x-axis (in standard position) is represented by the coordinates  $(\cos \theta, \sin \theta)$ . This demonstrates the periodic and symmetric nature of the sine and cosine functions, visually and algebraically.

The area under a curve between two points can be calculated by finding the definite integral between the two points.

### Mokowā | Space

At a single point, the derivative of a function equals the gradient of the tangent line. This can be interpreted as the rate of change of the function at that point.

Tangent lines are local approximations of a function. Near a specific point, the tangent line and the function have approximately the same graph.

The gradient of the tangent line at a local maximum or minimum of a function is 0 or undefined.

*I know how to:*

- find square roots of negative numbers
- calculate the number of possible arrangements in a set, both when the order of the arrangements does matter and when the order does not matter.

*I know how to:*

- solve algebraic equations using the zero-product property (where several expressions multiply to be zero, then one of them must be zero)
- determine the nature of the roots of a quadratic equation
- operate on algebraic fractions
- algebraically solve systems of three linear equations in three-dimensional space, and geometrically interpret the solutions
- predict the effect on the representations of a function when it undergoes a transformation
- use the chain rule to differentiate composite functions.

*I know how to:*

- use simple trigonometric identities (e.g.  $\sin^2(\theta) + \cos^2(\theta) = 1$ ) to simplify calculations
- apply calculus to trigonometric functions
- approximate the area under a curve using rectangles or trapeziums, and improve the approximation
- use derivatives and integrals to solve kinematic problems involving displacement, velocity, and acceleration.

*I know how to:*

- find and graph tangents of a function
- describe curves and circles using parametric equations
- explore and prove conjectures about functions (e.g., about the nature of their graphs, their rates of change, and the area under their curves).

## Do

### Te tūhura pūāhua | Investigating situations

*I can:*

- pose a question for investigation
- find entry points for addressing a question, identifying relevant prior knowledge, givens, assumptions, constraints, relationships, and concepts
- plan an investigation pathway and follow it in a systematic and organised way
- monitor and evaluate progress, adjusting the investigation pathway if needed
- make sense of outcomes or conclusions in light of a given situation and context.

### Te whakaata pūāhua | Representing situations

*I can:*

- use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns, variations, and trends
- use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments
- select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations
- use visualisation to mentally represent and manipulate relationships, objects, and ideas.

### Te tūhono pūāhua | Connecting situations

*I can:*

- suggest connections between ideas and approaches
- suggest connections between different representations
- connect new ideas to things I already know
- make connections to ideas in other learning areas and in a range of cultural, linguistic, and historical contexts.

## Understand

*Kei hopu tōu ringa ki te aka tāepa, engari kia mau ki te aka matua.*

The world is characterised by change and variation that we use mathematics and statistics to understand.

*Ko te pae tawhiti whāia kia tata, ko te pae tata whakamaua kia tina.*

Mathematical and statistical logic and reasoning enable us to identify and explain relationships and to justify conclusions.

## Know

### Tauanga | Statistics

Sourcing data and learning from it are at the heart of evidence-based decision making. Data-based information is used to inform and influence decisions, behaviours, policies, and opinions.

How we collect data affects the applicability, quality, diversity, and quantity of the data, as well as the conclusions we draw from it. We need to take care about sources of bias.

For statistical inference, data is sometimes obtained using random sampling, to enable and justify sample-to-population inferences. Sometimes experiments are used to justify cause-and-effect inferences.

Randomisation tests are simulation methods that can be used with experiments to assess the strength of evidence for the existence of treatment differences.

Bootstrap confidence intervals are simulation methods that can be used to allow for uncertainties in estimation due to sampling error.

Data can be used to develop and train algorithmic predictive-models. Predictions from these models have a distribution that changes as the conditioning or predictor variables are changed.

#### *I know how to:*

- use ethical and responsible data practices when designing and conducting studies, storing and transferring data, and evaluating studies
- determine whether a random sample, an experiment, a secondary data source, or another approach is most suitable for answering an investigative question
- design and implement a simple randomised experiment to compare two groups
- create, manipulate, and merge data from a variety of sources manually and using statistical software, creating a range of relevant data visualisations for it
- select, use, and evaluate appropriate statistical models for making predictions (including regression, time-series, and classification models)
- use the results from analyses to form and communicate conclusions, acknowledge uncertainty, and make new conjectures
- critique data- and chance-based information, data visualisations, embedded statistics, and claims from a variety of sources, including the media.

### Tūponotanga | Probability

There are situations in life that involve uncertainty. Probability can help us think about these situations and make decisions on them.

Probabilities can be estimated from gathered (empirical) data or from theoretical models; both approaches make assumptions that might not be valid.

Generating data from a probability model through simulation can demonstrate what outcomes are likely or unlikely under certain conditions, as well as the variability of the outcomes.

Uniform, binomial, Poisson, and normal probability distributions have recognisable key features and can be used to model situations.

#### *I know how to:*

- conduct large-scale simulations to model probability outcomes
- predict and explain effects caused by changing the parameters of probability distributions
- estimate and calculate probabilities of independent, combined, and conditional events
- calculate, interpret, and evaluate risk
- calculate, interpret, and evaluate the expected value of a numerical random variable
- make and critique claims for reports on polls by taking into account the informal margin of error and considering possible sources of bias.

## Do

### Te whakatauwhānui i ngā kitenga | Generalising findings

#### *I can:*

- recognise and explore patterns and make conjectures and draw conclusions about them
- identify relationships including similarities, differences, and new connections
- look for patterns and regularities that can be applied in another situation or are always true
- make and test conjectures, using reasoning and counterexamples to decide if they are true or not
- use appropriate symbols to express generalisations.

### Te whakamārama me te parahau i ngā kitenga | Explaining and justifying findings

#### *I can:*

- make statements and give explanations inductively based on observations or data
- make statements and give explanations deductively based on knowledge, definitions, and rules
- critically reflect on others' thinking, distinguishing between correct and flawed logic and asking questions to clarify and understand
- use evidence, reasoning, and proofs to explain why I agree or disagree with statements
- develop collective understandings by sharing, comparing, contrasting, critiquing, and building on ideas with others
- present reasoned, coherent explanations and arguments for an idea, solution, or process.



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Curriculum Centre

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**Te Tāhuhu o  
te Mātauranga**  
Ministry of Education

**Te Kāwanatanga  
o Aotearoa**  
New Zealand Government

# Mathematics and statistics

What you told us and how we responded



# Contents

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## Purpose of this report

**The Ministry is two years into a six-year programme to refresh The New Zealand Curriculum.**

The refreshed curriculum will support every ākonga (learner) to experience success in their learning. It will give effect to Te Tiriti o Waitangi, and will be inclusive, clear about the learning that matters, and easy to use from year 0 to year 13.

This will support schools and kura to create rich and responsive learning that helps every ākonga realise their goals and aspirations.

Te Mātaiaho | the Curriculum Framework is currently draft and the final version will be released in Term 3, 2023.

The refreshed NZC will continue the same eight learning areas as in the existing 2007 Curriculum. These are being developed and released in phases, and schools will have until the beginning of 2027 to start using the fully refreshed curriculum.

The mathematics and statistics learning area is part of the second set of learning areas to be refreshed, along with the English learning area.

The social sciences learning area was released ready for use in November of 2022. The remaining curriculum learning areas will be refreshed over 2023 and 2024.

In late 2022, we asked the education sector for feedback on the draft mathematics and statistics content.

This report summarises what we heard, and the actions we took to respond to that feedback.

# Developing the mathematics and statistics learning area

The refreshed mathematics and statistics curriculum content was developed by the mathematics and statistics Writing Group<sup>1</sup> working with the Ohu Arataki, the coherence group.

The learning area draft has been updated following feedback gathered through engagement in late 2022 and early 2023.

The writing group focused on ensuring the content enacts Te Mātaaitipu | the draft Vision for Young People, and explicitly weaves mātauranga Māori, literacy, numeracy, values, and the key competencies throughout, as was done in the development of Aotearoa New Zealand's histories and the social sciences learning area.

## Who we consulted with

Since the Curriculum Refresh began in 2021, the Ministry of Education has engaged widely with the education sector in the development of Te Mātaiaho and refreshing the learning areas.

In late 2022 we asked kaiako, teachers, tumuaki, school leaders, and government organisations and agencies with a strong interest in mathematics and statistics for feedback to improve the draft curriculum content.

This was to ensure it reflects the learning that matters in mathematics and statistics.

We supported tumuaki and school leaders to engage with the draft curriculum content and discuss it with their colleagues. They provided their feedback to us via an online survey. We received 213 survey responses on the draft curriculum content from 114 kura and schools.

The feedback was independently analysed and reported back to us by the New Zealand Council for Educational Research (NZCER).

## What we heard

The feedback we heard centred around three key themes:

- 1 Many people felt the important ideas of mathematics were not clear enough in the learning area Understands.
- 2 People wanted more consistency across the learning areas (both in terms of the level of detail and the structure of the content) for ease of use and clarity.
- 3 People were pleased to see the practices of mathematics represented in DO element of the learning area, though some wanted to see how these developed over the phases of the curriculum.

<sup>1</sup> The Writers Group was: Fiona Ell, Pania Te Maro, Marina McFarland, Dorothea Collier, Julie Roberts, Jodie Hunter, Ann Leach, Josephina Tamatoa, Kim Madden, Cami Sawyer, and Clare O'Connell.

# How we responded

## 1 – Clarifying the big ideas of mathematics

- The UNDERSTANDS have been rewritten to more explicitly address fundamental concepts in mathematics and statistics.
- Many of the ideas about the purpose and value of mathematics and statistics have been moved to the learning area Purpose statement.

### Understand | Big ideas

#### From

- Mātauranga Māori and mathematics and statistics help make sense of the world.
- Mathematics and statistics have a continuous, evolving human history.
- Mathematics and statistics are elegant, explorative, creative, and powerful.
- Mathematics and statistics help us in our everyday lives and decisions and are key to many areas of knowledge and practice.
- Mathematics and statistics reward persistence and positivity.

#### To

- The world is full of patterns and structures that we use mathematics and statistics to understand.
- The world is characterised by change and variation that we use mathematics and statistics to understand.
- Mathematical and statistical logic and reasoning enable us to identify and explain relationships and to justify conclusions.
- The interface between mātauranga Māori and mātauranga mathematics and statistics offers opportunities for insights that uphold the integrity of each knowledge system.
- Mathematics and statistics have a continuous, evolving human history.

## 2 – More consistency across the learning areas

- Content from the previous draft's KNOWs has been reorganised, clarified, and made more consistent with the curriculum refresh model.
- The KNOW and the DO progressions now more closely resemble the progression in social sciences and English. While the number of words used has been reduced, the learning area still adds detail and clarity.

### Know | Contexts

#### From

Progress outcome typically by Year 3

Mātauranga tau | Number

Whole numbers

I know:

- Our number system is based on groups of ten, groups of 100, groups of 1000, and so on (base

ten)

- Numbers can be put together and taken apart in different ways (composing and decomposing)
- Multiplication and division involve recognising and working with groups, the number of groups, and the total

- Numbers can include groups that I can use to find patterns, by, for example, skip counting or using pairs of numbers that add to 10
- Instead of counting I can use number patterns and an understanding of addition, subtraction, multiplication, and division to solve problems.

**I know how to:**

- estimate discrete and continuous quantities (i.e., separate items and amounts of something) and explain my estimate
- recognise, read, write, and order whole numbers up to 10 000
- group, partition, and recombine whole numbers up to 1 000
- solve addition and subtraction with two- and three-digit numbers
- solve multiplication problems involving two one-digit numbers, and a one- and a two-digit number
- solve division problems involving whole numbers, a one-digit divisor, and no remainders
- use the rules and patterns for operations that I have learned in algebra when working with numbers.

**Rational numbers**

**I know:**

- Fractions are a way to show pieces of a whole
- We need to know what the whole is to understand the size of a fraction
- The bottom number of a fraction (denominator)

shows how many pieces a whole has been equally split into, and the top number (numerator) shows how many of those parts the fraction represents

- The bigger the bottom number (denominator) of a fraction, the smaller the pieces
- When fractions have the same denominator, I can add them together by seeing how many pieces I have in total and putting the total over the size of the pieces (the denominator)
- When a fraction has a 1 as the numerator, it is called a unit fraction (in the same way that we call 1 a 'unit')
- I can find a unit fraction by dividing a whole into the number of pieces in the denominator and taking one of those pieces
- whole can be an area, a measurement, or a set of objects
- The same amount, like a half or a quarter, can be shown by different (equivalent) fractions because the
- relationship between the numerator and the denominator is the same.

**I know how to:**

- recognise, read, write, represent, and order halves, thirds, quarters, fifths, sixths, and eighths
- identify or find an equivalent fraction involving halves and quarters
- add unit fractions with like denominators
- find a unit fraction of a whole.

**To**

**Progress outcome typically by Year 3**

**Mātauranga tau | Number**

**I know that:**

- In base 10, there are ten digit symbols and their value is defined by their position within a number. Digits in any column are worth ten times as much as those in the column to the right.
- Te reo Māori and other Pacific languages explicitly describe the logic of the base 10 numbering system.
- Numbers can be composed and decomposed in different ways by using patterns.
- Multiplication and division involve recognising and working with groups, the number of groups, and the total.
- Fractions show parts of a whole in a region, a measurement, or a set of objects. The same amount (e.g., a half or a quarter) can be shown by equivalent fractions.

**I know how to:**

- recognise, read, write, and order whole numbers up to 10,000
- group, partition, and recombine whole numbers up to 1,000
- add and subtract two- and three-digit numbers
- multiply two single-digit numbers or multiply a single-digit and a two-digit number
- divide whole numbers with a single-digit divisor and no remainders
- recognise, read, write, represent, and order halves, thirds, quarters, fifths, sixths, and eighths
- find a unit fraction of a whole (e.g., a region, measurement, or set of objects), and add unit fractions with like denominators.

# 3 – Building on the DOs strength

- The DOs have been rewritten to allow for progression. The addition of a new DO, “Explaining, justifying, and evaluating mathematical and statistical findings”, will allow for more focus on critical thinking and evaluation.

## Do | Practices

### From

#### Progress outcome typically by Year 3

##### Te whakaaro ararau | Reasoning

In my learning in mathematics and statistics, I can:

- build statements and explanations inductively from observations or data, considering the context
- build statements and explanations deductively from mathematical and statistical knowledge, definitions, and rules
- make conjectures, using reasoning and counter examples to decide if they are true or not
- critically reflect on statements based on mathematical and statistical ideas, distinguishing correct logic from flawed logic and asking questions to clarify and understand
- use evidence, reasoning, and proofs to explain why I agree or disagree with statements or why something will always be true
- develop collective understandings with others by sharing ideas and comparing, contrasting, and building on them.

#### Progress outcome typically by Year 6

##### Te whakaaro ararau | Reasoning

In my learning in mathematics and statistics, I can:

- build statements and explanations inductively from observations or data, considering the context
- build statements and explanations deductively from mathematical and statistical knowledge, definitions, and rules
- make conjectures, using reasoning and counter examples to decide if they are true or not
- critically reflect on statements based on mathematical and statistical ideas, distinguishing correct logic from flawed logic and asking questions to clarify and understand
- use evidence, reasoning, and proofs to explain why I agree or disagree with statements or why something will always be true
- develop collective understandings with others by sharing ideas and comparing, contrasting, and building on them.

### To

#### Progress outcome typically by Year 3

##### Te whakamārama me te parahau i ngā kitenga | Explaining and justifying findings

I can:

- make statements and give explanations about what I notice and wonder
- make statements and give explanations deductively based on prior knowledge
- ask questions to clarify and understand others' thinking
- use evidence and reasoning to explain why I agree or disagree with statements
- develop collective understandings by sharing and building on ideas with others
- present basic explanations and arguments for an idea, solution, or process.

#### Progress outcome typically by Year 6

##### Te whakamārama me te parahau i ngā kitenga | Explaining and justifying findings

I can:

- make statements and give explanations inductively based on observations or data
- make statements and give explanations deductively based on knowledge, definitions, and rules
- critically reflect on others' thinking, evaluating their logic and asking questions to clarify and understand
- use evidence, reasoning, and proofs to explain why I agree or disagree with statements
- develop collective understandings by sharing, comparing, contrasting, critiquing, and building on ideas with others
- present reasoned explanations and arguments for an idea, solution, or process.

# Where to find content, guidance, and resources

The refreshed mathematics and statistics learning area content can be found on the Curriculum Refresh website: [curriculumrefresh.education.govt.nz](https://curriculumrefresh.education.govt.nz)

A range of guidance and resources are being developed to support the implementation of the refreshed mathematics and statistics learning area.

These will be released progressively over time.

The refreshed mathematics and statistics learning area content and all supporting guidance and resources will be transitioned to Tahurangi | the Online Curriculum Hub once it is ready.

Ongoing support and resources for local curriculum design and the implementation of the refreshed NZC have also been developed and is available on the Curriculum Refresh website.

## Timeframes

Although the refreshed learning area is now available, the only immediate requirement for schools and kura, is to implement the new Aotearoa New Zealand's histories curriculum content from the beginning of 2023.

For the rest of the refreshed curriculum, schools will have until the beginning of 2027 to explore and get familiar with the framework and the learning areas as they are refreshed, in order to be able to implement them in their local curriculum.







DRAFT

Te Mātaiaho

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# The New Zealand Curriculum

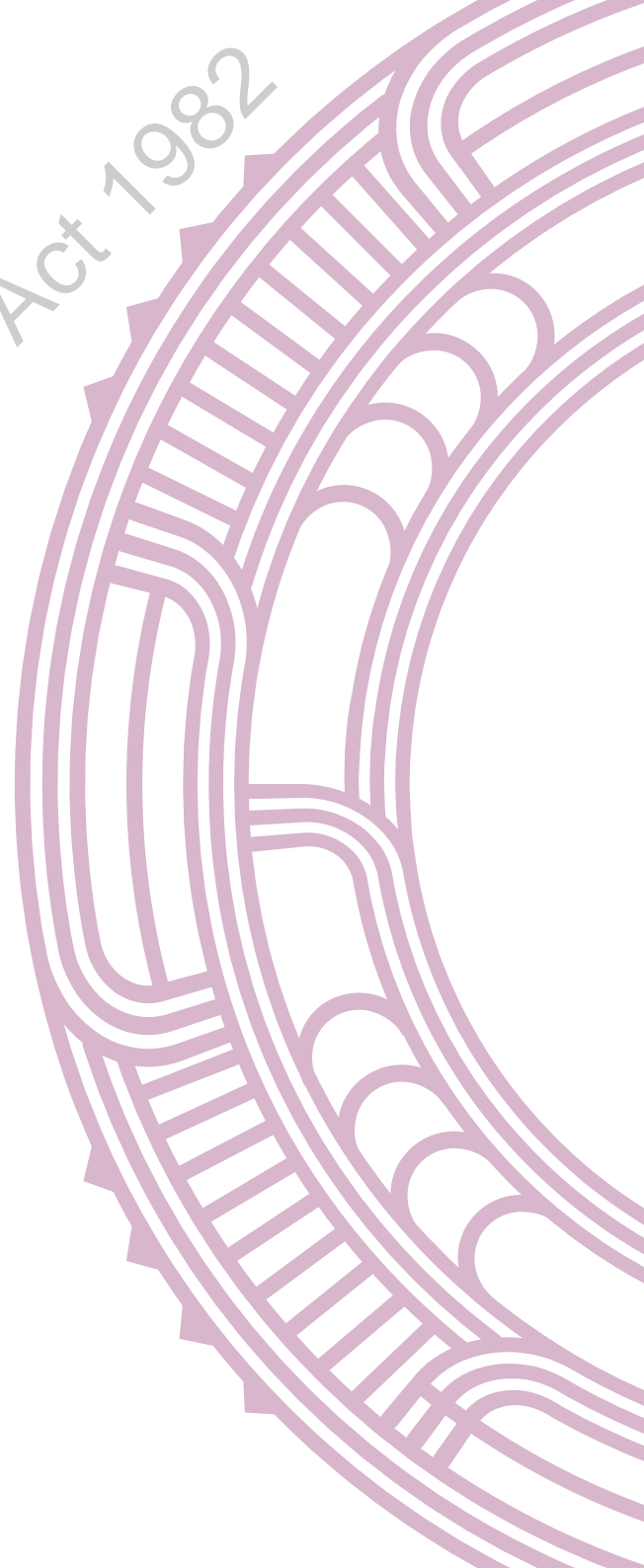
## MATHEMATICS AND STATISTICS YEARS 0-8



Te Tāhuhu o  
te Mātauranga  
Ministry of Education

Te Kāwanatanga  
o Aotearoa  
New Zealand Government

Released under the Official Information Act 1982



Released under the Official Information Act 1982

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In the maths curriculum, the tables in the teaching sequences for phases 1 and 2 are spread across two pages. Therefore, on screen, the curriculum is best viewed using the ‘double pages’ document, so that each double page spread is visible.

If you are printing the document to view it, print the ‘single pages’ version, backed. Then staple or bulldog clip the left-hand side of the printout, so that as you go through the document, each double spread is visible when you turn the pages.

## Purpose statement

*Ānō me he whare pūngāwerewere.  
Behold, it is like the web of a spider.*

In the mathematics and statistics learning area, students study, learn, and appreciate the power and beauty of abstraction, reasoning, and symbolic representation. They learn to investigate, interpret, and explain patterns and relationships in quantity, space, time, data, and uncertainty. Mastering mathematical concepts means students can accurately and efficiently use mathematics and statistics (maths) as a foundation for new learning and to solve problems, reason, and make connections between concepts. The above whakataukī points towards the connections between different concepts, knowledge, and processes that are central to mathematics and statistics.

Students discover personal enjoyment and satisfaction in the learning area as a result of engaging teaching methods that build confidence through incremental challenges and that promote curiosity through the exploration of maths concepts. They appreciate that maths provides tools for powerful problem solving and discover how these tools can improve their lives and the lives of others.

Mathematics and statistics often serve as a universal language fostering collaboration, innovation, and mutual understanding. Students participate when they engage in discussions about their maths thinking and the thinking of others. They also discuss important social matters such as the ethical gathering, interpretation, and communication of data, and challenging misinformation and disinformation.

Mathematics and statistics have a history that involves many cultures contributing to innovation and shaping our thinking today. Students come to understand the value of mathematical modelling and statistical investigation as a lens for resolving collective global challenges.

Through the learning area, students develop fluency and mastery in maths, which leads to pathways into a wide range of industries that rely on maths knowledge and reasoning. Fluency and mastery also allow students to participate fully in an increasingly technology- and information-rich world of work.

Learning in maths builds both literacy and numeracy. Maths contributes to students' literacy by developing their skills in oral and written communication, reasoning, and comprehension. The learning area supports the use of specific vocabulary and symbols and the understanding of tables, graphs, and diagrams, as well as critical thinking about the quality of data and stories told about it.

The learning area embodies a structured approach to mathematics that encompasses:

- › a clearly sequenced year-by-year curriculum
- › a progressive and cumulative approach to the acquisition of knowledge, skills, and competencies in order to build student mastery
- › teaching guidance on effective practice.

The learning area will be supported by high-quality, curriculum-aligned instructional resources.

# Understand-Know-Do Overview



## Understand Big ideas

Understand describes the deep and enduring big ideas and themes that students develop over phases 1-5.

UNDER DEVELOPMENT

The Understand-Know-Do Overview will be completed when all five learning phases have been completed.

# Know

## Content and concepts

Know comprises the meaningful and important content, concepts, and topics that exemplify and enrich students' understanding of the big ideas.

UNDER DEVELOPMENT

The Understand-Know-Do Overview will be completed when all five learning phases have been completed.

## Do Practices

Do describes the practices (skills, strategies, processes, and competencies) that are particular to each learning area and that bring rigour to learning.

UNDER DEVELOPMENT

The Understand-Know-Do Overview will be completed when all five learning phases have been completed.

## Phases

1

Years 0–3

2

Years 4–6

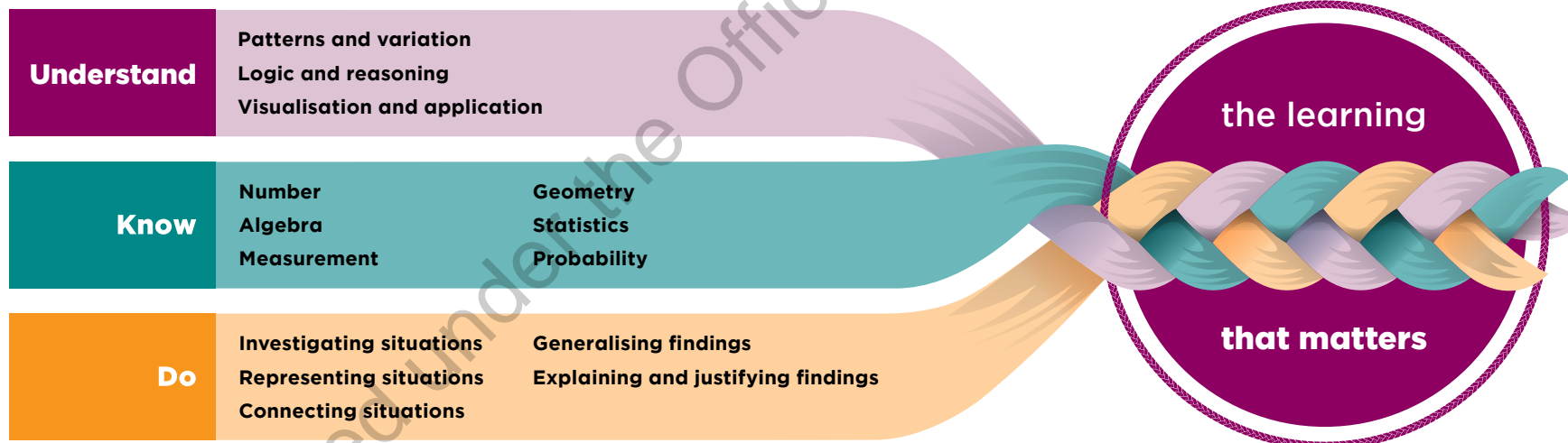
3


Years 7–8

# Mathematics and statistics learning area structure

As the Overview on the preceding pages shows, there are three elements in the mathematics and statistics learning area: Understand, Know, and Do.

- › **Understand** helps connect school maths with the wider world and identifies critical learning in maths. Understand comes alive as students engage in the practices of Do to learn the knowledge embedded in Know.
- › **Know** is broken down into six strands that represent the key learning in maths: number, algebra, measurement, geometry, statistics, and probability.
- › **Do** includes processes that provide ways of presenting, applying, investigating, manipulating, and connecting the mathematical knowledge in the curriculum. These processes are central to how students learn and apply maths knowledge.





The year-by-year teaching sequences are organised in line with the strands of Know. The statements that begin with ‘*Use the mathematical processes to:*’ describe explicit teaching for the Do practices that supports the learning of key concepts, procedures, and skills. They Intentionally incorporate literacy skills through the use of verbal representations, vocabulary, word problems, and student explanations and justifications to enrich the learning.

Each statement in a sequence varies in the amount of teaching time it requires. As an overall guide, the weighting for each strand and sub-strand is indicated by the total number of statements. Some statements are repeated across multiple years, allowing more time for progression, consolidation, and emphasis in other areas.

The teaching methods within the teaching sequences describe recommended teaching representations or approaches for specific statements. They support students to develop fluency, to apply their knowledge in creative ways and in varied contexts, and to problem solve.

### **The science of learning**

The science of learning provides insights into how to sequentially structure teaching and learning programmes to ensure progressive mastery of maths concepts. It also provides pedagogical principles that support student learning in maths.

Maths learning is cumulative in nature, with each new concept building on others that have already been learned. Therefore, students’ mastery of concepts is critical for ongoing learning and success in maths. Mastery in maths combines both conceptual and procedural knowledge. It is important, therefore, that students comprehend what they are doing and why they are doing it (a conceptual focus) as well as developing fluency and automaticity (a procedural focus). This enables them to accurately and efficiently use their maths knowledge as a foundation for new learning.

Learning in maths imposes a substantial cognitive load. Cognitive load is reduced when teachers break down learning into small steps, making sure that each step is mastered and automaticity is achieved before progressing.

Mastery of maths is supported by pedagogical approaches that use concrete and pictorial representations to introduce new maths concepts, before moving on to abstract representations. Students also require repeated opportunities to practise new knowledge and skills in different contexts and through a variety of tasks. They need timely feedback to correct any misconceptions before those misconceptions are encoded in long-term memory, at which point they are much harder to correct.

As students progress in their maths learning and become more procedurally fluent, they should be given opportunities to apply the concepts and procedures they have mastered. Applying knowledge to new situations, and engaging in more complex tasks such as problem-solving activities, requires and develops higher-order thinking skills.

## Phases

1

Years 0–3

2

Years 4–6

3

Years 7–8

## Teaching guidance

### A comprehensive learning programme

A comprehensive mathematics and statistics programme has the following elements:

#### 1. Positive relationships with maths

To support students in developing a positive relationship with maths, focus on building their confidence by setting high, positive expectations and ensuring every student experiences success. Engage with students' interests outside the classroom to create meaningful learning experiences, and reinforce that maths knowledge, skills, and processes can develop with consistent effort. Provide manageable challenges and teach perseverance, valuing mistakes as learning opportunities. Encourage risk-taking and exploration of maths in real-world contexts, such as designing a school garden or collecting data for decision making.

Create an inclusive learning environment where all students feel supported to contribute and take risks. Teach active listening, reflection, and collaboration skills, and connect maths tasks to students' languages and cultures. Work with students' families, inviting them to be involved in everyday maths activities and classroom maths work.

Model personal engagement in maths processes, so that you build knowledge, curiosity, and enjoyment in maths while teaching.

#### 2. Maths mastery and a structured approach to teaching and learning

Maths mastery is a teaching approach that builds deep conceptual understandings by carefully sequencing mathematics and statistics content. Students are supported to master content through the use of material and pictorial representations, deliberately designed consolidation, and opportunities to apply concepts to new situations. When students have mastered a maths concept, they can accurately and efficiently use it as a foundation for new learning and to solve problems, reason, and make connections between concepts.

When planning and teaching maths units and lessons using the maths mastery approach, it is important to consider the purpose of learning and to include a balance of practices across the week.

### 3. Effective use of technology

Carefully chosen technology can enhance the teaching and learning of maths. Such technology includes digital applications, calculators, and software. Plan to use technology to make maths engaging and accessible for students, supporting their understanding of concepts.

Where appropriate, use technology to:

- › represent and explore patterns and functions
- › draw graphs and data visualisations
- › perform calculations
- › visualise shapes, space, and transformations
- › collect, analyse, and interpret data
- › organise and communicate findings and reasoning.

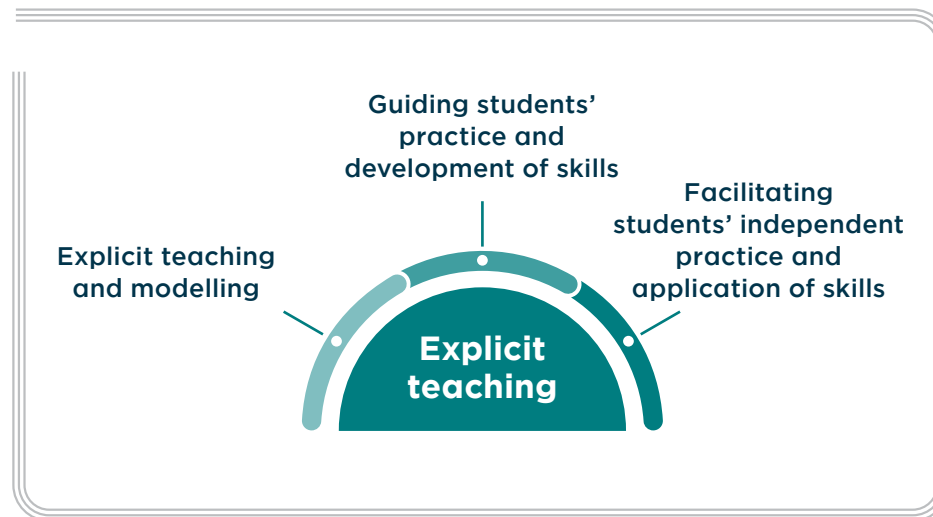
Students should be taught to use technology accurately, appropriately, and efficiently as a tool to support their purpose. This is an important skill in maths. However, this does not mean that the use of technology replaces drawn diagrams or written and mental calculations. Students need the ability to estimate and reason to ensure they can evaluate whether findings generated by technology are reasonable and effective.

### 4. Explicit teaching

Intentional teaching is purposeful and planned. It involves teachers constantly noticing, recognising, and responding to students' learning to make sure that they are achieving the learning purposes.

Explicit teaching is an important aspect of intentional teaching. It means that teachers understand and explain concepts thoroughly and clearly, with planned and defined purposes for each lesson. They do not expect students to work out these concepts naturally or intuitively on their own.

Explicit teaching is a valuable approach when learning is new, and when misunderstandings and gaps have been identified. It is not all about teacher talk. Instead, teachers gradually hand over to students the responsibility for their own learning. Beginning with teaching and modelling, teachers move to guided practice and skill development, and finally to facilitating students' independent practice. Students are active learners throughout this process. This process is repeated as students encounter new learning.



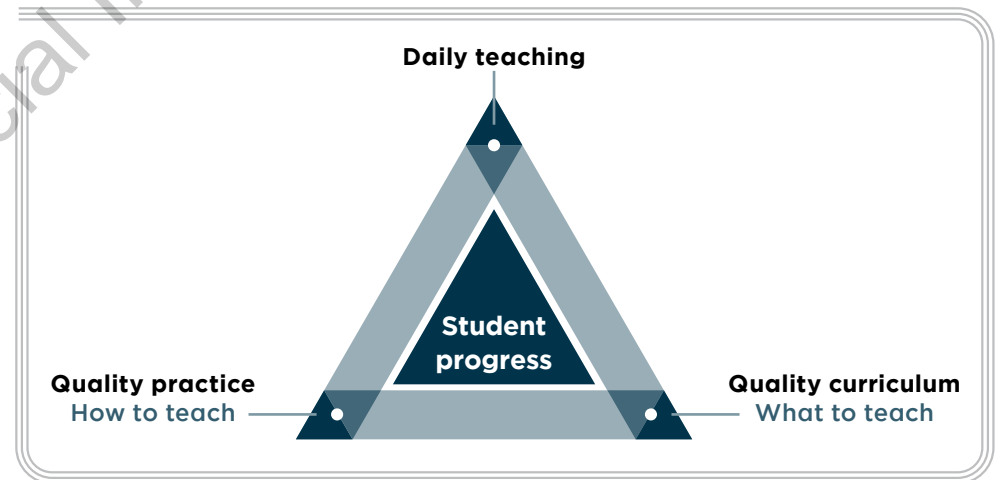
## Planning

Teaching and learning plans are developed for a syndicate, year, topic or unit, week, and lesson, and differentiated to meet the needs of individual students. The following considerations are critical when planning.

- › Design organised and focused plans in order to make optimal use of instructional time.
- › Identify the critical content that will provide rigorous learning for all students by combining teaching sequence statements from across strands (e.g., statements from number, geometry, and measurement).
- › Sequence the knowledge and practices in ways that take into account such things as easier and harder aspects, highly frequent to less frequent usage, prerequisites, and associated knowledge and practices.
- › Break down complex skills, strategies, and knowledge into smaller instructional units.
- › Use assessment information to identify particular learning and teaching strategies that reflect students' prior knowledge and that will extend and deepen this knowledge. Provide learner-focused feedback and reflect on and evaluate students' responses to learning to plan future teaching.
- › Use flexible groups within a lesson, based on the purpose of learning for the lesson (e.g., working as a whole class to demonstrate and discuss, before moving into smaller groups to investigate a situation or solve a problem).

### An hour a day of maths

Students progress best when they have daily opportunities to learn through evidence-informed, high-quality approaches and teaching practices.



Teach maths for an average of an hour a day, including a dedicated lesson using the structure in the following table. Integrate maths instruction across a range of learning areas, and, if appropriate, spread the hour of dedicated instruction across the day to maximise students' engagement. Space the learning of a skill, concept, or process over several lessons so that students have multiple opportunities to learn and to consolidate their learning by applying it to new topics.

## Structure for dedicated maths lessons

### A 'getting started' introduction

Plan to:

- › link to prior learning to ensure all students can access and understand new concepts or processes
- › check students' understanding as you extend previous learning
- › introduce new concepts using a focus activity, group challenge, problem solving, or a task that activates prior knowledge and interests.

### Working time

Depending on the purpose for the lesson, aim to include one or more experiences. As students are working, take time to notice, recognise, and respond to their learning.

Plan to:

- › explicitly teach, leading back and forth interactions and including explanations, demonstrations, questioning, short tasks, and discussion, to enable students to think, reason, and apply their knowledge to solve problems
- › provide whole-class, small-group, paired, or individual work opportunities where students develop or consolidate concepts and practices through investigations, problem solving, tasks, or games
- › provide additional explicit teaching based on the learning needs of individual students
- › help students organise new knowledge in ways that connect with their prior knowledge
- › provide independent or guided practice and activities supporting active retrieval
- › use review activities to consolidate learning and inform next steps.

### A 'connect and reflect' summary

Plan to summarise the lesson:

- › making clear connections back to the purpose of the lesson
- › reviewing learning by discussing, sharing, and analysing learning pathways and findings
- › taking pre-teaching opportunities to prime students for the next lesson.

Collect information about what was learnt through:

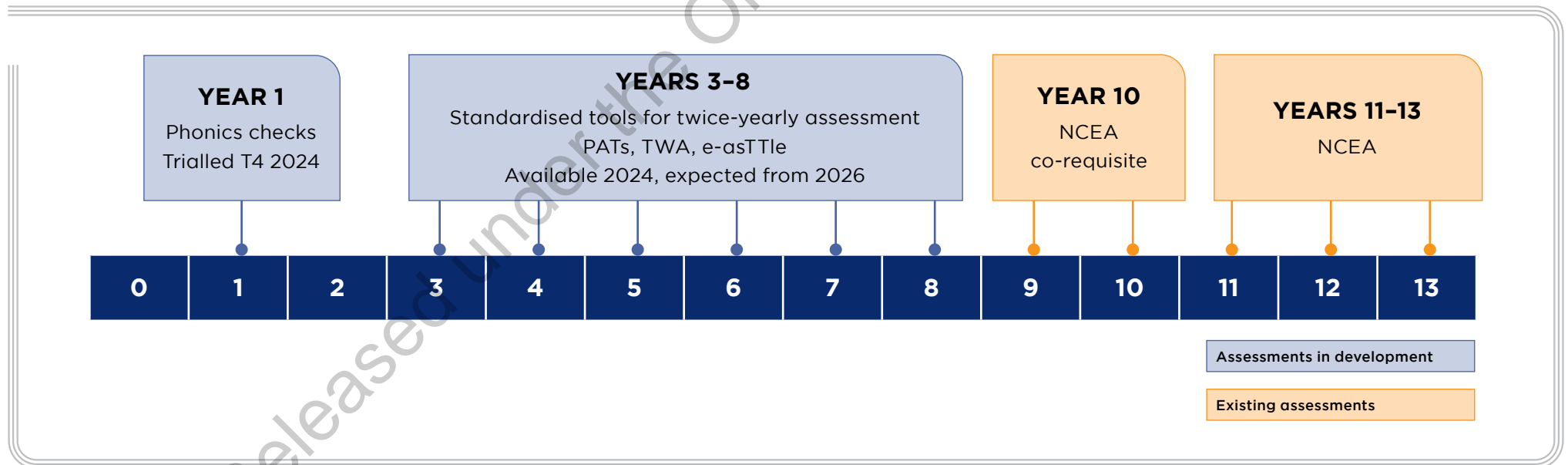
- › self and peer assessment, with students reflecting on goals and identifying next steps
- › checking conceptual understanding, with an emphasis on procedures, vocabulary, and representations
- › noticing and highlighting curiosity, resilience, perseverance through challenges, and progress.

## Use of standardised assessment tools

Schools are expected to use the assessment tools shown in the table below so that we have a nationally consistent approach to assessment and a common set of assessment tools. Teachers determine when the assessments take place.

The tools reflect the shifts in the focus of mathematics and statistics learning throughout the schooling pathway. Assessment information from the tools will help teachers to make decisions about responding to the learning needs of students. This will help school leaders to prioritise and monitor urgent action when it is required to support classroom teaching. Where teaching needs to be targeted and intensified to meet specific needs for finite periods, leaders can draw on a breadth of available supports as required.

### Comprehensive system for assessment and aromatawai from 2025



# Phase

# 1

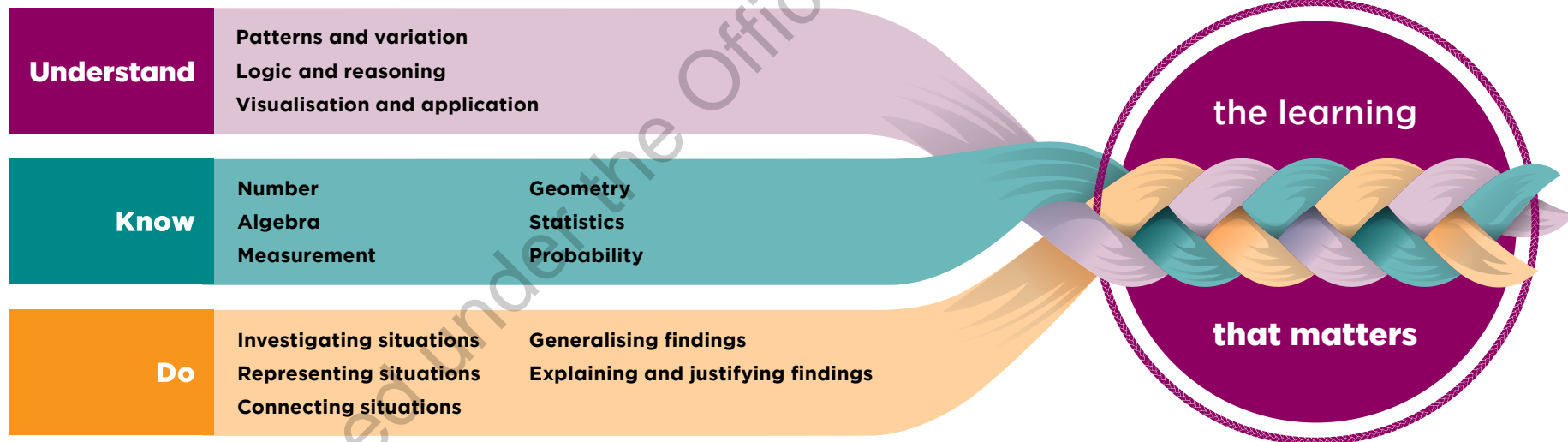
Years 0–3

*Thriving in environments rich in literacy and numeracy*

*Te tupu pāhautea i te taiao ako e haumako ana i te reo matatini me te pāngarau*

## Progress outcome by the end of year 3 (Foundation)

In phase 1, students weave together Understand, Know, and Do and use logic and reasoning to investigate, classify, and describe patterns and variations in quantities, shapes, and data. They use materials, number lines, and pictures to visualise these concepts, make connections between representations, and explain their reasoning. They begin to generalise and to understand the properties of numbers and shapes.



## Understand Big ideas

As they develop their maths capabilities and knowledge, students come to understand that the world has structures useful for noticing, exploring, and describing **variation** and different types of **patterns** and relationships. Students are developing **logical** reasoning and critical thinking skills that enable them to seek information, ask questions, and share their mathematical reasoning.

Statistical **reasoning** allows them to begin to explore their immediate world and make statements about what they observe. Students are developing the ability to **visualise, which allows them** to understand and interact with maths ideas. They **apply** these ideas to explore everyday situations and to support decision making and the communication of ideas.

## Know Content and concepts

### Number | Mātauranga tau

By the end of this phase, students know that our **number system is base 10** with ten digit symbols. The place value of a digit in a number depends on its position; as we move to the left, each column is worth ten times more. Zero is used as a placeholder. Students know that they can use subitising patterns to support **estimations** and calculations. They know that numbers can be **partitioned and recombined** in different ways. Addition is putting parts together to find the total or whole. Subtraction takes parts away from the whole; it is also the difference or distance between numbers. Multiplication and division involve recognising and working with **equal groups**: how many are in each group, the number of groups, and the total amount in all groups.

Students know that **fractions are numbers** that can be described using words, pictures, or symbols. When fractions are represented symbolically, the bottom number (denominator) shows how many pieces a whole has been equally split into, and the top number (numerator) shows how many of those parts the fraction represents. The bigger the denominator of a fraction, the smaller the pieces. **Fractions show parts of a whole** region, set of objects, or measurement; they also show the division of two numbers (the quotient).

### Algebra | Taurangi

Students come to know that committing **maths facts** to memory allows them to be recalled fluently, so that attention is freed for working on more complex tasks. The **equal sign is relational** in that it shows that the two sides of an equation represent the same quantity. Students notice properties in basic operations: **inverse operations** undo each other (addition and subtraction, multiplication and division), the **commutative property** applies to addition and multiplication, the additive **identity** is 0, and the multiplicative **identity** is 1.

Students know that **patterns** can be made of elements, including numeric or spatial elements, in a sequence governed by a rule. Repeating patterns have a unit of repeat; growing patterns can increase or decrease. Students also know that an **algorithm** is a set of step-by-step instructions to complete a task or solve a problem.

### Measurement | Ine

By the end of this phase, students know that **systems of measurement** have a history and that different cultures use different approaches to measuring. Students know that they can **measure and compare various attributes**, such as length, area, volume, capacity, mass (weight), temperature, duration, and turn, using informal or standard units. When measuring, the measurement units must remain the same and join up continuously with no gaps or overlaps. The measurement is the total number of units used from start to end. Students also know that the distance around the edge of a two-dimensional shape gives perimeter, covering a surface gives area, and filling a three-dimensional shape gives capacity or volume.

## Geometry | Āhuahanga

During this phase, students come to know that **patterns and regularities in shapes** can be used to compare, classify, and predict. Two- and three-dimensional shapes have **features** that can be observed and described using **geometric language**. Shapes and objects can flip (reflect), turn (rotate), slide (translate), and be used to create patterns. Objects can be rotated in space and may appear different from other **perspectives**. Students know that **maps** are two-dimensional representations of places in the world with symbols to show locations and landmarks. The **position** of a location can be described relative to another location, including a known environmental feature.

## Statistics | Tauanga

By the end of this phase, students know that **data** is information about the world, that it comes in many forms, and that it helps them to learn about people, their lives, and their environment. They know that a **statistical enquiry cycle** can be used to investigate a group using questions that they ask of the data. A variable refers to an attribute of the data, such as height, number of children, or colour. **Sorting and organising** variables helps to make sense of data and to answer **summary investigative** questions. **Data visualisations** are representations of all available values of one or more variables that reveal relationships or tell a story.

## Probability | Tūponotanga

Students come to know that a chance-based situation has a set of possible outcomes that can be arranged into events. The probability of an event is the chance of it occurring.

## Do Practices

### Investigating situations | Te tūhura pūāhua

By the end of this phase, students can work with others to pose a question for investigation, find entry points for addressing a question, plan an investigation pathway, and follow it. They can identify relationships and relevant prior experience and knowledge to support the investigation. They can describe progress on the investigation pathway and work with others to make sense of outcomes or conclusions in light of a given situation and context.

### Representing situations | Te whakaata pūāhua

Students can use representations to explore, find, and illustrate patterns. They use representations to learn new ideas and explain ideas to others, and they use visualisations to mentally represent and manipulate groups and shapes. They select or create appropriate mental, oral, physical, or virtual representations.

### Connecting situations | Te tūhono pūāhua

Students can suggest connections between ideas, approaches, and different representations. They connect new ideas to things they already know. They make connections with ideas in other learning areas and with familiar local contexts.

### Generalising findings | Te whakatauwānui i ngā kitenga

Students can recognise and explore patterns, and make conjectures and draw conclusions about them. They can identify relationships, including similarities, differences, and new connections. They look for patterns and regularities that might be applied in another situation or always be true. They make and test conjectures, using reasoning and counterexamples to decide if they are true or not. They use words and pictures to express generalisations.

### Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

Students can make statements, give explanations about what they notice and wonder, and make deductions based on prior knowledge. They ask questions to clarify and understand others' thinking and use evidence and reasoning to explain why they agree or disagree with statements. They develop collective understandings by sharing and building on ideas with others, and they present basic explanations and arguments for an idea, solution, or process.

## Phase

# 1

Years 0–3

## Teaching sequence

*Thriving in environments rich in literacy and numeracy*


*Te tupu pāhautea i te taiao ako e haumako ana i te reo matatini me te pāngarau*

Throughout phase 1, students should experience teaching that encourages curiosity and fosters success, as they explore environments and contexts rich in number and spatial elements. Students engage in active, hands-on experiences that involve them doing maths through meaningful tasks that engage their interests and reflect the world outside the classroom.

Continuously monitor students' reasoning, questions, representations, and use of materials, and act quickly to respond to any misconceptions. Ensure your teaching builds on what students already understand, know, and can do.

Throughout each day, provide opportunities for students to:

- › **develop positive relationships with maths** – plan meaningful problem solving and investigations into familiar situations, as well as the exploration of rich mathematical situations
- › **'have a go' and take risks** – reinforce the idea that mistakes help us learn as we try new skills or contribute their ideas
- › **experience explicit teaching of concepts and practices** – use worked examples with new learning broken down into clearly explained, manageable steps
- › **listen to the teacher 'think out loud'** – voice your thinking to show your decision making about which numbers or operations to use as you demonstrate a skill or process

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- › **use numbers, materials, and pictorial representations (diagrams, pictures)** – select representations that support the purpose of learning and that will help students represent their thinking and reasoning and learn new ideas. Over the phase, move students towards using symbols and showing operations as number equations. Number lines are a key representation in this phase for representing, ordering, and comparing numbers (including fractions) and demonstrating operations
  - › **build vocabulary** – use games, songs, word walls, books, and online tools, and be intentional about the vocabulary you use. This supports students to connect their informal language to reasoning, using maths vocabulary. Draw on your students’ first and heritage languages
  - › **engage in the maths practices to connect learning within and across contexts** – point out connections within concepts (e.g., “If I know  $3 + 4$ , then I know  $4 + 3$ ), and teach connected statements together (e.g., quarters, quarter turns, and quarter hours). Encourage students to connect with maths outside school by bringing in photos resources, books, and cultural artefacts from home that link to maths learning
  - › **visualise and identify patterns, connections, and structures in shapes and numbers** – engage students in sorting, classifying, grouping, and partitioning, discussing what they have noticed and what they are wondering about
  - › **engage with you and their peers** – support interactions that allow students to discuss, explain their maths ideas, summarise, ask questions, and make suggestions. Over phase one, help students to recall and connect maths learning using questions, materials, and verbal or visual prompts
  - › **consolidate maths learning to develop fluency with time-sensitive statements in the teaching sequence** – plan a range of tasks, such as using songs, families of facts, games, and apps to practise skip counting, addition, subtraction, multiplication, and division facts.

The highlighted cells in the teaching sequence show critical steps that alert teachers to specific aspects of learning that are essential and time-sensitive. They support teachers to notice, recognise, and respond to student learning in a timely way.

## Number

	<b>6 months</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 1</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 2</b> <i>Informed by formative assessment, teach students to:</i>
<b>Number structure</b>	› subitise (recognise instantly) the number of objects in a group of up to 5	› subitise (recognise instantly) the number of objects in a group of up to 10 objects, including combining two patterns of 1-5 objects	› group objects in a pattern of at least 10 objects, subitise the number of objects in each part, and find the total number in the pattern using the parts
	› count to 10 and beyond to 20, forwards and backwards, from any number	› count to 20 and beyond to 100, forwards and backwards in 1s, 2s, and 10s, from any number	› count to 100, forwards and backwards, from any number, in 1s, 2s, 5s, and 10s
		› recognise and represent the ten-and-ones structure of the 'teen' numbers 11-19	› recognise and represent the base ten structure of numbers up to 100
	› identify, read, and write whole numbers up to at least 10	› identify, read, and write whole numbers up to at least 20	› identify, read, and write whole numbers up to at least 100
	› compare and order whole numbers up to at least 10 and ordinal numbers (1st, 2nd, 3rd), using words	› compare and order whole numbers up to at least 20 and ordinal numbers (1st, 2nd, 3rd), using words or numerals with suffixes	› compare and order whole numbers up to at least 100
	› partition up to 5 objects, and then up to 10 objects, using a systematic approach and noticing patterns in the sequence	› partition and regroup up to 20 objects in different ways, using a systematic approach and noticing patterns	› partition and regroup whole numbers up to at least 100, using a systematic approach and noticing patterns

<b>Year 3</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
	<i>Use the following when representing number structure:</i>
<ul style="list-style-type: none"> <li>› estimate to the nearest 10 the number of objects in a collection of less than 100</li> </ul>	<ul style="list-style-type: none"> <li>› dot patterns, 10s frames, fingers (years 0–2)</li> <li>› materials that can be grouped in 10 (e.g., iceblock sticks) (year 3)</li> </ul>
<ul style="list-style-type: none"> <li>› count to 1,000, forwards and backwards in 1s, 2s, 3s, 5s, 10s, and 100s, from any number</li> </ul>	<ul style="list-style-type: none"> <li>› number lines, 100s boards, number flip charts, 1,000s books</li> </ul>
<ul style="list-style-type: none"> <li>› recognise and represent the base ten structure of numbers up to 1,000</li> </ul>	<ul style="list-style-type: none"> <li>› 10s frames</li> <li>› ice block sticks, place-value (PV) blocks, PV money, PV houses, arrow cards</li> </ul>
<ul style="list-style-type: none"> <li>› identify, read, and write whole numbers up to at least 1,000</li> </ul>	<ul style="list-style-type: none"> <li>› 100s boards, PV houses, number fans, number cards, words and numerals</li> </ul>
<ul style="list-style-type: none"> <li>› compare and order whole numbers up to at least 1,000</li> </ul>	<ul style="list-style-type: none"> <li>› 100s boards (years 0–2)</li> <li>› number flip chart, PV houses, number cards, digit cards, number lines</li> </ul>
<ul style="list-style-type: none"> <li>› partition and regroup whole numbers up to at least 1,000, using a systematic approach and noticing patterns</li> </ul>	<ul style="list-style-type: none"> <li>› multilink cubes to break into groups, bead strings, 10s frames (years 0–1)</li> <li>› an abacus (year 2)</li> <li>› PV blocks, PV money (year 3)</li> <li>› partitioning diagrams, tables for recording patterns</li> </ul>

	<b>6 months</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 1</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 2</b> <i>Informed by formative assessment, teach students to:</i>
<b>Number structure</b>	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- generalise patterns and structures to quantify groups without counting</li> <li>- investigate different ways numbers can be partitioned</li> <li>- explain and justify using vocabulary that identifies quantities when ordering and comparing numbers and patterns (e.g., more than, same as, less than, between)</li> <li>- connect ordinal numbers with the counting sequence (year 1)</li> </ul> </li> </ul>		<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect with algebra number patterns and te reo Māori or other languages with an explicit base 10 number structure</li> <li>- generalise the PV structure to compare and order numbers</li> <li>- investigate different ways numbers can be partitioned and recorded</li> <li>- explain and justify the structure of numbers using PV language</li> </ul> </li> </ul>
<b>Operations</b>		› use estimation to predict and to check the reasonableness of calculations	› use estimation to predict and to check the reasonableness of calculations
			› identify the nearest tens to any whole number up to 100
	› join and separate groups of up to a total of 10 objects, and find the result by grouping and counting	› join and separate groups of up to a total of 20 objects, and find the difference between groups by grouping and counting (e.g., $9 + 6$ ; $7 + \_ = 11$ )	› add and subtract numbers up to 100 without renaming (e.g., $53 + 21$ ; $55 - 32$ )
		› multiply and divide by making equal groups and using grouping or counting	› multiply and divide by grouping and skip counting

<b>Year 3</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>– connect with algebra number patterns and te reo Māori or other languages with an explicit base 10 number structure</li> <li>– generalise the PV structure to compare and order numbers</li> <li>– investigate different ways numbers can be partitioned and recorded</li> <li>– explain and justify the structure of numbers using PV language</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› discrete materials, PV materials</li> <li>› digit cards, patterns</li> <li>› number lines, words, equations.</li> </ul>
	<b><i>Use the following when representing operations:</i></b>
<ul style="list-style-type: none"> <li>› use estimation to predict and to check the reasonableness of calculations</li> </ul>	<ul style="list-style-type: none"> <li>› language for support (e.g., more or less than, close to)</li> </ul>
<ul style="list-style-type: none"> <li>› round whole numbers up to 1,000 to the nearest hundreds and tens</li> </ul>	<ul style="list-style-type: none"> <li>› number lines marked with the multiples of 10 or 100, progressing to unmarked number lines</li> <li>› 100s boards (year 2)</li> </ul>
<ul style="list-style-type: none"> <li>› add and subtract 2- and 3-digit numbers without renaming and without a change-unknown (e.g., <math>148 - 23</math>; <math>235 + 121</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› discrete materials, number lines (years 0-2)</li> <li>› horizontal and vertical methods (year 3)</li> </ul>
<ul style="list-style-type: none"> <li>› multiply a 1- or 2-digit number by a 1-digit number, without renaming (e.g., <math>4 \times 6</math>; <math>2 \times 23</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› pictures and diagrams (years 1-3)</li> <li>› discrete materials and number lines with grouping (years 1-2)</li> <li>› arrays, PV materials (year 3)</li> </ul>
<ul style="list-style-type: none"> <li>› divide whole numbers by a 1-digit divisor with no remainders, by grouping and using the inverse relationship with multiplication (e.g., <math>32 \div 4</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› pictures, diagrams, number lines</li> <li>› bar models, multilink cubes</li> </ul>

	<b>6 months</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 1</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 2</b> <i>Informed by formative assessment, teach students to:</i>
<b>Operations</b>	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect and use addition and subtraction in a range of situations</li> <li>- generalise the key ideas of counting when quantifying and when finding the total, difference, fair share, and comparisons</li> <li>- investigate word problems and the language used to describe an operation</li> <li>- explain and justify ways of quantifying, including counting, subitising, groupings, and sharing</li> </ul> </li> </ul>		<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect and use addition, subtraction, multiplication, and division in a range of situations</li> <li>- generalise the use of the commutative property when solving addition problems</li> <li>- investigate word problems and identify an operation to use</li> <li>- explain and justify ways of quantifying, including estimation, groupings, and known efficient methods</li> </ul> </li> </ul>
<b>Rational numbers</b>		<ul style="list-style-type: none"> <li>› identify and represent halves and quarters as fractions of sets and regions, using equal parts of the whole</li> </ul>	<ul style="list-style-type: none"> <li>› identify, read, write (using symbols and words), and represent halves, thirds and quarters as fractions of sets and regions, using equal parts of the whole</li> </ul>
			<ul style="list-style-type: none"> <li>› directly compare two fractions involving halves, thirds, and quarters</li> </ul>
		<ul style="list-style-type: none"> <li>› find a half or quarter of a set using equal sharing and grouping</li> </ul>	<ul style="list-style-type: none"> <li>› find a half, quarter, or third of a set by identifying groups and patterns (rather than sharing by ones)</li> </ul>
			<ul style="list-style-type: none"> <li>› identify, from part of a set or shape, the whole set or shape</li> </ul>

<b>Year 3</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>– connect and use addition, subtraction, multiplication, and division in a range of situations</li> <li>– generalise the use of the commutative property when solving addition problems</li> <li>– investigate word problems and identify an operation to use</li> <li>– explain and justify ways of quantifying, including estimation, groupings, and known efficient methods</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› materials, pictures</li> <li>› number lines, arrays</li> <li>› word problems, equations.</li> </ul>
	<b><i>Use the following when representing rational numbers:</i></b>
<ul style="list-style-type: none"> <li>› identify, read, write, and represent halves, thirds, quarters, fifths, sixths, and eighths as fractions of sets and regions, using equal parts of the whole</li> </ul>	<ul style="list-style-type: none"> <li>› a range of continuous materials (bar models) and discrete materials (sets of objects)</li> <li>› words, fraction symbols</li> </ul>
<ul style="list-style-type: none"> <li>› compare and order fractions involving halves, quarters, and eighths and identify when two fractions are equivalent</li> </ul>	<ul style="list-style-type: none"> <li>› a range of continuous materials (bar models, fraction tiles) and discrete materials (sets of objects)</li> </ul>
<ul style="list-style-type: none"> <li>› find a unit fraction of a whole (e.g., <math>\frac{1}{3}</math> of 15)</li> </ul>	<ul style="list-style-type: none"> <li>› discrete materials</li> <li>› bar models (including paper strips) to show a whole and fractions to show partitions (year 3)</li> </ul>
<ul style="list-style-type: none"> <li>› identify, from a unit fraction part of a set or amount, the whole set or amount</li> </ul>	<ul style="list-style-type: none"> <li>› discrete materials</li> <li>› bar models (year 3)</li> </ul>
<ul style="list-style-type: none"> <li>› add unit fractions with the same denominator (e.g., <math>\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› fraction tiles, paper fraction strips, equations</li> </ul>

	<b>6 months</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 1</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 2</b> <i>Informed by formative assessment, teach students to:</i>
<b>Rational numbers</b>		<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect fractions in measurement and geometry situations</li> <li>- investigate practical situations involving sharing, partitioning and identifying fractions</li> <li>- explain and justify ways to equal share.</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect a unit fraction of a quantity to division by a denominator</li> <li>- investigate different ways fractions can be represented and partitioned</li> <li>- explain that in a fraction the denominator indicates the number of parts a whole has been divided into, and the numerator the number of fractional parts</li> </ul> </li> </ul>
<b>Financial maths</b>			<ul style="list-style-type: none"> <li>› recognise and order NZ denominations up to \$20 according to their value, make groups of 'like' denominations, and calculate their value</li> </ul>
			<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect to place value</li> <li>- investigate appropriate financial situations.</li> </ul> </li> </ul>

<b>Year 3</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect a unit fraction of a quantity to division by a denominator</li> <li>- investigate different ways fractions can be represented and partitioned</li> <li>- explain that in a fraction the denominator indicates the number of parts a whole has been divided into, and the numerator the number of fractional parts</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› discrete and continuous materials</li> <li>› words, pictures, symbols, number lines.</li> </ul>
	<b><i>Use the following when representing financial maths:</i></b>
<ul style="list-style-type: none"> <li>› make amounts of money using one- and two-dollar coins and 5-, 10-, 20-, 50-, and 100-dollar notes</li> </ul>	<ul style="list-style-type: none"> <li>› play money (coins and notes)</li> </ul>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect to addition and subtraction when calculating amounts</li> <li>- investigate appropriate financial situations.</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› play money (coins and notes).</li> </ul>

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# Algebra

	<b>6 months</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 1</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 2</b> <i>Informed by formative assessment, teach students to:</i>
<b>Generalising number properties</b>		<ul style="list-style-type: none"> <li>› identify addition facts up to 10 and their corresponding subtraction facts (families of facts), including doubles and halves</li> </ul>	<ul style="list-style-type: none"> <li>› recall addition facts up to 10, and identify addition facts up to 20 and their corresponding subtraction facts (families of facts), including doubles and halves</li> </ul>
		<ul style="list-style-type: none"> <li>› explore adding 0 to or subtracting 0 from a number</li> </ul>	<ul style="list-style-type: none"> <li>› explore multiplying a number by 0 and 1 and dividing a number by 1</li> </ul>
		<ul style="list-style-type: none"> <li>› explore the commutative property of addition (e.g., <math>5 + 4 = 4 + 5</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› identify the commutative property of addition (e.g., <math>5 + 4 = 4 + 5</math>)</li> </ul>
			<ul style="list-style-type: none"> <li>› identify the relationship between skip counting and multiplication facts for 2s, 5s, and 10s</li> </ul>
		<ul style="list-style-type: none"> <li>› use the mathematical processes to investigate the relationship between addition and subtraction</li> </ul>	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>– generalise subtraction problems beyond recalled facts by looking for patterns</li> <li>– investigate patterns using choral counting, materials, the recording of multiples, and the relationships between skip counting and multiplication and division facts</li> </ul> </li> </ul>

<b>Year 3</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
	<i>Use the following when representing number properties:</i>
<ul style="list-style-type: none"> <li>› recall addition facts up to 20 and their corresponding subtraction facts (families of facts), including doubles and halves</li> </ul>	<ul style="list-style-type: none"> <li>› materials, including 10s frames and multilink cubes</li> <li>› games</li> </ul>
<ul style="list-style-type: none"> <li>› explore dividing a number by itself, and why we cannot divide by 0 (e.g., by trying to solve <math>0 \times \_ = 5</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› word problems</li> <li>› materials</li> </ul>
<ul style="list-style-type: none"> <li>› use the additive identity (e.g., <math>4 + 0 = 4</math> and <math>5 - 0 = 5</math>), multiplicative identity (e.g., <math>5 \times 1 = 5</math> and <math>4 \div 1 = 4</math>), and commutative property</li> </ul>	<ul style="list-style-type: none"> <li>› materials, including 10s frames and blocks</li> <li>› the concept of equality (years 1-2)</li> <li>› word problems and materials (year 3)</li> </ul>
<ul style="list-style-type: none"> <li>› recall multiplication and corresponding division facts for 2s, 3s, 5s, and 10s</li> </ul>	<ul style="list-style-type: none"> <li>› 100's boards patterns, choral counting, games and number lines (year 2)</li> <li>› games, families of facts, table grids (year 3)</li> </ul>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- generalise subtraction problems beyond recalled facts by looking for patterns</li> <li>- investigate patterns using choral counting, materials, the recording of multiples, and the relationships between skip counting and multiplication and division facts</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› materials, pictures, word problems, equations, families of facts (to show addition and subtraction, multiplication and division).</li> </ul>

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	<b>6 months</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 1</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 2</b> <i>Informed by formative assessment, teach students to:</i>
<b>Equations and relationships</b>		<ul style="list-style-type: none"> <li>› solve true or false number sentences and open number sentences involving addition and subtraction of 1-digit numbers, using an understanding of the equal sign (e.g., <math>9 - 6 = 8 - \underline{\quad}</math>; <math>7 - 5 = 6 - 4</math> (T or F?))</li> </ul>	<ul style="list-style-type: none"> <li>› solve true or false number sentences and open number sentences involving addition and subtraction of 1- and 2-digit numbers, using an understanding of the equal sign (e.g., <math>18 + \underline{\quad} = 17 + 6</math>; <math>17 = 25</math> (T or F?))</li> </ul>
	<ul style="list-style-type: none"> <li>› copy, continue, create, and describe a repeating pattern with two elements</li> </ul>	<ul style="list-style-type: none"> <li>› copy, continue, create, and describe a repeating pattern with three elements, and identify missing elements in a pattern</li> </ul>	<ul style="list-style-type: none"> <li>› recognise and describe the unit of repeat in a repeating pattern, and use it to predict further elements using the ordinal position</li> </ul>
	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- generalise when noticing that repeated patterns constructed in different ways are the same pattern (e.g., 'red, blue, red, blue' and 'hop, jump, hop, jump' are both ABAB patterns)</li> <li>- investigate repeating patterns in a range of contexts</li> <li>- explain and justify how a pattern is repeating</li> </ul> </li> </ul>		<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- generalise using the unit of repeat and ordinal position to identify further elements in a pattern</li> <li>- investigate repeating patterns in a range of contexts</li> <li>- explain and justify how a pattern is repeating or growing, and predict future terms in the pattern</li> </ul> </li> </ul>
<b>Algorithmic thinking</b>		<ul style="list-style-type: none"> <li>› sort objects into two groups, following a simple rule</li> </ul>	<ul style="list-style-type: none"> <li>› follow a set of instructions to sort numbers or objects according to a simple rule</li> </ul>
			<ul style="list-style-type: none"> <li>› give step-by-step instructions, and identify and correct errors as they are followed</li> </ul>
		<ul style="list-style-type: none"> <li>› use the mathematical processes to investigate appropriate situations.</li> </ul>	

<b>Year 3</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
	<b><i>Use the following when representing equations and relationships:</i></b>
<ul style="list-style-type: none"> <li>› solve true or false number sentences and open number sentences involving addition and subtraction, using an understanding of the equal sign</li> </ul>	<ul style="list-style-type: none"> <li>› balance scales and discrete materials (year 1)</li> <li>› 10s frames and discrete materials (year 2)</li> <li>› word problems with comparisons (year 3)</li> </ul>
<ul style="list-style-type: none"> <li>› recognise, continue, and create growing patterns, and describe a rule to explain a pattern</li> </ul>	<ul style="list-style-type: none"> <li>› a range of materials with attributes (e.g., size, colour, texture, shape, movement, sound)</li> <li>› tables (years 2–3)</li> <li>› discrete objects, counters, blocks (year 3)</li> </ul>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>– generalise using the unit of repeat and ordinal position to identify further elements in a pattern</li> <li>– investigate repeating patterns in a range of contexts</li> <li>– explain and justify how a pattern is repeating or growing, and predict future terms in the pattern</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› materials, words, symbols, movements, drawings, tables.</li> </ul>
	<b><i>Use the following when representing algorithmic thinking:</i></b>
<ul style="list-style-type: none"> <li>› follow, and create patterns from, rules or simple algorithms</li> </ul>	<ul style="list-style-type: none"> <li>› discrete objects (year 1)</li> <li>› drawing and describing a representation of instructions</li> <li>› direction cards to form a sequence</li> <li>› flow diagrams (years 2–3)</li> </ul>
<ul style="list-style-type: none"> <li>› formulate a familiar routine or basic task as a set of precise, step-by-step instructions (i.e., an algorithm)</li> </ul>	<ul style="list-style-type: none"> <li>› drawing and describing a representation of instructions</li> <li>› direction cards to form a sequence, sequence steps, flow diagrams</li> </ul>
<ul style="list-style-type: none"> <li>› use the mathematical processes to investigate appropriate situations.</li> </ul>	<ul style="list-style-type: none"> <li>› verbal instructions, symbols, drawings</li> <li>› direction cards.</li> </ul>

# Measurement

	<b>6 months</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 1</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 2</b> <i>Informed by formative assessment, teach students to:</i>
<b>Measuring</b>	› directly compare two objects by an attribute (e.g., length, mass (weight), capacity)	› compare the length, mass (weight), temperature, volume, and capacity of objects directly and indirectly (e.g., by comparing each of them with another object and using the object repeatedly)	› estimate and use an informal unit repeatedly to measure the length, mass (weight), volume, or capacity of an object
			› compare and order several objects using informal units of length, mass (weight), volume, or capacity
			› turn, and describe how far an object or person has turned, using half and quarter turns as benchmarks
	› use the mathematical processes to: <ul style="list-style-type: none"> <li>- investigate ways to directly and indirectly compare</li> <li>- explain and justify, using the language of comparison (more, less, longer, shorter, heavier, lighter)</li> </ul>		› use the mathematical processes to: <ul style="list-style-type: none"> <li>- connect to ordering and comparing numbers</li> <li>- investigate a range of practical measurement situations, including ways of measuring by different cultures</li> <li>- explain and justify, using the same informal unit when measuring</li> </ul>

<b>Year 3</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
	<i>Use the following when representing measuring:</i>
<ul style="list-style-type: none"> <li>› estimate and then reliably measure length, capacity, and mass (weight), using metric units (e.g., from tools with labelled markings)</li> </ul>	<ul style="list-style-type: none"> <li>› physical comparisons (years 0-1)</li> <li>› balance scales, a range of capacity containers (years 1-2)</li> <li>› rulers, measuring jugs and containers, scales (year 3)</li> </ul>
<ul style="list-style-type: none"> <li>› compare and order objects using metric units of length, mass (weight), or capacity</li> </ul>	<ul style="list-style-type: none"> <li>› identical units (e.g., blocks, hands, paper clips, cups (year 2)</li> <li>› metric units on appropriate tools (year 3)</li> </ul>
<ul style="list-style-type: none"> <li>› turn, and describe how far an object or person has turned, using half, quarter, and three-quarter turns as benchmarks</li> </ul>	<ul style="list-style-type: none"> <li>› physical objects</li> <li>› themselves</li> </ul>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect to base 10 place value, ordering and comparing numbers</li> <li>- investigate a range of practical measurement situations, including ways of measuring by different cultures</li> <li>- explain and justify, using the labelled markings on tools</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› physical objects, progressing from informal units to appropriate metric tools.</li> </ul>

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	<b>6 months</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 1</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 2</b> <i>Informed by formative assessment, teach students to:</i>
<b>Perimeter, area, and volume</b>			› visualise, estimate, and measure the perimeter and area of 2D shapes, using informal units
			› use the mathematical processes to: <ul style="list-style-type: none"> <li>- connect with groupings, addition, and known multiplication facts</li> <li>- investigate practical familiar contexts</li> <li>- explain and justify the importance of using the same unit when measuring</li> </ul>
<b>Time</b>	› connect days of the week to familiar events and daily routines (e.g., the class timetable)	› identify how the passing of time is measured in years, months, weeks, days, hours, minutes, and seconds › name and order the days of the week, and sequence events in a day using everyday language of time	› name and order the months and seasons › describe duration using months, weeks, days, and hours › use a calendar to identify the date and to determine the number of days in each month
		› tell the time to the hour using the language of 'o'clock'	› tell the time to the hour and half-hour, using the language of 'past' and 'o'clock'
	› use the mathematical processes to: <ul style="list-style-type: none"> <li>- connect daily routines and familiar events to days of the week and months of the year</li> <li>- investigate a calendar (its days, weeks, and months) and how long it takes to do tasks (i.e., duration).</li> </ul>		› use the mathematical processes to: <ul style="list-style-type: none"> <li>- connect half past, quarter to, and quarter past to fractions; and daily routines and familiar events to days of the week and months of the year</li> <li>- investigate calendars (their days, weeks, and months).</li> </ul>

<b>Year 3</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
	<i>Use the following when representing perimeter, area, and volume:</i>
<ul style="list-style-type: none"> <li>› visualise, estimate, and measure:               <ul style="list-style-type: none"> <li>– the perimeter of polygons using metric units</li> <li>– the area of 2D shapes covered with squares of identical size</li> <li>– the volume of rectangular prisms (cuboids) by filling them with identical units</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› 2D shapes as a unit of measure for perimeter (e.g., ice block sticks) (year 2)</li> <li>› informal units for measuring area (e.g., blocks, squares, sticky notes)</li> <li>› square grids and rulers (year 3)</li> </ul>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>– connect with groupings, addition, and known multiplication facts</li> <li>– investigate practical familiar contexts</li> <li>– explain and justify the importance of using the same unit when measuring</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› materials, square grids, blocks, rulers.</li> </ul>
	<i>Use the following when representing time:</i>
<ul style="list-style-type: none"> <li>› use a calendar to work out the number of days, weeks, or months until important events</li> </ul>	<ul style="list-style-type: none"> <li>› pictorial timetables (years 0-1)</li> <li>› cards for ordering days</li> <li>› calendars</li> <li>› the classroom daily timetable</li> </ul>
<ul style="list-style-type: none"> <li>› tell the time to the hour, half hour, and quarter past and quarter to the hour</li> </ul>	<ul style="list-style-type: none"> <li>› analogue and digital clocks</li> </ul>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>– connect half past, quarter to, and quarter past to fractions; and daily routines and familiar events to days of the week and months of the year</li> <li>– investigate calendars (their days, weeks, and months).</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› timetables</li> <li>› analogue and digital clocks</li> <li>› a calendar.</li> </ul>

# Geometry

	<b>6 months</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 1</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 2</b> <i>Informed by formative assessment, teach students to:</i>
<b>Shapes</b>	<ul style="list-style-type: none"> <li>› identify, sort by one feature, and describe familiar 2D shapes</li> </ul>	<ul style="list-style-type: none"> <li>› identify, describe, and classify familiar 2D and 3D shapes presented in different orientations, including triangles, circles, rectangles (including squares), cubes, cylinders, and spheres</li> </ul>	<ul style="list-style-type: none"> <li>› identify, describe, and classify the properties of 2D and 3D shapes including ovals, semicircles, polygons (e.g., hexagons, pentagons), rectangular prisms (cuboids), pyramids, hemispheres, and cones, using the properties of shapes</li> </ul>
	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect 2D and 3D shapes in the environment</li> <li>- investigate ways of sorting 2D and 3D shapes into groups</li> <li>- explain, justify, and compare how shapes have been grouped</li> </ul> </li> </ul>		<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect right angles to square corners in shapes and objects</li> <li>- investigate properties of 2D and 3D shapes, including lines of symmetry</li> <li>- explain and justify the classification of shapes into groups based on their properties</li> </ul> </li> </ul>
<b>Spatial reasoning</b>	<ul style="list-style-type: none"> <li>› compose by trial and error an outlined target shape using smaller shapes, and decompose a shape into smaller shapes</li> </ul>	<ul style="list-style-type: none"> <li>› anticipate which smaller shapes might be used to compose a target shape, and then check by making the shape</li> </ul>	<ul style="list-style-type: none"> <li>› anticipate which smaller shapes might be used to compose and decompose a target shape, and then check by making the shape</li> </ul>
		<ul style="list-style-type: none"> <li>› slide, flip, and turn 2D shapes to make a pattern</li> </ul>	<ul style="list-style-type: none"> <li>› recognise lines of symmetry in patterns or pictures, and create or complete symmetrical pictures or patterns</li> </ul>

<b>Year 3</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
	<i>Use the following when representing shapes:</i>
<ul style="list-style-type: none"> <li>› visualise, identify, compare, and classify 2D and 3D shapes using the properties of shapes including lines of symmetry</li> </ul>	<ul style="list-style-type: none"> <li>› a range of 2D and 3D shapes</li> <li>› tactile materials</li> <li>› digital tools</li> </ul>
<ul style="list-style-type: none"> <li>› identify right angles in shapes and objects</li> </ul>	<ul style="list-style-type: none"> <li>› 2D and 3D shapes</li> <li>› objects in the environment</li> </ul>
<ul style="list-style-type: none"> <li>› the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect right angles to square corners in shapes and objects</li> <li>- investigate properties of 2D and 3D shapes, including lines of symmetry</li> <li>- explain and justify the classification of shapes into groups based on their properties</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› materials</li> <li>› digital tools</li> <li>› vocabulary.</li> </ul>
	<i>Use the following when representing spatial reasoning:</i>
<ul style="list-style-type: none"> <li>› compose and decompose 2D shapes using the properties of shapes (e.g., lines of symmetry), other shapes, side lengths, and angles</li> </ul>	<ul style="list-style-type: none"> <li>› pattern blocks, attribute shapes, paper shapes, playdough, tangrams</li> <li>› discrete target shapes (year 0)</li> <li>› continuous target shapes (years 1-3)</li> </ul>
<ul style="list-style-type: none"> <li>› predict the result of a one-step transformation on 2D shapes</li> </ul>	<ul style="list-style-type: none"> <li>› 2D shapes, paper folding</li> <li>› mirrors</li> <li>› symmetrical object outlines (discrete and continuous)</li> <li>› painting, art-related tasks</li> <li>› predicting the result of a transformation</li> </ul>

	<b>6 months</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 1</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 2</b> <i>Informed by formative assessment, teach students to:</i>
<b>Spatial reasoning</b>	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- investigate how shapes can be flipped and turned to make patterns</li> <li>- explain and justify how new shapes can be created, using the names and properties of the shapes and spatial vocabulary</li> </ul> </li> </ul>		<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect quarter, half, and three-quarter turns to fractions</li> <li>- generalise about 2D shapes (e.g., how they can be partitioned into smaller shapes, and how, when orientated in different directions (flip, turn), their properties do not change)</li> <li>- investigate transformation (flip, slide, turn) and lines of symmetry in pictures, patterns, and the environment</li> <li>- explain and justify how shapes have been used to create new shapes</li> </ul> </li> </ul>
<b>Pathways</b>	<ul style="list-style-type: none"> <li>› follow instructions to move to a familiar location or locate an object</li> </ul>	<ul style="list-style-type: none"> <li>› follow and give instructions to move to a familiar location or locate an object</li> </ul>	<ul style="list-style-type: none"> <li>› follow and give instructions to move people or objects to a different location, using direction, distances (e.g., number of steps), and half and quarter turns</li> </ul>
		<ul style="list-style-type: none"> <li>› use pictures, diagrams, or stories to describe the positions of objects and places</li> </ul>	<ul style="list-style-type: none"> <li>› interpret diagrams to describe the positions of objects and places in relation to other objects and places</li> </ul>
	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- investigate ways of moving to different locations by following verbal instructions and simple diagrams and maps.</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect half and quarter turns with fractions</li> <li>- investigate ways of moving to different locations by following verbal instructions and simple diagrams and maps.</li> </ul> </li> </ul>	

<b>Year 3</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect quarter, half, and three-quarter turns to fractions</li> <li>- generalise about 2D shapes (e.g., how they can be partitioned into smaller shapes, and how, when orientated in different directions (flip, turn), their properties do not change)</li> <li>- investigate transformation (flip, slide, turn) and lines of symmetry in pictures, patterns, and the environment</li> <li>- explain and justify how shapes have been used to create new shapes</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› 2D shapes.</li> </ul>
	<b><i>Use the following when representing pathways:</i></b>
<ul style="list-style-type: none"> <li>› follow and create a sequence of step-by-step instructions (an algorithm) for moving people or objects to a different location</li> </ul>	<ul style="list-style-type: none"> <li>› familiar locations</li> <li>› speaking frames for simple instructions (e.g., sentence starters)</li> <li>› spatial language (to support following and giving instructions)</li> <li>› directions, distance, turns (years 2–3)</li> </ul>
<ul style="list-style-type: none"> <li>› interpret, draw, and use simple maps to locate objects and places relative to other objects and places</li> </ul>	<ul style="list-style-type: none"> <li>› story books, familiar locations</li> <li>› spatial language (to support following and giving instructions)</li> <li>› simple maps</li> <li>› modelling how to draw physical representation of a simple map (year 3)</li> </ul>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect quarter, half, and three-quarter turns to fractions</li> <li>- investigate ways of moving to different locations by following verbal instructions and simple diagrams and maps.</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› spatial vocabulary</li> <li>› symbols</li> <li>› simple maps.</li> </ul>

# Statistics

	<b>6 months</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 1</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 2</b> <i>Informed by formative assessment, teach students to:</i>
<b>Problem</b>		<ul style="list-style-type: none"> <li>› pose summary investigative questions that classify objects or individuals into groups or categories (e.g., colour, brand), and anticipate what the data might show</li> </ul>	<ul style="list-style-type: none"> <li>› pose summary investigative questions about a group for which the data will have categorical variables, and anticipate what the data might show (e.g., which outcomes might be more frequent than others)</li> </ul>
		<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- pose summary investigative questions about a group and for which the data will have categorical variables</li> <li>- investigate an area of interest and things students are curious about</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- pose an investigative question with support</li> <li>- investigate an area of interest and things students are curious about</li> </ul> </li> </ul>
<b>Plan</b>		<ul style="list-style-type: none"> <li>› collect data for one variable by making observations or questioning others, and discuss how the data-gathering process might affect other people</li> </ul>	<ul style="list-style-type: none"> <li>› use survey and data-collection questions to collect data, identify who and what the data measures, and discuss how the data-gathering process might affect other people</li> </ul>
		<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- plan ways of collecting data and survey questions, with support</li> <li>- investigate different survey questions and how they can be interpreted by others</li> </ul> </li> </ul>	
<b>Data</b>		<ul style="list-style-type: none"> <li>› collect categorical data for one variable</li> </ul>	<ul style="list-style-type: none"> <li>› collect categorical data for more than one variable</li> </ul>
		<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- collect data using data cards, recording, and tally sheets</li> <li>- investigate different ways of collecting data</li> </ul> </li> </ul>	

<b>Year 3</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<ul style="list-style-type: none"> <li>› pose summary investigative questions about everyday situations, using categorical data and discrete numerical (whole number) data, including about identifying the variable and the group of interest, and anticipate what the data might show</li> </ul>	Support students to pose questions about an area of interest.
<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- pose an investigative question with support</li> <li>- investigate an area of interest and things students are curious about</li> </ul> </li> </ul>	
<ul style="list-style-type: none"> <li>› use survey and data-collection questions to collect data, identify who and what the data measures, and discuss how the data-gathering process might affect other people</li> </ul>	Demonstrate data collection methods.
<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- plan ways of collecting data and survey questions, with support</li> <li>- investigate different survey questions and how they can be interpreted by others</li> </ul> </li> </ul>	
<ul style="list-style-type: none"> <li>› collect, record, and sort data, or use secondary data sources provided by someone else</li> </ul>	Represent data using data cards, recording sheets, and tally sheets.
<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- collect data using data cards, recording, and tally sheets</li> <li>- investigate different ways of collecting data</li> </ul> </li> </ul>	

	<b>6 months</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 1</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 2</b> <i>Informed by formative assessment, teach students to:</i>
<b>Analysis</b>		<ul style="list-style-type: none"> <li>› create and describe data visualisations (e.g., picture graphs, physical dot plots) for categorical data, giving the frequency for each category</li> </ul>	<ul style="list-style-type: none"> <li>› create and describe data visualisations (e.g., picture graphs, dot plots) for categorical data, comparing the frequencies of categories</li> </ul>
		<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- investigate how different representations (e.g., a picture graph and dot plot) show the same information</li> <li>- explain and justify what a graph shows using 'I notice ...' statements</li> </ul> </li> </ul>	
<b>Conclusion</b>		<ul style="list-style-type: none"> <li>› choose statements that best answer the investigative question</li> </ul>	
		<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- connect descriptions with data visualisations and analysis questions with features of the visualisations</li> <li>- investigate ways of reflecting on findings to determine if they make sense with what they already know</li> <li>- explain why some statements answer the investigative question and some do not</li> </ul> </li> </ul>	
<b>Statistical literacy</b>		<ul style="list-style-type: none"> <li>› agree or disagree with others' statements about simple data visualisations (e.g., pictographs, physical dot plots).</li> </ul>	<ul style="list-style-type: none"> <li>› match statements made by others with features in simple data visualisations, and agree or disagree with the statements.</li> </ul>

<b>Year 3</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<ul style="list-style-type: none"> <li>› create and describe data visualisations (e.g., picture graphs, dot plots, bar graphs) for categorical and discrete numerical data</li> </ul>	<p>Demonstrate creating data visualisations and describing what a graph shows.            Use data cards, picture graphs, frequency tables, and dot plots (years 1-2).            Use frequency tables, pictographs, and bar graphs (year 3).            Use 'I notice' statements.</p>
<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- represent data using data cards, frequency tables, picture graphs, pictographs, dot plots, and bar graphs</li> <li>- investigate how different representations (e.g., a picture graph and dot plot) show the same information</li> <li>- explain and justify what a graph shows using 'I notice ...' statements</li> </ul> </li> </ul>	
<ul style="list-style-type: none"> <li>› choose statements that best answer the investigative question, reflect on findings, and compare them with anticipated outcomes</li> </ul>	<p>Demonstrate making statements about data visualisations.            Show the structure of a comparative statement from data.</p>
<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- connect descriptions with data visualisations and analysis questions with features of the visualisations</li> <li>- investigate ways of reflecting on findings to determine if they make sense with what they already know</li> <li>- explain why some statements answer the investigative question and some do not</li> </ul> </li> </ul>	
<ul style="list-style-type: none"> <li>› identify relevant features in others' data visualisations, connect these to descriptive statements, agree or disagree with the statements, and suggest improvements</li> </ul>	<p>Demonstrate making claims about data visualisations, including giving reasons for agreeing or disagreeing with a claim.</p>
<ul style="list-style-type: none"> <li>› use the statistical processes to explain and justify, using agree-with and disagree-with descriptive statements, and suggest ways to improve.</li> </ul>	

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# Probability

	<b>6 months</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 1</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 2</b> <i>Informed by formative assessment, teach students to:</i>
<b>Probability investigations</b>		<ul style="list-style-type: none"> <li>› engage in stories or games that involve chance-based situations and:               <ul style="list-style-type: none"> <li>- decide if something will happen, won't happen, or might happen</li> <li>- identify possible and impossible outcomes (e.g., what might happen next)</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› engage in chance-based investigations about games and everyday situations to:               <ul style="list-style-type: none"> <li>- identify possible outcomes</li> <li>- collect and record data</li> <li>- create visualisations for frequencies of outcomes (e.g., lists, picture, graphs)</li> <li>- describe what these data visualisations show</li> <li>- answer chance-based investigative questions</li> <li>- notice variations in outcomes (e.g., how often each of the numbers on a dice come up)</li> </ul> </li> </ul>
<b>Critical thinking in probability</b>			<ul style="list-style-type: none"> <li>› agree or disagree with the statements made by others about chance situations</li> </ul>
		<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- connect relative frequency in words (e.g., two out of three) to fractions (e.g., <math>\frac{2}{3}</math>)</li> <li>- investigate games of chance and list possible outcomes</li> <li>- use the statistical enquiry cycle (PPDAC) for chance-based investigations</li> <li>- explain, justify, and use the language of probability (impossible, unlikely, possible, likely, certain) and its ordering from impossible to certain.</li> </ul> </li> </ul>	

<b>Year 3</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<ul style="list-style-type: none"> <li>› engage in chance-based investigations about games and everyday situations to:               <ul style="list-style-type: none"> <li>- anticipate what might happen</li> <li>- identify possible outcomes</li> <li>- collect and record data</li> <li>- create data visualisations for frequencies of possible outcomes</li> <li>- describe what these visualisations show</li> <li>- answer investigative questions</li> <li>- reflect on anticipated outcomes</li> <li>- notice variations in outcomes</li> </ul> </li> </ul>	<p>Play games of chance using physical objects (e.g., dice, coins, spinners, pulling things out of a hat).</p> <p>Create tables for frequencies, lists, and outcomes.</p>
<ul style="list-style-type: none"> <li>› explain and question statements about chance-based situations, with reference to data</li> </ul>	<p>Demonstrate making claims about data visualisations, including giving reasons for agreeing or disagreeing with a claim.</p>
<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- connect relative frequency in words (e.g., two out of three) to fractions (e.g., <math>\frac{2}{3}</math>)</li> <li>- investigate games of chance and list possible outcomes</li> <li>- use the statistical enquiry cycle (PPDAC) for chance-based investigations</li> <li>- explain, justify, and use the language of probability (impossible, unlikely, possible, likely, certain) and its ordering from impossible to certain.</li> </ul> </li> </ul>	<p>Represent probability using materials, dice, coins, spinners, and items in a bag.</p> <p>Represent outcomes using drawings, tally charts, lists, and tables.</p>

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# Phase

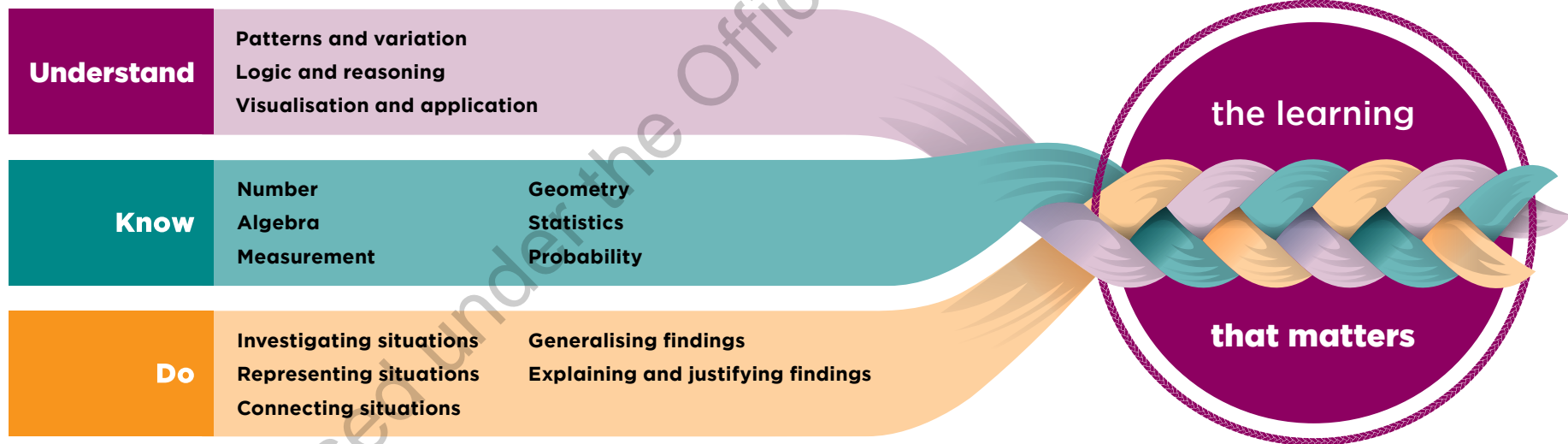
# 2

Years 4-6

*Expanding horizons of knowledge and collaborating  
Te whakawhānui i ngā pae o te mātauranga, me te mahi tahi*

## Progress outcome by the end of year 6

In phase 2, students weave together Understand, Know, and Do and use a variety of visual representations to model number operations and solve word problems. They connect and extend their reasoning about whole numbers to fractions and decimals, and they visualise and classify angles, using benchmarks to justify their classifications. They also apply their understanding of number operations to reason about perimeter and area and apply comparative reasoning to investigate variations in shapes and data.



## Understand Big ideas

As they develop their maths capabilities and knowledge, students build on their understanding that the world is full of **patterns** and of relationships in which **variation** occurs. They use the structures of mathematics and statistics to support noticing, exploring, and describing different types of patterns and relationships, enabling insights, generalisations, and predictions. By engaging with maths concepts, students develop **logical** reasoning and critical thinking skills that enable them to evaluate information, question assumptions, and present reasoning with clarity.

Statistical **reasoning** from observation allows students to explore what is probable and to draw reasonable conclusions. They continue to develop the ability to **visualise** maths ideas in order to understand and interact with abstract concepts. They **apply** maths ideas to help understand familiar situations and to support decision making and the communication of ideas.

## Know Content and concepts

### Number | Mātauranga tau

Students know that in our **number system** each place value is a power of 10, and this continues infinitely. To the right, the system continues beyond ones, to create decimals (tenths, hundredths, thousandths); the decimal point is placed between the ones column and the tenths column. **Estimation and rounding** support checking the reasonableness of solutions of operations involving whole numbers, fractions, and decimals.

Students know that to **calculate expressions** that have more than one operation, operations inside brackets (grouped together) are done first, then powers or exponents. If there are multiplication and division, these are done in left-to-right order; addition and subtraction are also done in left-to-right order. Students use the mnemonic GEMA: grouped, exponents, multiplication, addition. Division can be partitive (the number of shares is known) or quotative (the size of the shares is known).

Students also know that **fractions** can represent one number divided by another, operate on quantities, and be larger than 1. Improper fractions can also be written as a mixed number represented as a whole number and a fraction, combined with a hidden addition. In simplified fractions, the numerator and denominator have no common factors; if the denominator of a simplified fraction is 1, then it can be written as a whole number. **Decimals** are fractions that have powers of 10 as their denominators and that can be written as numbers using a decimal point. A **percentage** is a fraction with a denominator of 100.

### Algebra | Taurangi

Students know the **properties** of operations: the commutative and associative properties only work for addition and multiplication (not for subtraction and division), the associative property applies to addition

and multiplication, and the distributive property applies to multiplication over addition and subtraction. Students know that the equal ( $=$ ) and inequality ( $<$ ,  $>$ ) signs show relationships and that applying the same operation to both sides of an equation preserves the balance of the equation.

Students know that in a **pattern**, the relationship between the **ordinal position** and the corresponding element can be used for finding the pattern rule. Any element can be found by knowing the position, and any position can be found from the element. Tables and XY graphs provide a way of organising the positions and elements of a pattern to reveal relationships or rules. An **algorithm** is an ordered list of instructions for solving a problem.

### Measurement | Ine

Students know that, like our place-value number system, the **metric measurement system** is based on powers of 10 and that appropriate metric units are used to quantify length, area, volume, capacity, mass (weight), temperature, and duration. Measurements can include whole units and parts of units. Different **measurement tools and scales** use different-sized units, and the unit must be recorded with the amount. If a measurement starts at 0, the point on the scale where it ends tells us the measurement. Angles measure parts of a full turn and use the unit of degrees.

### Geometry | Āhuahanga

Students know that two- and three-dimensional shapes have consistent **properties** that can be used to define, compare, classify, predict, and identify relationships between shapes. Shapes can be **transformed** by rotation, reflection, translation, and resizing (when they are enlarged or reduced). Lines of symmetry can

be horizontal, vertical, and diagonal. **Three-dimensional shapes** can be composed of connected two- or three-dimensional shapes. Students also know that **position** can be described using known environmental features and signs from the natural world. Maps can use grid references to specify the position of locations, scales to show distances, and connections to show pathways.

### Statistics | Tauanga

Students know that data about people and the natural world must be collected, used, and stored carefully. The **statistical enquiry cycle** (PPDAC) can be used in **summary, comparison, and time-series investigations**. A comparison investigation compares similarities and differences for a variable across two or more groups, and a time-series investigation considers a variable that changes over time. **Numerical variables** can be counted or measured; discrete numerical variables are counted, continuous numerical variables are measured. A **prediction** or assertion involves thinking about what data will show before it is collected or analysed. Data is not always accurately recorded; it needs to be checked for errors and may need correcting. Alternative **data visualisations** for the same data can lead to different insights.

### Probability | Tūponotanga

Students know that the statistical enquiry cycle (PPDAC) can be used for **chance-based investigations**. Probabilities and the language of probability are associated with values between 0 or 0% (impossible) and 1 or 100% (certain), can be used to describe situations that involve **uncertainty**, and help make decisions. The **probability of an outcome** in a chance-based investigation is the number of times the outcome occurs divided by the total number of outcomes, where all possible outcomes can be listed (theoretical probability), or the relative frequency of the outcome from a probability experiment (estimated probability).

## Do Practices

### Investigating situations | Te tūhura pūāhua

By the end of this phase, students can pose a question for investigation, find entry points for addressing a question, plan an investigation pathway, and follow it step by step. They can identify relevant prior knowledge, givens, and relationships to support the investigation. They can monitor and evaluate progress, adjusting the investigation pathway if necessary, and make sense of outcomes or conclusions in light of a given situation and context.

### Representing situations | Te whakaata pūāhua

Students can use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns and variations. They use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments. They select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations. They use visualisations to mentally represent and manipulate objects and ideas.

### Connecting situations | Te tūhono pūāhua

Students can suggest connections between ideas, approaches, and different representations. They connect new ideas to things they already know. They make connections with ideas in other learning areas and with familiar cultural, linguistic, and historical contexts.

### Generalising findings | Te whakatauwānui i ngā kitenga

Students can recognise and explore patterns, and make conjectures and draw conclusions about them. They can identify relationships, including similarities, differences, and new connections. They look for patterns and regularities that might be applied in another situation or always be true. They make and test conjectures, using reasoning and counterexamples to decide if they are true or not. They use appropriate symbols to express generalisations.

### Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

Students can make statements, give explanations inductively based on observations or data, and make deductions based on knowledge, definitions, and rules. They critically reflect on others' thinking, evaluating their logic and asking questions to clarify and understand. They use evidence, reasoning, and proofs to explain why they agree or disagree with statements. They develop collective understandings by sharing and building on ideas with others, and they present reasoned explanations and arguments for an idea, solution, or process.

## Teaching sequence

*Expanding horizons of knowledge, and collaborating*


*Te whakawhānui i ngā pae o te mātauranga, me te mahi tahi*

Throughout phase 2, encourage students to see themselves as capable, confident, and competent maths thinkers whose ideas are valued, who treat mistakes as part of the learning process, and whose capabilities in maths can develop over time with consistent effort. Confidence is built through experiencing success and developing competence and understanding. Over phase 2, students collaborate with others to expand their knowledge and understanding. Support this by working with the class to establish expectations and responsibilities when it comes to working together as peers, sharing thinking, and agreeing or disagreeing about maths learning.

Continuously monitor students' reasoning, questions, and use of representations, and act quickly to respond to any misconceptions. Ensure your teaching builds on what students already understand, know, and can do.

Throughout each day, provide opportunities for students to:

- › **develop positive relationships with maths** – explore rich mathematical situations and maths tasks that are useful and meaningful to the class or community
- › **actively listen to, reflect, and build on each other's thinking and learning** – use discourse-based tools and a range of open questions to facilitate productive discussions. Over the phase, encourage students to use evidence and examples to justify their claims and findings
- › **experience explicit teaching of concepts and practices** – use worked examples and break down new learning into clearly explained manageable steps. Use and explain maths symbols and notation conventions, and how they work. Teach efficient written methods, including column methods for operations

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- › **select appropriate representations to share their working and reasoning**
    - over the phase, move students towards using pictures, diagrams, and mathematical notation such as equations and inequalities. Teach students which representations are most effective for visualising different types of information (e.g., number lines are important for visualising operations, differences, the comparative size of numbers, and rounding conventions)
  - › **actively learn maths vocabulary** – help students to connect the correct vocabulary to the learning purpose and problem. Ask them to use the correct vocabulary when explaining their findings and reasoning, and draw on their first and heritage languages
  - › **connect maths learning within and across contexts** – teach connected skills and concepts together (e.g., multiplication and division with area and volume). Make connections obvious and explicit by highlighting skills or concepts students have applied in other learning areas
  - › **visualise quantities, patterns, shapes, measurements, and space** – support students to visual by estimating the number of items in a group, using rounding or known benchmarks to make estimations, and noticing how a shape has been rotated, reflected, or is composed of other shapes
  - › **consolidate maths learning to develop fluency with time-sensitive statements in the teaching sequence** – use warm-up routines as a form of active recall that connects back to prior learning (e.g., a quick challenge, curly question, or game). Plan for students to develop fluency through practice, using a range of approaches
  - › **record learning in their maths book** – encourage students to record vocabulary, problem solving, summaries, goals, and reflections on their maths learning. This could include words, mathematical notation, and a range of representations.

The highlighted cells in the teaching sequence show critical steps that alert teachers to specific aspects of learning that are essential and time-sensitive. They support teachers to notice, recognise, and respond to student learning in a timely way.

## Number

	<b>Year 4</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 5</b> <i>Informed by formative assessment, teach students to:</i>
<b>Number structure</b>	<ul style="list-style-type: none"> <li>› count to and within 1,000, from any multiple of 100, forwards and backwards in 25s and 50s</li> </ul>	
	› recognise the base ten structure of numbers up to 10,000	› recognise the base ten structure of numbers up to 100,000
	› identify, read, write, compare, and order whole numbers up to 10,000	› identify, read, write, compare, and order whole numbers up to 100,000
		› identify factors of numbers up to 100
	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect with metric units that are powers of 10, and with decimal place value</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect with metric units that are powers of 10, and with decimal place value</li> <li>- investigate factors and multiples</li> </ul> </li> </ul>
<b>Operations</b>	› use rounding and estimation to predict and to check the reasonableness of calculations	› use rounding and estimation to predict and to check the reasonableness of calculations
	› round whole numbers to the nearest thousand, hundred, or ten, and round tenths to the nearest whole number	› round whole numbers to a specified power of 10, and round tenths and hundredths to the nearest whole number
	› add and subtract 2- and 3-digit numbers	› add and subtract whole numbers up to 10,000
	› multiply a 2-digit by 1-digit number and two 1-digit whole numbers (e.g., $5 \times 46$ ; $8 \times 7$ )	› multiply a 3-digit by 1-digit number and two 2-digit whole numbers (e.g., $6 \times 248$ ; $37 \times 84$ )

<b>Year 6</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
	<i>Use the following when representing number structure:</i>
	<ul style="list-style-type: none"> <li>› patterns in a 100s board and 1,000s book</li> <li>› choral counting, captured and recorded on the board</li> </ul>
<ul style="list-style-type: none"> <li>› recognise the base ten structure of numbers up to 1,000,000</li> </ul>	<ul style="list-style-type: none"> <li>› place-value (PV) houses and materials</li> </ul>
<ul style="list-style-type: none"> <li>› identify, read, write, compare, and order whole numbers up to 1,000,000</li> </ul>	<ul style="list-style-type: none"> <li>› marked number lines, PV houses, materials</li> </ul>
<ul style="list-style-type: none"> <li>› identify square numbers and factors of numbers up to 125</li> </ul>	<ul style="list-style-type: none"> <li>› arrays, lists of factor pairs, multiplication charts</li> </ul>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect with metric units that are powers of 10, and with decimal place value</li> <li>- investigate factors and square numbers</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› verbal and written numbers</li> <li>› PV materials, number lines.</li> </ul>
	<i>Use the following when representing operations:</i>
<ul style="list-style-type: none"> <li>› use rounding and estimation to predict and to check the reasonableness of calculations</li> </ul>	<ul style="list-style-type: none"> <li>› known facts</li> <li>› benchmarks</li> </ul>
<ul style="list-style-type: none"> <li>› round whole numbers to a specified multiple of powers of 10, and round tenths and hundredths to the nearest whole number or one decimal place</li> </ul>	<ul style="list-style-type: none"> <li>› number lines</li> </ul>
<ul style="list-style-type: none"> <li>› add and subtract any whole numbers</li> </ul>	<ul style="list-style-type: none"> <li>› materials</li> <li>› horizontal and vertical methods</li> </ul>
<ul style="list-style-type: none"> <li>› multiply multi-digit whole numbers (e.g., <math>54 \times 112</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› jumps on a number line (year 4)</li> <li>› area-model and vertical methods</li> </ul>

	<b>Year 4</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 5</b> <i>Informed by formative assessment, teach students to:</i>
<b>Operations</b>	› divide whole numbers by a 1-digit divisor, with no remainders (e.g., $65 \div 5$ )	› divide whole numbers by a 1-digit divisor, with a remainder (e.g., $83 \div 5 = 16$ , remainder 3)
	› use the mathematical processes to: <ul style="list-style-type: none"> <li>- connect multiplication, division, and factors with area, volume, and perimeter</li> <li>- connect multiplication and division with proportional reasoning</li> <li>- generalise the use of inverse operations and the commutative and distributive properties, to check findings</li> <li>- investigate comprehending and solving word problems, deciding which operation to use and why</li> <li>- explain and justify findings, by connecting to estimates and other checking methods</li> </ul>	
<b>Rational numbers</b>	› identify, read, write, and represent tenths as fractions and decimals	› identify, read, write, and represent tenths and hundredths as fractions and decimals
	› compare and order tenths as fractions and decimals, and convert decimals to fractions	› compare and order tenths and hundredths as fractions and decimals, and convert decimals to fractions
	› divide whole numbers by 10 to make decimals	› divide whole numbers by 10 and 100 to make decimals and whole numbers
	› for fractions with related denominators of 2, 4, and 8, 3 and 6, or 5 and 10: <ul style="list-style-type: none"> <li>- compare and order the fractions</li> <li>- identify when two fractions are equivalent</li> <li>- represent the fractions in their simplest form</li> </ul>	› for fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100: <ul style="list-style-type: none"> <li>- compare and order the fractions</li> <li>- identify when two fractions are equivalent</li> <li>- represent the fractions in their simplest form</li> </ul>
	› convert, using number lines, between improper fractions and mixed numbers for fractions with denominators of 2, 3, 4, 5, 6, and 10	› convert between improper fractions and mixed numbers for fractions with denominators up to 10

<b>Year 6</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<ul style="list-style-type: none"> <li>› divide whole numbers by a 1-digit divisor, with a remainder (e.g., <math>198 \div 7</math>; <math>4154 \div 8</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› diagrams, known facts (year 4)</li> <li>› jumps on a number line</li> <li>› vertical method</li> </ul>
<ul style="list-style-type: none"> <li>› use the order of operations rule GEMA with grouping, addition, subtraction, multiplication, and division</li> </ul>	<ul style="list-style-type: none"> <li>› step-by-step layouts with only one equal sign per line</li> </ul>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>– connect multiplication, division, and factors with area, volume, and perimeter</li> <li>– connect multiplication and division with proportional reasoning</li> <li>– generalise the use of inverse operations and the commutative and distributive properties, to check findings</li> <li>– investigate comprehending and solving word problems, deciding which operation to use and why</li> <li>– explain and justify findings, by connecting to estimates and other checking methods</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› diagrams</li> <li>› number lines</li> <li>› area models</li> <li>› vertical and horizontal methods.</li> </ul>
<b><i>Use the following when representing rational numbers:</i></b>	
<ul style="list-style-type: none"> <li>› identify, read, write, and represent fractions, decimals (to two places), and percentages</li> </ul>	<ul style="list-style-type: none"> <li>› equivalence materials, number lines, decimal PV houses</li> </ul>
<ul style="list-style-type: none"> <li>› compare and order fractions, decimals (to two places), and percentages and convert decimals, and percentages to fractions</li> </ul>	<ul style="list-style-type: none"> <li>› double number lines</li> <li>› blank 100s boards</li> </ul>
<ul style="list-style-type: none"> <li>› multiply and divide numbers by 10 and 100 to make decimals and whole numbers</li> </ul>	<ul style="list-style-type: none"> <li>› PV houses</li> </ul>
<ul style="list-style-type: none"> <li>› for fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:               <ul style="list-style-type: none"> <li>– compare and order the fractions</li> <li>– identify when two fractions are equivalent</li> <li>– represent the fractions in their simplest form</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› fraction walls (equivalence materials) and benchmarks, to show the relationship between two denominators for simplifying</li> <li>› number lines, for comparing and ordering</li> </ul>
<ul style="list-style-type: none"> <li>› convert between improper fractions and mixed numbers</li> </ul>	<ul style="list-style-type: none"> <li>› fraction walls and tiles</li> <li>› number lines</li> </ul>

	<b>Year 4</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 5</b> <i>Informed by formative assessment, teach students to:</i>
<b>Rational numbers</b>	› find a unit fraction of a whole number, using multiplication or division facts and where the answer is a whole number (e.g., $\frac{1}{5}$ of 40)	› find a fraction of a whole number, using multiplication and division facts and where the answer is a whole number (e.g., $\frac{2}{3}$ of 24)
	› identify, from a unit fraction part of a set, the whole set	› identify, from a fractional part of a set, the whole set
	› add and subtract fractions with the same denominators to make up to one whole or less than one whole (e.g., $\frac{3}{8} + \frac{3}{8} + \frac{2}{8} = 1$ )	› add and subtract fractions with the same denominators, including to make more than one whole
	› add and subtract decimals to one decimal place	› add and subtract decimals to two decimal places
	› use doubling or halving to scale a quantity (e.g., to double or half a recipe)	› use known multiplication facts to scale a quantity
	› use the mathematical processes to: <ul style="list-style-type: none"> <li>- connect equivalent fractions and decimals</li> <li>- connect decimal place value and operations with whole number place value and operations</li> <li>- connect decimals with measurement</li> <li>- investigate appropriate situations</li> <li>- explain and justify equivalent fractions</li> <li>- convert between mixed numbers and improper fractions</li> <li>- generalise that multiplying or dividing a number by a power of ten changes the position of the digits on a PV chart (years 5–6)</li> </ul>	

<b>Year 6</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
› find a fraction or percentage of a whole number where the answer is a whole number (e.g., $\frac{3}{8}$ of 48; 30% of \$150)	› bar models › benchmarks (years 5–6)
› identify, from a fractional part of a set, the whole set	› discrete materials (year 4) › bar models
› add and subtract fractions with the same or related denominators (e.g., $\frac{1}{4} + \frac{1}{8}$ )	› fraction tiles, number lines (years 4–5) › fraction tiles and number lines, to demonstrate renaming fractions with related denominators (year 6)
› add and subtract whole numbers and decimals to two decimal places	› horizontal methods (including number lines) and vertical methods
› use known multiplication and division facts to scale a quantity	› diagrams, known facts, horizontal and vertical methods
› use the mathematical processes to: <ul style="list-style-type: none"> <li>– connect equivalent fractions and decimals</li> <li>– connect decimal place value and operations with whole number place value and operations</li> <li>– connect decimals with measurement</li> <li>– investigate appropriate situations</li> <li>– explain and justify equivalent fractions</li> <li>– convert between mixed numbers and improper fractions</li> <li>– generalise that multiplying or dividing a number by a power of ten changes the position of the digits on a place value chart (years 5–6)</li> </ul>	› discrete and continuous materials.

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	<b>Year 4</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 5</b> <i>Informed by formative assessment, teach students to:</i>
<b>Financial maths</b>	<ul style="list-style-type: none"> <li>› make amounts of money using dollars and cents (e.g., to make 3 dollars and 70 cents)</li> </ul>	<ul style="list-style-type: none"> <li>› represent money values in multiple ways using notes and coins</li> </ul>
	<ul style="list-style-type: none"> <li>› estimate and calculate the total cost and change for items costing whole dollar amounts</li> </ul>	<ul style="list-style-type: none"> <li>› estimate the cost to the nearest dollar of items costing dollars and cents, and the change from the nearest ten dollars</li> </ul>
	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect to rounding and addition and subtraction</li> <li>- investigate making amounts of money, using different denominations</li> <li>- investigate financial plans and decisions.</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect to rounding and addition and subtraction of decimals to two places</li> <li>- investigate making amounts of money, using different denominations</li> <li>- investigate financial plans and decisions.</li> </ul> </li> </ul>

## Algebra

	<b>Year 4</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 5</b> <i>Informed by formative assessment, teach students to:</i>
<b>Generalising number properties</b>	<ul style="list-style-type: none"> <li>› use inverse operations to solve multiplication and division problems</li> </ul>	<ul style="list-style-type: none"> <li>› use inverse operations to solve multiplication and division problems</li> </ul>
	<ul style="list-style-type: none"> <li>› explore the associative property with addition and multiplication</li> </ul>	<ul style="list-style-type: none"> <li>› explore why the commutative and associative properties do not work for subtraction and division</li> </ul>
	<ul style="list-style-type: none"> <li>› recall multiplication and corresponding division facts for 4s, 6s, 9s, and 10s</li> </ul>	<ul style="list-style-type: none"> <li>› recall multiplication facts to <math>10 \times 10</math> and corresponding division facts</li> </ul>

<b>Year 6</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
	<i>Use the following when representing financial maths:</i>
<ul style="list-style-type: none"> <li>› solve problems involving purchases (e.g., ensuring they have enough money)</li> <li>› create simple financial plans ( e.g., shopping lists, a family budget)</li> </ul>	<ul style="list-style-type: none"> <li>› play money</li> <li>› spreadsheets</li> <li>› tables</li> </ul>
<ul style="list-style-type: none"> <li>› calculate 10%, 25%, and 50% of whole dollar amounts (e.g., 50% of \$280)</li> </ul>	<ul style="list-style-type: none"> <li>› known facts and rounding</li> <li>› bar models and benchmarks, to find percentages (year 6)</li> <li>› division, then subtracting from the whole (year 6)</li> </ul>
<ul style="list-style-type: none"> <li>› use the mathematical processes to: <ul style="list-style-type: none"> <li>- connect to rounding, addition and subtraction of decimals to two places, and calculating a percentage of a whole amount</li> <li>- investigate making amounts of money, using different denominations</li> <li>- investigate financial plans and decisions.</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› play money</li> <li>› written and mental methods.</li> </ul>

<b>Year 6</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
	<i>Use the following when representing number properties:</i>
<ul style="list-style-type: none"> <li>› use inverse operations to solve multiplication and division problems</li> </ul>	<ul style="list-style-type: none"> <li>› families of facts</li> <li>› bar models</li> <li>› equations, demonstrating working backwards</li> </ul>
<ul style="list-style-type: none"> <li>› use commutative, associative, and identity properties, deciding which operations they work for and which they don't</li> </ul>	<ul style="list-style-type: none"> <li>› array and area models for the commutative and associative properties</li> </ul>
<ul style="list-style-type: none"> <li>› recall multiplication facts to at least <math>10 \times 10</math> and corresponding division facts</li> </ul>	<ul style="list-style-type: none"> <li>› families of facts, multiplication grids, arrays, number lines</li> </ul>

	<b>Year 4</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 5</b> <i>Informed by formative assessment, teach students to:</i>
<b>Generalising number properties</b>	› explore the distributive property of multiplication over addition (e.g., $7 \times 8 = 7 \times (5 + 3) = (7 \times 5) + (7 \times 3)$ )	› explore the distributive property of multiplication over addition and subtraction (e.g., $6 \times 18 = 6 \times (20 - 2) = (6 \times 20) - (6 \times 2)$ )
	› use the mathematical processes to: <ul style="list-style-type: none"> <li>- generalise multiplication problems beyond recalled facts, by looking for patterns</li> <li>- investigate patterns in the multiples of times tables</li> </ul>	
<b>Equations and relationships</b>	› form and solve true or false number sentences and open number sentences involving multiplication and division, using understanding of the equal sign (e.g., $5 \times \_ = 20$ ; $\_ \div 3 = 6$ )	› form and solve true or false number sentences and open number sentences involving all four operations (e.g., $674 + 56 - k = 671$ )
	› recognise and describe the rule for a growing pattern using words, tables, and diagrams, and predict further elements in the pattern	› use tables to recognise the relationship between the ordinal position and its corresponding element in a growing pattern, develop a rule in words, and predict further elements in the pattern
	› use the mathematical processes to: <ul style="list-style-type: none"> <li>- investigate inverse operations to find missing numbers in equations and growing patterns (e.g., tivaevae)</li> <li>- explain and justify the relationship between the ordinal position and its corresponding element to find a pattern's rule.</li> </ul>	
<b>Algorithmic thinking</b>		

<b>Year 6</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<ul style="list-style-type: none"> <li>› use the distributive, commutative, and associative properties</li> </ul>	<ul style="list-style-type: none"> <li>› arrays, area models</li> </ul>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- generalise multiplication problems beyond recalled facts, by looking for patterns</li> <li>- investigate patterns in the multiples of times tables</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› arrays, area models</li> <li>› families of facts.</li> </ul>
<b><i>Use the following when representing equations and relationships:</i></b>	
<ul style="list-style-type: none"> <li>› form and solve true or false number sentences and open number sentences involving all four operations, using equality or inequality (e.g., <math>8 \times 7 &lt; 8 \times 5 + 8</math> (T or F?))</li> </ul>	<ul style="list-style-type: none"> <li>› things that balance</li> <li>› known facts</li> <li>› inverse operations</li> <li>› equal and inequality symbols</li> </ul>
<ul style="list-style-type: none"> <li>› use tables, XY graphs, and diagrams to recognise relationships in a linear pattern, develop a rule in words that identifies the constant amount of change between consecutive elements or terms in the pattern, and predict further elements in the pattern</li> </ul>	<ul style="list-style-type: none"> <li>› visually growing patterns, recording the position and term for each element in the sequence</li> </ul>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- investigate inverse operations to find missing numbers in equations and growing patterns (e.g., tivaevae)</li> <li>- explain and justify the relationship between the ordinal position and its corresponding element to find a pattern's rule</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› diagrams, words</li> <li>› tables, graphs.</li> </ul>
<b><i>Use the following when representing algebraic thinking:</i></b>	
<ul style="list-style-type: none"> <li>› create and use algorithms for making decisions that involve clear choices (e.g., formulating a familiar routine as a set of step-by-step instructions)</li> </ul>	<ul style="list-style-type: none"> <li>› flowcharts</li> <li>› numbered instructions</li> </ul>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect to algorithms for operations</li> <li>- investigate situations that involve making decisions.</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› step-by-step instructions</li> <li>› diagrams, flow charts.</li> </ul>

# Measurement

	<b>Year 4</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 5</b> <i>Informed by formative assessment, teach students to:</i>
<b>Measuring</b>	<ul style="list-style-type: none"> <li>› develop personal benchmarks for estimation and measure length, area, mass (weight), capacity, and duration, using appropriate metric units</li> </ul>	<ul style="list-style-type: none"> <li>› estimate and then accurately measure length, mass (weight), capacity, temperature, and duration, using appropriate metric units or a combination of units</li> </ul>
	<ul style="list-style-type: none"> <li>› use appropriate units to describe length, mass (weight), capacity, and time</li> </ul>	<ul style="list-style-type: none"> <li>› use the appropriate unit and tool for the task and the attribute being measured</li> </ul>
	<ul style="list-style-type: none"> <li>› use the metric measurement system to explore relationships between units</li> </ul>	<ul style="list-style-type: none"> <li>› use the metric measurement system based on powers of ten to explore relationships between units, including benchmark fractions and decimals</li> </ul>
	<ul style="list-style-type: none"> <li>› recognise that angles can be measured in degrees, using 90, 180, and 360 degrees as benchmarks</li> </ul>	<ul style="list-style-type: none"> <li>› describe an angle using the terms acute, right, obtuse, straight, and reflex, by comparing the angle with benchmarks of 90, 180, and 360 degrees</li> </ul>
	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect measuring with place value and decimals, angles with fractions of a circle and degrees of turn, and benchmark fractions with measurements (e.g., <math>500\text{ml} = \frac{1}{2}\text{L}</math>)</li> <li>- investigate, using practical measuring situations (e.g., using scaled measurement instruments, reading angles using geometric software and protractors)</li> <li>- explain and justify the use of appropriate metric units for a given situation</li> </ul> </li> </ul>	

<b>Year 6</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
	<i>Use the following when representing measuring:</i>
› estimate and then accurately measure length, mass (weight), capacity, temperature, and duration, using appropriate metric units or a combination of units	› rulers, scales, timers, thermometers, measuring jugs (always ensuring accurate use)
› select and use the appropriate unit and tool for the task and the attribute being measured	› units that match the situation
› convert between common metric units for length, mass (weight), and capacity; and use decimals to express parts of wholes in measurements	› PV and decimal PV houses
› visualise, measure, and draw (to the nearest degree), the amount of turn in angles up to 360 degrees	› protractors, to demonstrate measuring and drawing (year 6)
› use the mathematical processes to: <ul style="list-style-type: none"> <li>- connect measuring with place value and decimals, angles with fractions of a circle and degrees of turn, and benchmark fractions with measurements (e.g., 500ml = <math>\frac{1}{2}</math> L)</li> <li>- investigate, using practical measuring situations (e.g., using scaled measurement instruments, reading angles using geometric software and protractors)</li> <li>- explain and justify the use of appropriate metric units for a given situation</li> </ul>	› measurement tools › PV materials.

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	<b>Year 4</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 5</b> <i>Informed by formative assessment, teach students to:</i>
<b>Perimeter, area, and volume</b>	<ul style="list-style-type: none"> <li>› visualise, estimate, and calculate:               <ul style="list-style-type: none"> <li>- the perimeter of polygons using metric units</li> <li>- the area of shapes covered with squares or half squares</li> <li>- the volume of shapes filled with centicubes, taking note of layers and stacking</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› visualise, estimate, and calculate:               <ul style="list-style-type: none"> <li>- the perimeter of polygons</li> <li>- the area of shapes covered with squares or partial squares</li> <li>- the volume of rectangular prisms, taking note of layers and stacking</li> </ul> </li> </ul>
	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect area with multiplication arrays and the commutative property of multiplication</li> <li>- generalise the formula for finding the area and volume of rectangles and rectangular prisms</li> <li>- investigate practical contexts for finding perimeter, area, and volume</li> </ul> </li> </ul>	
<b>Time</b>		<ul style="list-style-type: none"> <li>› describe the differences in duration between units of time (e.g., days and weeks, months and years)</li> </ul>
	<ul style="list-style-type: none"> <li>› tell the time to the nearest 5 minutes, using the language of minutes past the hour and to the hour</li> </ul>	<ul style="list-style-type: none"> <li>› solve duration-of-time problems involving 'am' and 'pm' notation</li> </ul>
	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect units of time to fractions</li> <li>- investigate calendars, timetables, and schedules to work out the duration between events, or the start and end times for events.</li> </ul> </li> </ul>	

<b>Year 6</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
	<i>Use the following when representing perimeter, area, and volume</i>
<ul style="list-style-type: none"> <li>› visualise, estimate, and calculate the area of rectangles and right-angled triangles and the volume of rectangular prisms, by applying multiplication</li> </ul>	<ul style="list-style-type: none"> <li>› square grids</li> <li>› rulers</li> <li>› 2D shapes</li> </ul>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect area with multiplication arrays and the commutative property of multiplication</li> <li>- generalise the formula for finding the area and volume of rectangles and rectangular prisms</li> <li>- investigate practical contexts for finding perimeter, area, and volume</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› written methods with clearly laid out working.</li> </ul>
	<i>Use the following when representing time:</i>
<ul style="list-style-type: none"> <li>› convert between units of time and solve duration-of-time problems, in both 12- and 24-hour time systems</li> </ul>	<ul style="list-style-type: none"> <li>› word problems, for converting weeks and months into days and vice versa</li> <li>› digital and analogue clocks (year 4)</li> <li>› subtracting time to calculate duration</li> <li>› inclusive counting (e.g., for the number of days between now and next Tuesday, start counting from today) (years 5-6)</li> </ul>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect units of time to fractions</li> <li>- investigate calendars, timetables, and schedules to work out the duration between events, or the start and end times for events.</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› digital and analogue clocks.</li> </ul>

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# Geometry

	<b>Year 4</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 5</b> <i>Informed by formative assessment, teach students to:</i>
<b>Shapes</b>	<ul style="list-style-type: none"> <li>› identify, classify, and describe the properties of polygons (including triangles and quadrilaterals) using properties of shapes, including line and rotational symmetry</li> <li>› compare and classify angles in 2D shapes equal to, smaller than, or larger than a right angle</li> </ul>	<ul style="list-style-type: none"> <li>› identify, classify, and describe the properties of:               <ul style="list-style-type: none"> <li>- regular and irregular polygons, using edges, vertices, and angles</li> <li>- prisms, using the cross section, faces, edges, and vertices</li> </ul> </li> <li>› identify and describe parallel and perpendicular lines, including those forming the sides of polygons</li> </ul>
	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect angles with turns</li> <li>- investigate the properties of triangles and polygons</li> <li>- investigate line and rotational symmetry</li> <li>- explain and justify whether lines are parallel and shapes are regular</li> <li>- explain and justify the value of unknown angles in triangles and quadrilaterals</li> </ul> </li> </ul>	
<b>Spatial reasoning</b>	<ul style="list-style-type: none"> <li>› identify the 2D shapes that compose 3D shapes (e.g., a triangular prism is made up of two triangles and three rectangles)</li> <li>› visualise, predict, and identify which shape is a reflection, rotation, or translation of a given 2D shape</li> </ul>	<ul style="list-style-type: none"> <li>› visualise and connect 3D shapes with their nets, their 2D diagrams, verbal descriptions of them, and the same shapes drawn from different perspectives</li> <li>› resize a 2D shape so that it is either bigger or smaller</li> </ul>
	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect enlargement with simple grid references or coordinates and with doubling and halving</li> <li>- generalise the properties of shapes that do not change when transformed</li> <li>- investigate nets that fold together, shapes that tessellate, and transformations</li> </ul> </li> </ul>	

<b>Year 6</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
	<i>Use the following when representing shapes:</i>
<ul style="list-style-type: none"> <li>› classify, identify, and explain similarities and differences between:               <ul style="list-style-type: none"> <li>- 2D shapes, including types of triangle</li> <li>- prisms and pyramids</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› a range of 2D and 3D shapes, including tactile materials, diagrams, and digital tools</li> </ul>
<ul style="list-style-type: none"> <li>› identify and describe the interior angles of triangles and quadrilaterals</li> </ul>	<ul style="list-style-type: none"> <li>› angle benchmarks (year 4)</li> <li>› diagrams</li> <li>› protractors</li> </ul>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect angles with turns</li> <li>- investigate the properties of triangles and polygons</li> <li>- investigate line and rotational symmetry</li> <li>- explain and justify whether lines are parallel and shapes are regular</li> <li>- explain and justify the value of unknown angles in triangles and quadrilaterals</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› notation</li> <li>› vocabulary (e.g., obtuse, acute, reflex, and right angles).</li> </ul>
	<i>Use the following when representing spatial reasoning:</i>
<ul style="list-style-type: none"> <li>› visualise and draw nets for rectangular prisms</li> </ul>	<ul style="list-style-type: none"> <li>› sketching and constructing 3D shapes</li> </ul>
<ul style="list-style-type: none"> <li>› visualise, create, and describe 2D geometric patterns and tessellations using rotation, reflection, and translation, and identifying the properties of shapes that do not change</li> </ul>	<ul style="list-style-type: none"> <li>› grids for resizing (year 5)</li> <li>› 2D shapes, squared paper, and tracing paper to predict and test transformations (years 4 &amp; 6)</li> </ul>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect enlargement with simple grid references or coordinates and with doubling and halving</li> <li>- generalise the properties of shapes that do not change when transformed</li> <li>- investigate nets that fold together, shapes that tessellate, and transformations</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› 3D shapes seen from different perspectives</li> <li>› grids.</li> </ul>

	<b>Year 4</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 5</b> <i>Informed by formative assessment, teach students to:</i>
<b>Pathways</b>	<ul style="list-style-type: none"> <li>› use grid references to identify regions and to plot positions on a grid map</li> <li>› interpret and describe pathways, including half and quarter turns and the distance travelled</li> </ul>	<ul style="list-style-type: none"> <li>› interpret and create a grid map to plot positions and pathways, using grid references and directional language, including the four main compass points</li> </ul>
	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect compass points with angles and turns, and grid references with graphing skills</li> <li>- investigate different types of maps.</li> </ul> </li> </ul>	

## Statistics

	<b>Year 4</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 5</b> <i>Informed by formative assessment, teach students to:</i>
<b>Problem</b>	<ul style="list-style-type: none"> <li>› investigate summary and comparison situations with categorical and discrete numerical data, using multivariate (2 or more variables) data, by               <ul style="list-style-type: none"> <li>- posing summary and comparison investigative questions that can be answered with data</li> <li>- making predictions or assertions about expected findings</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› investigate summary and comparison situations with categorical and discrete numerical data, using multivariate data by               <ul style="list-style-type: none"> <li>- posing summary and comparison investigative questions that can be answered with data</li> <li>- making predictions or assertions about expected findings</li> </ul> </li> </ul>
	<ul style="list-style-type: none"> <li>› use the statistical processes to investigate school-related issues of interest</li> </ul>	
<b>Plan</b>	<ul style="list-style-type: none"> <li>› plan how to collect primary data to support answering an investigative question, including:               <ul style="list-style-type: none"> <li>- deciding on the group of interest</li> <li>- deciding the variable(s) for which data will be collected</li> <li>- taking account of ethical practices in data collection</li> </ul> </li> </ul>	

<b>Year 6</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<ul style="list-style-type: none"> <li>› interpret and create grid references and simple scales on maps</li> <li>› use directional language, including the four main compass points, turn (in degrees), and distance (in m, km), to locate and describe positions and pathways</li> </ul>	<p data-bbox="1131 247 1915 279"><i>Use the following when representing represent pathways:</i></p> <ul style="list-style-type: none"> <li>› maps with coordinates and compass points</li> </ul>
<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect compass points with angles and turns, and grid references with graphing skills</li> <li>- investigate different types of maps.</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› graphs and maps on grids.</li> </ul>

<b>Year 6</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<ul style="list-style-type: none"> <li>› investigate summary, comparison, and time-series situations, using multivariate data to:               <ul style="list-style-type: none"> <li>- pose investigative questions that can be answered with data</li> <li>- make predictions or assertions about expected findings</li> </ul> </li> </ul>	<p>Demonstrate posing investigative questions, and support students to write their own.</p>
<ul style="list-style-type: none"> <li>› use the statistical processes to investigate school-related issues of interest</li> </ul>	<p>Demonstrate the use of context analysis diagrams to represent school-related issues.</p>
<ul style="list-style-type: none"> <li>› plan how to collect primary data or how to use provided data, including identifying the variables of interest and, for provided data:               <ul style="list-style-type: none"> <li>- identifying who the data was collected from</li> <li>- identifying the original investigator's purpose for collecting the data</li> <li>- deciding if the source is reputable, by checking if any survey questions appear to be biased towards a particular point of view</li> </ul> </li> </ul>	<p>Demonstrate asking evaluation questions about sources and ethical practices.</p>

	<b>Year 4</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 5</b> <i>Informed by formative assessment, teach students to:</i>
<b>Plan</b>	<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- investigate topics of interest</li> <li>- explain and justify primary and secondary data, sensitive topics or questions, and ethical practices for data collection and use</li> </ul> </li> </ul>	
<b>Data</b>	› use a variety of tools to collect data, and check for errors in the data	› use a variety of tools to collect data, check for errors in the data, and correct errors by re-collecting the data, if possible
	› use the statistical processes to investigate methods for collecting secondary data	
<b>Analysis</b>	› create and describe data visualisations for summary and comparison investigations that make meaning from the data, with statements including the name of the variable	› create and describe data visualisations for summary and comparison investigations that make meaning from the data, with statements including the names of the variable and group of interest
	<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- investigate appropriate situations</li> <li>- explain and justify using 'I notice' statement about data visualisations, selecting the visualisation that best represents the data</li> </ul> </li> </ul>	
<b>Conclusion</b>	› choose the best descriptive statements to answer the investigative question, reflecting on findings and how they compare with initial predictions or assertions	› answer the investigative question, comparing findings with initial predictions or assertions and their existing knowledge of the world
	<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- connect statements with data visualisations to answer an investigative question, and to connect initial predictions or assertions with actual findings</li> <li>- investigate appropriate situations</li> </ul> </li> </ul>	
<b>Statistical literacy</b>	› check the statements that others make about data to see if they make sense, using information to clarify or correct statements where needed.	› check and, if needed, improve the statements others make about data, including data from two or more sources.
	› use the statistical processes to investigate, interpret, critique, and check the claims made about data presented in tables, pictographs, bar graphs, line graphs, and pie charts.	

<b>Year 6</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>– investigate topics of interest</li> <li>– explain and justify primary and secondary data, sensitive topics or questions, and ethical practices for data collection and use</li> </ul> </li> </ul>	Demonstrate how to represent data collection methods, who to measure, what to measure, and how to use a planning tool.
<ul style="list-style-type: none"> <li>› describe information about variables in secondary data by using publisher-provided data dictionaries (e.g., how data was collected for them and possible outcomes for them)</li> </ul>	Demonstrate a range of data collection methods and what errors in data 'look like'.
<ul style="list-style-type: none"> <li>› use the statistical processes to investigate methods for collecting secondary data</li> </ul>	Demonstrate data collecting, conducting observations, and designing surveys and questionnaires.
<ul style="list-style-type: none"> <li>› create and describe a variety of data visualisations that make meaning from the data, identifying features, patterns, and trends in context, including the variable and group of interest</li> </ul>	Demonstrate constructing and analysing data visualisations.
<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>– investigate appropriate situations</li> <li>– explain and justify using 'I notice' statement about data visualisations, selecting the visualisation that best represents the data</li> </ul> </li> </ul>	Demonstrate representing data using dot plots, bar graphs, frequency tables, and time-series graphs.
<ul style="list-style-type: none"> <li>› answer the investigative question, comparing findings with initial predictions or assertions and their existing knowledge of the world</li> </ul>	Demonstrate making statements about data that match investigative questions.
<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>– connect statements with data visualisations to answer an investigative question, and to connect initial predictions or assertions with actual findings</li> <li>– investigate appropriate situations</li> </ul> </li> </ul>	Demonstrate the use of concise statements.
<ul style="list-style-type: none"> <li>› identify, explain, check, and, if needed, improve features in others' data investigations (e.g., survey questions, misleading information or statements).</li> </ul>	Demonstrate identifying misleading data, matching data visualisations, and checking the claims of investigations.
<ul style="list-style-type: none"> <li>› use the statistical processes to investigate, interpret, critique, and check the claims made about data presented in tables, pictographs, bar graphs, line graphs, and pie charts.</li> </ul>	Draw on data visualisations from a variety of sources.

# Probability

	<b>Year 4</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 5</b> <i>Informed by formative assessment, teach students to:</i>
<b>Probability investigations</b>	<ul style="list-style-type: none"> <li>› engage in chance-based investigations with equally likely outcomes by:               <ul style="list-style-type: none"> <li>- posing investigative questions</li> <li>- anticipating what might happen</li> <li>- identifying possible outcomes for the investigative questions</li> <li>- generating all possible ways to get each outcome (a theoretical approach) or undertaking a probability experiment and recording the occurrences of each outcome</li> <li>- creating data visualisations for possible outcomes</li> <li>- describing what these visualisations show</li> <li>- finding probabilities as fractions</li> <li>- answering investigative questions</li> <li>- reflecting on anticipated outcomes</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› engage in chance-based investigations, including those with not equally likely outcomes, by:               <ul style="list-style-type: none"> <li>- posing investigative questions</li> <li>- anticipating what might happen</li> <li>- identifying possible outcomes for the investigative questions</li> <li>- generating all possible ways to get each outcome (a theoretical approach) or undertaking a probability experiment and recording the occurrences of each outcome</li> <li>- creating data visualisations for possible outcomes</li> <li>- describing what these visualisations show</li> <li>- finding probabilities as fractions</li> <li>- answering investigative questions</li> <li>- reflecting on anticipated outcomes</li> </ul> </li> </ul>
<b>Critical thinking in probability</b>	<ul style="list-style-type: none"> <li>› agree or disagree with others' conclusions about chance-based investigations.</li> </ul>	<ul style="list-style-type: none"> <li>› agree or disagree with others' conclusions about chance-based investigations, with justification.</li> </ul>
	<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- connect the chance of an outcome occurring with fractions, decimals, and percentages</li> <li>- investigate everyday chance-based situations using physical activities and technology.</li> </ul> </li> </ul>	

<b>Year 6</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<ul style="list-style-type: none"> <li>› engage in one-stage, chance-based investigations, including those with not equally likely outcomes, by:               <ul style="list-style-type: none"> <li>– posing investigative questions</li> <li>– anticipating what might happen</li> <li>– identifying possible outcomes for the investigative questions</li> <li>– generating all possible ways to get each outcome (a theoretical approach) or undertaking a probability experiment and recording the occurrences of each outcome</li> <li>– creating data visualisations for possible outcomes</li> <li>– describing what these visualisations show</li> <li>– finding probabilities as fractions</li> <li>– answering investigative questions</li> <li>– reflecting on anticipated outcomes</li> <li>– comparing findings from probability experiments and associated theoretical probabilities, if the theoretical model exists</li> </ul> </li> </ul>	<p>Play games of chance (e.g., Biased Bingo).</p> <p>Demonstrate creating systematic records of possible outcomes.</p> <p>Demonstrate using data visualisations.</p>
<ul style="list-style-type: none"> <li>› interrogate statements that others make about one-stage, chance-based situations, referring to evidence.</li> </ul>	<p>Demonstrate matching claims to data visualisations, using evidence from the data.</p> <p>Support students to interpret data and construct a response (e.g., with sentence starters, writing frames) (year 6).</p>
<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>– connect the chance of an outcome occurring with fractions, decimals, and percentages</li> <li>– investigate everyday chance-based situations using physical activities and technology.</li> </ul> </li> </ul>	<p>Demonstrate representing outcomes using lists, tables, tree diagrams, tally charts, distributions, visualisations, words, and numbers.</p>

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# Phase

# 3

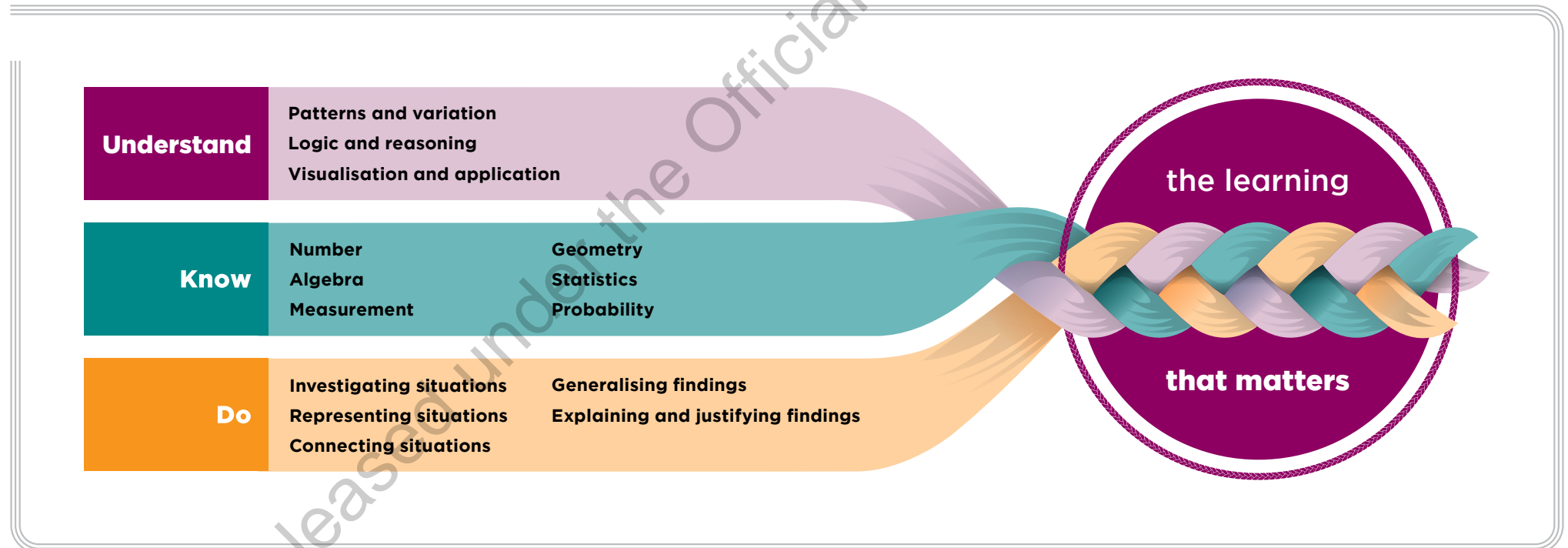
Years 7–8

*Seeing ourselves in the wider world and advocating with and for others*  
*Te aro atu ki te ao whānui me te kōkiri kaupapa hei hāpai tahi i ētahi atu*

## Progress outcome by the end of year 8

In phase 3, students weave together Understand, Know, and Do and share their logic and reasoning using maths notation, conventions, and vocabulary to clearly explain and justify their problem-solving approaches. They use number lines to visualise and extend their reasoning about positive integers to negative integers, enabling application to situations

such as temperature change and financial transactions. They also use tables and graphs to represent linear patterns, and generalise by forming equations. They reason about unknown angles, explore variations in shapes through transformation, and use data visualisations to investigate claims and make predictions.



## Understand Big ideas

As they further develop their maths capabilities and knowledge, students use **patterns** and a multitude of relationships in which change and **variation** occur. They draw on the structures of mathematics and statistics to support noticing, exploring, and describing different types of patterns and relationships, generating insights and generalisations and making predictions. By engaging with maths concepts, students develop **logical** reasoning and critical thinking skills that enable them to critically evaluate information, question assumptions, and present arguments with clarity.

Statistical **reasoning** from observation and theory allows them to differentiate what is probable from what is possible and to draw reliable conclusions about what is reasonable. Students further develop the ability to **visualise** maths ideas in order to perceive, understand, and interact with abstract concepts. They **apply** maths ideas to understand familiar and unfamiliar situations, facilitating better decision making and communication of ideas.

## Know Content and concepts

### Number | Mātauranga tau

By the end of this phase, students know that some **numbers have special properties**, such as primes, composites, squares, square roots, and cubes. A fraction can describe a **proportional relationship** between two amounts. Every fraction can be represented by an infinite set of equivalent fractions that occupy the same point on the number line. Multiplying a fraction by an **equivalent form** of 1, such as  $\frac{3}{3}$ , results in an equivalent fraction that can be useful for comparing, adding, and subtracting. **Decimals** continue the place-value system using negative powers of ten. They can be terminating, repeating and infinite, or non-repeating and infinite.

Students know that **integers** are positive and negative whole numbers, as well as zero. On a number line, fractions and decimals occur between integers. There are real-life situations described by quantities less than zero, such as temperature, below sea level, or debt, and these quantities can be operated on.

Students know that when calculating or forming expressions, the **order of operations** is important. Operations inside brackets (grouped together) are done first, then powers or exponents. If there are multiplication and division, these are done in left-to-right order; addition and subtraction are also done in left-to-right order. Students use the mnemonic GEMA: grouped, exponents, multiplication, addition. Division can result in a **remainder** expressed as a whole number, fraction, or decimal.

### Algebra | Taurangi

By the end of this phase, students know that the **inverse property** applies to addition and multiplication. **Inequalities** can also include “or equal to” ( $\leq$ ,  $\geq$ ) to show a relationship that allows for the possibility of equality. In algebra, a **variable** can be used to represent an unknown number, a quantity that can vary or change (e.g.,  $y = 3x + 4$ ,  $A = bh$ ), or a specific unknown value to be solved for (e.g.,  $3a = 18$ ). In algebra, there are conventional ways of writing multiplication and division.

Students also know that **linear patterns** have a constant rate of change and their XY graphs are **straight lines**. Not all patterns are linear. **Algorithms** help solve problems in a systematic way. Their instructions are created, tested, and revised.

### Measurement | Ine

By the end of this phase, students know that in the **metric system** there are **base** measurements with **prefixes** added to show the size of units. A measurement can be converted from smaller to bigger units, and vice versa, by dividing or multiplying by powers of 10. Length is a **one-dimensional measure**, area is a **two-dimensional measure**, and volume is a **three-dimensional measure**. This is apparent in the notation of units, such as cm, cm<sup>2</sup>, and cm<sup>3</sup>. Shapes can be **decomposed** or **recomposed** to help us find measurements (e.g., for perimeters, areas, and volumes). When multiplying lengths or dividing an area or volume by a length, the result has a **derived unit**.

### Geometry | Āhuahanga

By the end of this phase, students know that **spatial properties** of simple **polygons** and **polyhedra** can also apply to more complex two- and three-dimensional shapes. Properties of two- and three-dimensional shapes that do not change under a transformation are called **invariant**. Unknown angles can be found using the properties of angles on a **straight line**, angles at a **point**, **vertically opposite** angles, and interior angles in triangles and quadrilaterals. Viewing objects from different angles gives **different perspectives**, which can be represented in models and diagrams. **Position**, **direction**, and **pathways** can be described using scale, compass points, and environmental features. **Coordinate systems** and maps can express position, direction, and pathways.

## Statistics | Tauanga

By the end of this phase, students know that data collection and use involves a responsibility to protect the rights of people in relation to their own data and that of others, and the **ethical use** of data, including methods, interpretation, and conclusions. People need to understand who they are giving data to and why, before they agree to contribute to a dataset. The **statistical enquiry cycle** (PPDAC) can be used to conduct data-based investigations about the wider community. There are different types of **questions** used when undertaking statistical investigations: investigative (**summary, comparison, relationship, or time-series**), survey, data-collection, interrogative, or analysis questions. **Data visualisations** show patterns, trends, and variations. Alternative visualisations of the same data can lead to different insights and communicate different information. A **distribution** is formed from all the possible values of a variable and their frequencies. A **relationship** investigation looks for a relationship between paired numerical or paired categorical variables. **Predictions** or **assertations** may not be reflected in the data.

## Probability | Tūponotanga

By the end of this phase, students know that a probability experiment involves repeated trials. Results from sets of repeated trials for the same probability experiment may vary. Some chance-based situations, such as the result of tossing a drawing pin, can only be explored by probability experiments. Estimates of probabilities from experiments should be based on a very large number of trials (the law of large numbers). The estimated probability of an event from an experiment equals the relative frequency for that event.

If all possible outcomes in a chance-based situation are **equally likely**, the probability of an event equals the number of ways the event can happen divided by the total number of possible outcomes. The statistical enquiry cycle (PPDAC) can be used to conduct experiments. For a given situation, **estimated probabilities** from **experiments** and **theoretical model** probabilities will differ. **Probability distributions** from experiments and probability distributions from theoretical models will also differ.

## Do Practices

### Investigating situations | Te tūhura pūāhua

By the end of this phase, students can pose a question for investigation, find entry points for addressing a question, plan an investigation pathway, and follow it step by step. They can identify relevant prior knowledge, givens, and relationships to support the investigation. They can monitor and evaluate progress, adjusting the investigation pathway if necessary, and make sense of outcomes or conclusions in light of a given situation and context.

### Representing situations | Te whakaata pūāhua

Students can use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns and variations. They use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments. They select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations. They use visualisations to mentally represent and manipulate objects and ideas.

### Connecting situations | Te tūhono pūāhua

Students can suggest connections between ideas, approaches, and different representations. They connect new ideas to things they already know. They make connections with ideas in other learning

areas and with familiar cultural, linguistic, and historical contexts.

### Generalising findings | Te whakatauhānui i ngā kitenga

Students can recognise and explore patterns, and make conjectures and draw conclusions about them. They can identify relationships, including similarities, differences, and new connections. They look for patterns and regularities that might be applied in another situation or always be true. They make and test conjectures, using reasoning and counterexamples to decide if they are true or not. They use appropriate symbols to express generalisations.

### Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

Students can make statements, give explanations inductively based on observations or data, and make deductions based on knowledge, definitions, and rules. They critically reflect on others' thinking, evaluating their logic and asking questions to clarify and understand. They use evidence, reasoning, and proofs to explain why they agree or disagree with statements. They develop collective understandings by sharing and building on ideas with others, and they present reasoned explanations and arguments for an idea, solution, or process.

## Teaching sequence


*Seeing ourselves in the wider world and advocating with and for others*  
*Te aro atu ki te ao whānui me te kōkiri kaupapa hei hāpai tahi i ētahi atu*

Throughout phase 3, demonstrate, highlight, and affirm an attitude of exploration, enthusiasm, and curiosity towards maths endeavour and challenge, holding high expectations for every student. Students should critically reflect on others' reasoning, evaluating their logic and asking questions to clarify. To promote this, facilitate ongoing discussions and reflections about established expectations for interactions in maths learning, reinforcing that all students are involved. Support increasing agency for students to make decisions about investigations and problem solving (e.g., while planning their approach, selecting representations, and justifying their findings).

Continuously monitor students' reasoning, questions, and use of representations, and act quickly to respond to any misconceptions. Ensure your teaching builds on what students already understand, know, and can do.

Throughout each day, provide opportunities for students to:

- › **develop positive relationships with maths** – provide authentic tasks that are meaningful to students' experiences, interests, and the wider world. Demonstrate and teach strategies for persistence (e.g., trying another way or drawing a diagram). Plan to balance teacher talk with opportunities for rich, extended student interactions and discussions
- › **actively listen to, reflect, and build on each other's thinking and learning** – use discourse-based tools and a range of open questions to facilitate productive and challenging discussions. Over the phase, encourage students to convert their observations into a conjecture or claim and to use evidence to justify their claims and findings

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- › **experience explicit teaching of concepts and practices** – use worked examples and break down new learning into clearly explained manageable steps. Show students efficient written methods. Where there may be an error, misconception, or missing step, use examples to support students to develop critical analysis and reasoning
  - › **select and use representations that best support the learning purpose** – encourage students to use graphs, tables and equations. Over the phase, they will increasingly use equations to represent their reasoning. Visualising a situation by drawing a diagram can give students a way into a problem
  - › **learn and use maths vocabulary and concepts** – ensure students connect the correct vocabulary to the learning purpose and problem (e.g., by using the Frayer model's four quadrants: definition, characteristics, example, and non-example). Draw on students' first and heritage languages
  - › **use visualisation to represent and manipulate relationships, shapes, and quantities** – encourage students to use visualisation when, for example, predicting or deducing the effect of a transformation, viewing a solid shape from different perspectives, using coordinate pairs and locations, and identifying terms in a growing pattern
  - › **actively recall learning, practise new skills and processes, and make connections with prior learning** – provide regular opportunities to practise, so that students maintain their automatic recall of facts and continue to develop procedural fluency and reasoning. Support students to consolidate what they have learned by repeating the process or task you have demonstrated, firstly in familiar situations and then in unfamiliar situations.

# Number

	<b>Year 7</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 8</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
			<b>Use the following when representing number structure:</b>
<b>Number structure</b>	› identify, read, write, compare, and order whole numbers using powers of 10 (e.g., 10,000 = 10 <sup>4</sup> )	› identify, read, write, compare, and order whole numbers and decimals using powers of 10 (e.g., 0.01 = $\frac{1}{100}$ = 10 <sup>-2</sup> )	› place-value (PV) houses, number lines, inequality symbols
	› find the highest common factor (HCF) of two numbers under 100, and find the least common multiple (LCM) of two numbers under 10	› use prime factorisation to represent a number and to find the HCF of two numbers	› factor trees, systematic lists
	› use exponents to notate repeated multiplication, and identify square roots of square numbers up to at least 100	› identify prime and composite numbers up to at least 100 and cube numbers up to at least 125	› divisibility tests for composite and prime numbers › demonstrations of exponent notation
	› use the mathematical processes to: <ul style="list-style-type: none"> <li>- connect with divisibility rules, simplifying fractions, area, and volume</li> <li>- generalise conjectures about prime or composite numbers</li> <li>- investigate appropriate situations</li> </ul>		› PV materials › factor trees › systematic lists.
			<b>Use the following when representing operations:</b>
<b>Operations</b>	› use rounding and estimation to predict and to check the reasonableness of calculations	› use rounding and estimation (including benchmarks) to predict and to check the reasonableness of calculations	› known facts › benchmarks
	› round whole numbers to any specified multiple of powers of 10, and round decimals to the nearest tenth, hundredth, or whole number		› number lines › visualising benchmarks

	<b>Year 7</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 8</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<b>Operations</b>	› multiply whole numbers		› horizontal and vertical methods
	› divide whole numbers by 1- or 2-digit divisors (e.g., $327 \div 5 = 65.4$ or $65 \frac{2}{5}$ )	› divide whole numbers (e.g., $327 \div 15 = 21.8$ or $21 \frac{4}{5}$ )	› horizontal and vertical methods
	› use the order of operations rule GEMA	› use the order of operations rule GEMA	› demonstrations of step-by-step layouts
	› order, compare, add, and subtract integers using tools	› order, compare, add, and subtract integers	› number lines, two sided counters, diagrams
	› use the mathematical processes to: <ul style="list-style-type: none"> <li>- investigate situations where integers are used (e.g., temperature, altitude, profit and loss)</li> <li>- explain and justify findings using estimation, and checking using inverse operations</li> </ul>		› number lines.
			<b><i>Use the following when representing rational numbers:</i></b>
<b>Rational numbers</b>	› identify, read, write, and represent fractions, decimals (to three places), and percentages	› identify, read, write, and represent fractions, decimals, and percentages	› bar models, number lines
	› compare, order, and convert between fractions, decimals (to three places), and percentages	› compare, order, and convert between fractions, decimals, and percentages	› double number lines › benchmarks for demonstrating conversion › renaming to tenths or hundredths, or finding a common denominator
	› multiply and divide numbers by powers of 10		› PV houses
	› find equivalent fractions, simplify fractions, and convert between improper fractions and mixed numbers		› demonstrations of simplifying fractions and finding equivalent fractions using division and multiplication
	› multiply fractions and decimals by whole numbers, and find a percentage of a whole number		› horizontal and vertical methods › demonstrations of finding a percentage using multiplication

	<b>Year 7</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 8</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<b>Rational numbers</b>	› find a whole amount, given a simple fraction or percentage (e.g., ‘25% is \$100, what is the original amount?’)	› find a whole amount, given a simple fraction or percentage (e.g., ‘75% is \$45, what is the original amount?’)	› bar models
	› add and subtract fractions with different denominators up to tenths (e.g., $\frac{3}{4} + \frac{1}{3}$ )	› add and subtract fractions with different denominators by using equivalent fractions	› renaming using HCFs and LCMs
	› add and subtract decimals to three decimal places, with an emphasis on estimating before calculating	› add, subtract, and multiply decimals, with an emphasis on estimating before calculating	› horizontal and vertical methods
	› use proportional reasoning to explore relationships between quantities (e.g., ‘If there are 3 red for every 7 blue balls, how many balls are there altogether when there are 18 red balls?’)	› use proportional reasoning to share in unequal proportions (e.g., ‘We have 100 stickers to share. for every 1 sticker I get, you get 3 stickers. How many do we each get?’)	› diagrams and comparison models
	› use the mathematical processes to: <ul style="list-style-type: none"> <li>– connect benchmarks (fractions, decimals, and percentages) and decimal operations with whole-number place values and operations</li> <li>– connect decimals with measuring</li> <li>– investigate HCFs and LCMs, the effect of multiplying and dividing decimals, situations where decimals are used and compared (e.g., sporting events), and proportional reasoning</li> <li>– explain and justify equivalence and which fraction is larger</li> </ul>	› continuous materials › bar models › written methods.	
			<b><i>Use the following when representing financial maths:</i></b>
<b>Financial maths</b>	› calculate costs, and change for any amount of money	› create and compare weekly, monthly, and yearly finance plans (e.g., saving plans, phone plans, budgets, and ‘buy now, pay later’ services)	› spreadsheets › practical examples

	<b>Year 7</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 8</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<b>Financial maths</b>	› calculate the percentage discounts of whole dollar amounts (e.g., 'What is 35% of \$180?')	› calculate percentage discounts	› demonstrations of finding a percentage and subtracting it from the whole, or of multiplying a whole by a required decimal fraction
	› use the mathematical processes to: <ul style="list-style-type: none"> <li>- connect negative numbers with debt</li> <li>- investigate practical financial decisions and statistics in the media about growth or loss.</li> </ul>		› digital tools (e.g., spreadsheets) › written and mental methods.

## Algebra

	<b>Year 7</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 8</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
			<i>Use the following when representing number properties:</i>
<b>Generalising number properties</b>	› explore multiplicative inverses (a number and its reciprocal) in multiplication		› fraction tiles, number lines › examples (for identifying trends)
	› explore additive inverses (pairs of opposites) in the addition and subtraction of positive and negative numbers (e.g., $-6 + 8 = -6 + 6 + 2$ )	› use commutative, associative, identity, and inverse properties with expressions, including those with negative numbers	› number lines
	› recall multiplication facts to at least $10 \times 10$ and identify and describe the divisibility rules for 2, 3, 5, 9, and 10	› identify and describe the properties of prime and composite numbers and explore divisibility rules	› pattern exploration in 100s boards › multiplication grids

	<b>Year 7</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 8</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<b>Generalising number properties</b>	<ul style="list-style-type: none"> <li>› describe and use the commutative, distributive, and associative properties of operations (e.g., <math>O \times \_ = \_ \times O</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› simplify algebraic expressions involving sums, products, and differences, including by expanding single brackets expressions using the distributive property (e.g., <math>2(x + 3) + 1 = 2x + 6 + 1 = 2x + 7</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› physical manipulatives (e.g., algebra tiles) and digital manipulatives</li> <li>› systematic expansion approaches, including expansion tables</li> </ul>
	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- represent algebraic expressions and equations using correct vocabulary and notation (e.g., <math>3 \times b = 3b</math>)</li> <li>- connect prime and composite numbers with factors, multiples, and divisibility rules</li> <li>- generalise relationships between positive and negative integers using the commutative, associate, and distributive properties of numbers</li> <li>- investigate appropriate situations</li> </ul> </li> </ul>		<ul style="list-style-type: none"> <li>› algebraic notation.</li> </ul>
			<b><i>Use the following when representing equations and relationships:</i></b>
<b>Equations and relationships</b>	<ul style="list-style-type: none"> <li>› form and solve 1-step linear equations (e.g., <math>t + 7 = 12</math> ; <math>2s = 14</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› form and solve 1- or 2-step linear equations (e.g., <math>5s - 3 = 17</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› word problems</li> <li>› demonstrations of equation-solving algorithms</li> </ul>
	<ul style="list-style-type: none"> <li>› find the value of an expression or formula given the values of variables (e.g., calculate <math>w + 12</math> when <math>w = 4</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› find the value of an expression or formula given the values of variables</li> </ul>	<ul style="list-style-type: none"> <li>› variable values in practical situations with familiar formulae (e.g., for area, volume)</li> </ul>
	<ul style="list-style-type: none"> <li>› identify the constant rate of change and fixed value for a linear pattern, writing the equation using variables and algebraic notation to represent the rule, and using the rule to make predictions</li> </ul>	<ul style="list-style-type: none"> <li>› determine if a pattern is linear and, if it is, write the equation for the pattern and use the equation</li> </ul>	<ul style="list-style-type: none"> <li>› tables and XY graphs</li> <li>› demonstrations of finding, and recording as an equation, the term-to-term and position-to-term rules for a sequence</li> </ul>
	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- connect to measurement formulae</li> <li>- generalise a rule for a pattern and use this to justify a prediction of a term</li> <li>- investigate the history and use of growing patterns in tukutuku and other well-known patterns (e.g., the Fibonacci sequence)</li> </ul> </li> </ul>		<ul style="list-style-type: none"> <li>› word problems</li> <li>› equations</li> <li>› tables and XY graphs.</li> </ul>

	<b>Year 7</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 8</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
			<i>Use the following when representing algorithmic thinking:</i>
<b>Algorithmic thinking</b>	› create, test, and revise algorithms involving a sequence of steps and decisions	› create, test, revise, and use algorithms to identify, interpret, and explain patterns	› flowcharts, numbered step-by-step instructions, digital tools
		› use the formula function of a spreadsheet to explore the effect of changing the value of a variable (e.g., hourly wages) on the results (i.e., cell values)	› spreadsheet cell conventions and formulae
	› use the mathematical processes to connect algorithms with methods for solving an operation.		› step by step instructions, diagrams, flow charts › variables on a spreadsheet.

## Measurement

	<b>Year 7</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 8</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
			<i>Use the following when representing measuring:</i>
<b>Measuring</b>	› estimate and then measure length, area, volume, capacity, mass (weight), temperature, data storage, time, and angle, using appropriate metric units		› rulers, scales, timers, protractors, thermometers, measuring jugs (always ensuring accurate use)
	› select and use an appropriate base measure (e.g., metre, gram, litre) within the metric system, along with a prefix (e.g., kilo, centi) to show the size of units		› practical measuring › units that are appropriate for the situation

	<b>Year 7</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 8</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<b>Measuring</b>	› convert between metric units of length, mass (weight), and capacity, using whole numbers and decimals to express parts of a unit (e.g., 724g = 0.724kg)	› convert between metric measurement units, including square units	› multiplying and dividing by powers of 10 › PV and decimal PV houses
	› find speed given distance and time	› find distance given speed and time, or time given distance and speed	› bar models showing relationships between speed, distance, and time
	› use the mathematical processes to: <ul style="list-style-type: none"> <li>- generalise equivalent measurements (e.g., 2.05L = 2050mL)</li> <li>- connect measurement conversions with multiplying and dividing by powers of 10</li> <li>- investigate practical measurement situations, including reading tools with scales</li> <li>- explain which measurement tools and units are appropriate in a given situation</li> </ul>		› measurement tools › PV materials.
			<b><i>Use the following when representing perimeter, area, and volume:</i></b>
<b>Perimeter, area, and volume</b>	› calculate the perimeter and area of compound shapes composed of triangles and rectangles	› calculate the volume of triangular prisms and shapes composed of rectangular prisms	› demonstrations of finding perimeter, area and volume in practical situations › clear layouts for working
	› use the mathematical processes to: <ul style="list-style-type: none"> <li>- generalise the formulae for finding the area of triangles and volume of triangular prisms</li> <li>- investigate practical contexts for finding perimeter, area, and volume</li> </ul>		› practical measuring › written methods with clearly laid out working.
			<b><i>Use the following when representing time:</i></b>
<b>Time</b>	› read, interpret, and use timetables and charts that present measurement information		› demonstrations using a range of examples, including online apps
	› convert between units of time and solve duration problems that involve fractions of time		› subtracting time to calculate for duration › inclusive counting

	<b>Year 7</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 8</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<b>Time</b>	<ul style="list-style-type: none"> <li>› use the mathematical processes to:               <ul style="list-style-type: none"> <li>- generalise units of time using base-60</li> <li>- investigate the duration of time in situations such as developing event schedules or planning journeys.</li> </ul> </li> </ul>		› calendars, timetables, and schedules.

## Geometry

	<b>Year 7</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 8</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
			<i>Use the following when representing shapes:</i>
<b>Shapes</b>	› classify shapes based on their properties, and name the resulting classes of shapes (e.g., triangles, pyramids)	› describe triangles, quadrilaterals, and other polygons in relation to their side, diagonal, and angle properties	› a range of 2D and 3D shapes, including shapes that draw on tactile materials, diagrams, and digital tools
	› identify and describe angles at a point, angles on a straight line, and vertically opposite angles	› reason about unknown angles in situations involving angles at a point, angles on a straight line, vertically opposite angles, interior angles of triangles, and polygons	› digital tools for exploring angles › measuring tools (e.g., rulers, protractors)
	› use the mathematical processes to: <ul style="list-style-type: none"> <li>- generalise using angle rules to find unknown angles</li> <li>- investigate diagonals and angles of polygons</li> <li>- explain and justify classifications using flowcharts, Venn diagrams, and tables</li> </ul>		› equations (to find unknown angles).

	<b>Year 7</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 8</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
			<b><i>Use the following when representing spatial reasoning:</i></b>
<b>Spatial reasoning</b>	› visualise, construct, and draw plan views for front, back, left, right, and top views of 3D shapes, using cube models, digital tools, and grid paper	› visualise and draw nets for prisms with a fixed cross section	› physical models or sketches, created using measurement tools
	› transform 2D shapes, including composite shapes, by resizing by a whole number or unit fraction of less than one	› recognise the invariant properties of 2D and 3D shapes under different transformations	› resizing using a centre of enlargement › 2D shapes and squared paper › tracing paper to predict and test transformations
	› use the mathematical processes to: <ul style="list-style-type: none"> <li>- investigate the meaning of kowhaiwhai patterns and other symbols from te ao Māori, and describe the use of transformations in these patterns</li> <li>- explain which properties of a shape will be affected by a given transformation</li> </ul>		› drawings and models › nets.
			<b><i>Use the following when representing pathways:</i></b>
<b>Pathways</b>	› interpret and communicate the location of positions and pathways using coordinates, angle measures, and the 8 main and halfway compass points (e.g., 45° E from N is NE)	› use map scales, compass points, distance, and turn to interpret and communicate positions and pathways in coordinate systems and grid reference systems	› demonstrations using 4-digit grid references and scale to calculate actual distance
	› use the mathematical processes to: <ul style="list-style-type: none"> <li>- connect map scales to proportional reasoning</li> <li>- connect angles and using a protractor with compass points</li> <li>- investigate the most efficient route between two destinations.</li> </ul>		› maps of familiar and unfamiliar locations › protractors and angle notation › diagrams of compass points › coordinate and grid references.

# Statistics

	<b>Year 7</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 8</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<b>Problem</b>	<ul style="list-style-type: none"> <li>› investigate, using multivariate datasets, summary, comparison, time-series, and relationship situations for paired categorical data by:               <ul style="list-style-type: none"> <li>- posing investigative questions about local community matters</li> <li>- making predictions or assertions about expected findings</li> </ul> </li> </ul>		Demonstrate posing investigative questions, and support students to write their own questions.
	<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- represent summary, comparison, relationship, and time-series investigative questions</li> <li>- investigate a broad area of interest before fine-tuning a specific investigative question</li> </ul> </li> </ul>		
<b>Plan</b>	<ul style="list-style-type: none"> <li>› plan how to collect or source data to answer investigative questions, including               <ul style="list-style-type: none"> <li>- determining or identifying the variables needed</li> <li>- planning how to collect data for each variable (e.g., how to measure them when collecting) or finding out how provided data was collected</li> <li>- identifying the group of interest or who the data was collected from</li> <li>- building awareness of ethical practices by strategic questioning of data collection methods</li> </ul> </li> </ul>		Demonstrate making a plan for how to collect and sort the data needed to answer an investigative question.
	<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- represent using a diagram who, what, and how to measure</li> <li>- investigate appropriate situations</li> <li>- explain and justify variables and groups of interest when working with secondary data</li> </ul> </li> </ul>		
<b>Data</b>	<ul style="list-style-type: none"> <li>› collect data, including               <ul style="list-style-type: none"> <li>- checking for errors, following up and correcting them when possible</li> <li>- creating data dictionaries that include information for others about the context</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› source ready-to-use data, and provide information about the variables using provided data dictionaries</li> </ul>	Demonstrate a range of data collection methods and what errors in data 'look like'.
	<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- represent data using a range of tools (e.g., spreadsheets, recording sheets)</li> <li>- investigate secondary data</li> <li>- explain errors in data and justify why they are errors</li> </ul> </li> </ul>		

	<b>Year 7</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 8</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<b>Analysis</b>	<ul style="list-style-type: none"> <li>› create and describe data visualisations for summary, comparison, relationships (paired categorical), and time-series investigations, including features and context in descriptions of distributions</li> </ul>	<ul style="list-style-type: none"> <li>› create and describe data visualisations for summary, comparison, relationships, and time-series investigations, using multiple visualisations to provide different views of the data and including features and context in descriptions of distributions</li> </ul>	Demonstrate constructing and analysing data visualisations.
	<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- represent data using dot plots, bar graphs, frequency tables, time-series graphs, two-way tables or graphs, scatter plots, fractions, proportions, and percentages</li> <li>- investigate how different data visualisations show different features of data and give different information</li> <li>- explain and justify patterns, trends, and features of data visualisations</li> </ul> </li> </ul>		
<b>Conclusion</b>	<ul style="list-style-type: none"> <li>› communicate findings in context to answer an investigative question, using evidence from analysis and comparing findings to initial predictions or assertions and existing knowledge of the world</li> </ul>	<ul style="list-style-type: none"> <li>› communicate findings in context to answer an investigative question, using evidence from analysis, considering possible explanations for findings, and comparing findings to initial predictions or assertions and existing knowledge of the world</li> </ul>	Demonstrate making statements about data and matching them with investigative questions.  Demonstrate reflecting on findings.
	<ul style="list-style-type: none"> <li>› use the statistical processes to:               <ul style="list-style-type: none"> <li>- connect statements with data visualisations to answer an investigative question</li> <li>- investigate appropriate situations</li> <li>- explain findings, and justify initial predictions or assertions given the findings</li> </ul> </li> </ul>		
<b>Statistical literacy</b>	<ul style="list-style-type: none"> <li>› examine the findings of others to check if their claims or statements are supported by the data visualisations they use</li> </ul>	<ul style="list-style-type: none"> <li>› examine the data-collection methods, data visualisations, and findings of others' statistical investigations to see if their claims are reasonable</li> </ul>	Demonstrate identifying misleading data, matching data visualisations, and checking the claims of investigations.
	<ul style="list-style-type: none"> <li>› use the statistical processes to explain and justify critiques of data visualisations and collection methods.</li> </ul>		

# Probability

	<b>Year 7</b> <i>Informed by formative assessment, teach students to:</i>	<b>Year 8</b> <i>Informed by formative assessment, teach students to:</i>	<b>Teaching methods</b>
<b>Probability investigations</b>	<ul style="list-style-type: none"> <li>› plan and conduct probability experiments for chance-based situations, including undertaking a large number of trials using technology, by:               <ul style="list-style-type: none"> <li>- posing investigative questions</li> <li>- identifying outcomes for the investigative question posed and anticipating what might happen</li> <li>- deciding on the number of trials, the tools to be used, and the recording method</li> <li>- collecting and recording data</li> <li>- creating data visualisations for the distribution of observed outcomes and (year 8) for all possible outcomes for theoretical probability models where they exist</li> <li>- describing what these visualisations show</li> <li>- finding the probability estimates for the different outcomes</li> <li>- proposing possible theoretical outcomes and associated probabilities for situations where no theoretical model exists</li> <li>- identifying similarities and differences between their findings and those of others</li> <li>- reflecting on anticipated outcomes</li> <li>- identifying similarities and differences between findings from probability experiments and associated theoretical probabilities, as appropriate</li> </ul> </li> </ul>		<p>Use probability experiments such as taking coloured blocks out of a bag, drawing a card, and flipping a coin, comparing theoretical with experimental probability.</p> <p>Demonstrate creating data visualisations that show outcomes.</p>
<b>Critical thinking in probability</b>	› agree or disagree with others' conclusions by interrogating their probability experiments		<p>Demonstrate and support students to interpret data and construct a response, using sentence starters or writing frames.</p> <p>Use 'true or false' or 'odd one out' challenges, asking students to support their ideas with reasons.</p>
		› agree with or challenge claims and identify misconceptions in relation to chance-based situations	
	› use the statistical processes to: <ul style="list-style-type: none"> <li>- represent outcomes using systematic approaches and technology</li> <li>- connect probabilities with proportional reasoning, fractions, and percentages</li> <li>- investigate games of chance, patterns in possible outcomes, and theoretical and experimental distributions</li> <li>- explain and justify probability estimates and claims about chance-based situations.</li> </ul>		



**Te Poutāhū**  
Curriculum Centre

**Te Mātaiaho**

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# **The New Zealand Curriculum**

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**MATHEMATICS AND  
STATISTICS YEARS 0-8**

*Mātai aho tāhūnui,  
Mātai aho tāhūroa,  
Hei takapau wānanga  
E hora nei.*

*Lay the kaupapa down  
And sustain it,  
The learning here  
Laid out before us.*



**Te Tāhuhu o  
te Mātauranga**  
Ministry of Education

**Te Kāwanatanga  
o Aotearoa**  
New Zealand Government

**OCTOBER 2024**

Released under the Official Information Act 1982

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There are two versions of the mathematics and statistics learning area. To ensure you can see the full teaching sequences, which spread across two pages:

- › for printing, print the ‘single pages’ version, backed; staple or bulldog clip the left-hand side of the printout
- › for reading on line, view the ‘double pages’ version.

Released under the Official Information Act 1982

# The New Zealand Curriculum – knowledge-rich, informed by the science of learning, and framed within the whakapapa of Te Mātaiaho

The New Zealand curriculum is knowledge-rich. It prioritises and explicitly describes what must be taught each year and is deliberately sequenced to enable students to build knowledge, skills, and competencies systematically over time. It supports teachers to design teaching programmes that bring learning to life in the classroom, using local, national, and global contexts.

The science of learning informs curriculum sequencing and teaching practice. The curriculum builds on scientific understanding to identify five characteristics of how we learn:

**We learn best when we experience a sense of belonging in the learning environment and feel valued and supported.**

Students bring with them different cultural identities, knowledge, belief systems, and experiences. They need to see that these are valued and reflected in a school environment characterised by strong relationships and mutual respect. Students' sense of belonging is enhanced by sensitivity to their individual needs, emotions, cultures, and beliefs.

**A new idea or concept is always interpreted through, and learned in association with, existing knowledge.**

The amount of existing knowledge students have, and the degree to which that knowledge is interconnected in long-term memory, influence both the quality and ease with which they can build on that knowledge. Recognising and drawing on students' prior knowledge therefore improves their learning.

**Establishing knowledge in a well-organised way in long-term memory reduces students' cognitive load when building on that knowledge. It also enables them to apply and transfer the knowledge.**

Establishing new knowledge and skill in long-term memory requires active engagement and multiple opportunities to engage with them, practise them, and connect them to existing knowledge structures. When knowledge is well organised in long-term memory, students are more likely to be able to build on it and apply it in novel ways. If knowledge is not well established in long-term memory, students' working memory is likely to be overloaded when they attempt to build on or apply it. This cognitive overload can cause confusion, anxiety, and disengagement.

**Our social and emotional wellbeing directly impacts on our ability to learn new knowledge.**

Social and emotional wellbeing reduces anxiety, which frees cognitive capacity to learn new knowledge and skills, leading to deeper, more durable learning. Conversely, anxiety and negative emotions inhibit students' ability to learn. The factors that impact positively or negatively on social and emotional wellbeing vary between students. The influence of these factors is dynamic – it fluctuates over time, even during the course of a single day.

**Motivation is critical for wellbeing and engagement in learning.**

Motivation develops when students feel that three basic needs are met: autonomy – developing increasing self-direction in learning; competence – experiencing success in learning and seeing oneself as a successful learner; social connection – belonging and contributing to a group from which one learns. Success in learning helps to build motivation.

# The New Zealand Curriculum – knowledge-rich, informed by the science of learning, and framed within the whakapapa of Te Mātaiaho

The design of this framework encompasses seven curriculum components. Te Mātaiaho as a whole weaves together these components, all of which begin with the word 'mātai', meaning to observe, examine, and deliberately consider.

## Mātaiahikā | Relationships with tangata whenua and local community

### Learning through relationships with tangata whenua and local communities

*Mātai kōrero ahiahi. | Keep the hearth occupied, maintain the stories by firelight.*

Poutama curves represent relationships with tangata whenua and the community.

## Mātaioho | National curriculum – contextualised

### The process by which schools bring the national curriculum to life through local, national, and global contexts

*Mātai oho, mātai ara, whītiki, whakatika. | Awaken, arise, and prepare for action.*

Unaunahi scales represent wealth of knowledge, purpose, and know-how.

## Mātaiaho | Learning areas

### The eight learning areas, which each include a purpose, big ideas, knowledge, and practices, year-by-year

*Mātai rangaranga te aho tū, te aho pae. | Weave the learning strands together.*

Taratara-a-kae niho notches represent diversity, resilience, and mana.

› Mathematics and statistics years 0-8

## Mātairangi | The guiding kaupapa

**The overarching kaupapa guiding the curriculum, based on the science of learning and ensuring excellent and equitable outcomes for students**

*Mātai ki te rangi, homai te kauhau wānanga ki uta, ka whiti he ora. | Look beyond the horizon, and draw near the bodies of knowledge that will take us into the future.*

The outer rings represent our guiding kaupapa.

## Mātainuku | Creating a foundation

**The curriculum principles (e.g., holding high expectations, and enabling all students to access the full scope of the curriculum)**

*Mātai ki te whenua, ka tiritiria, ka poupoua. | Ground and nurture the learning.*

The centre rings represent the foundation and calls to action.

## Mātaipū | Vision of young people

**The educational vision of young people, as conceived by young people**

*Mātaipū hei papa whenuakura. | Grow and nourish a thriving community.*

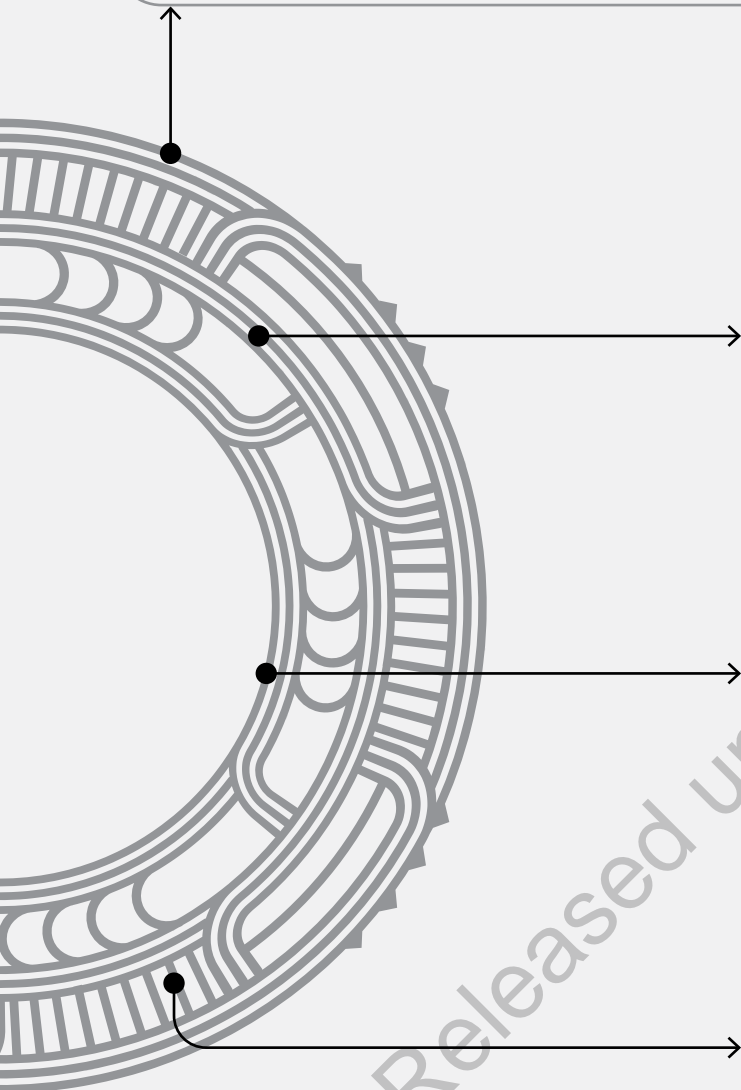
The inner rings and circular space represent the vision and students at the centre.

## Mātairea | Supporting progress

**The whole schooling pathway and the overarching focus for year-by-year learning and progress**

*Mātai ka rea, ka pihī hei māhuri. | Build and support progress.*

Niho kurī lines represent building and supporting the development of students.



# Learning areas

The curriculum has eight learning areas: English, the arts, health and physical education, learning languages, mathematics and statistics, science, social sciences, and technology. Together they provide the basis for a broad, general education for the first four phases of learning (years 0–10) and collectively lay a foundation for specialisation in phase 5 (years 11–13).

Each learning area is knowledge-rich. This knowledge has been carefully chosen to support all students in their schooling pathway and is framed using Understand, Know, and Do:

- › **Understand** – the deep and enduring big ideas and themes that students develop understanding of over the phases
- › **Know** – the meaningful and important content, concepts, and topics at each phase that enrich students' understanding of the big ideas and themes and that students study using the practices
- › **Do** – the practices (skills, strategies, and processes) that bring rigour to learning and support the development of the key competencies.

A **progression model** provides the structure that sequences the knowledge. It supports all students to develop greater:

- › breadth and depth of knowledge and understanding, through engaging with increasingly complex and ambiguous contexts
- › refinement and sophistication in their use of competencies, practices, strategies, processes, and skills
- › ability to connect, transfer, and apply new learning in meaningful contexts
- › knowledge and awareness of themselves as learners
- › effectiveness when working with others.

## Content of the learning areas

Knowledge and progression are reflected in how the learning areas are organised. Each learning area has the following main sections:

### Purpose statement and UKD overview

A purpose statement describes the learning area's contribution to the lives of students. It is followed by an overview of Understand, Know, and Do. This gives a view of the big ideas, themes, concepts, topics, and practices that underpin the learning area.

Teachers use the purpose statement and UKD overview to develop an understanding of the learning area, so that they can share its benefits with students.

### Learning area structure

For each learning area, this section outlines its structure and the changes it undergoes over five phases of learning, particularly in the final phase, where students specialise and choose from a range of subjects.

There are five phases of learning, spanning years 0–13. Each phase covers two to three years of schooling, which reflects how most schools organise learning across year levels.

A **critical focus** for each phase establishes a sustained, strengths-based, focus on the student and their social, emotional, and cognitive learning at this stage of their schooling journey. Each critical focus builds on the phase before and is reflected in the content of the learning area for the phase.

The critical focuses are:

- › **Phase 1** (years 0–3): Thriving in environments rich in literacy and maths
- › **Phase 2** (years 4–6): Expanding horizons of knowledge, and collaboration
- › **Phase 3** (years 7–8): Seeing ourselves in the wider world and advocating with and for others
- › **Phase 4** (years 9–10): Having a purpose and being empathetic and resilient
- › **Phase 5** (years 11–13): Navigating pathways and developing agency to help shape the future.

Teachers use the critical focus of each phase in their selection and design of topics and activities.

## Teaching guidance

Each learning area also draws from the science of learning and wider education theory to provide a knowledge base and guidance for teachers. Teachers use this to help them make purposeful decisions about how to teach the learning area's content in ways that are inclusive of all students.

The guidance is organised under three headings:

- › Designing a comprehensive teaching and learning programme
- › Using assessment to inform teaching
- › Planning.

## Progress outcomes


In each learning area, there is one comprehensive progress outcome for each phase.

The progress outcomes act as signposts that describe expectations for what students should sufficiently understand, know, and be able to do at key points in the schooling pathway.

The content of each progress outcome is organised using the Understand-Know-Do framework. While the Understand statements repeat across the five phases, students' depth of understanding increases as their knowledge of the learning area's content (Know) grows and their use of the practices (Do) develops.

When read alongside the progress outcomes for prior and subsequent phases, the progress outcome for a phase helps teachers maintain an overview of the learning they are building on and the learning they are preparing students for. Progress outcomes are therefore key for planning, along with the more detailed teaching sequences (described below).

Teachers also use the progress outcomes to help them form a comprehensive view of each student's progress, achievement, learning needs, and strengths. Schools can use information from twice-yearly, standardised assessment tools to help develop this view, which can also be used to report to parents.



In forming a view of progress and achievement, teachers should ask themselves:

- › **Are students using learning from the progress outcome of the previous phase to make sense of new learning in the current phase?** This demonstrates how well they can connect new learning to what they already know. It generally occurs in the first year of a phase.
- › **Are students consolidating the learning expressed in the progress outcome in a wide range of contexts?** This demonstrates how well and confidently they are using their new learning. This generally occurs in the second year of the phase.
- › **Are students secure in the learning described in the progress outcome within an increasingly complex range of contexts?** Are they showing greater depth of knowledge, understanding, and application as they use their new learning and prepare for the challenges of the next phase? This generally occurs towards the end of the final year of the phase.
- › **Are there gaps in learning that are going to restrict students' ability to make progress in the next phase of their learning?** This is a question teachers should ask across all years of the phase, drawing on the section *Using assessment to inform teaching* (page 23) to consider how to adapt their practices to meet students' learning needs.

Leaders must have a mechanism and strategies for prioritising and closely monitoring urgent action, when required, to support classroom teaching. Where teaching needs to be targeted and intensified to meet specific needs for finite periods, leaders draw on a breadth of available supports, as required.

## Teaching sequences

Each phase has a year-by-year teaching sequence. These sequences support teachers to know what to teach and when and how to teach it as students work towards the progress outcome for the phase. They have been organised to support students to revisit ideas, knowledge, and practices in ways that deepen their learning and enable them to use it at the next phase.

There are two parts in a teaching sequence: statements of **what** to teach, and 'teaching considerations' for **how** to teach:

- › the 'what to teach' statements are preceded by the stem 'Informed by prior learning ...', which reminds teachers to use their professional judgment and assessment information when selecting what content to teach
- › the teaching considerations help teachers to know 'how to teach' this content in response to students' prior knowledge, strengths, and experiences.

The teaching sequence tables should be viewed both vertically and horizontally. Looking down the columns helps teachers know what to plan for in a year's programme. Looking across the rows at the statements for the same concept in the preceding and following years helps teachers to recognise prior learning that students may come with and to consider how they might extend this year's learning. It also helps teachers to form a more detailed view of their students' progress, and it is a strong support when planning for mixed-level classes.

The approach of the year-by-year teaching sequences changes in phase 5, as the content becomes more discipline-focused.

Te Mātaiaho

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# The New Zealand Curriculum

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**MATHEMATICS AND  
STATISTICS YEARS 0-8**

## Board requirements

Schools and kura must give effect to the learning area *Mathematics and statistics Years 0–8*.

*Mathematics and Statistics Years 0–8* is published by the Minister of Education under section 90(1) of the Education and Training Act 2020 (the Act) as a foundation curriculum policy statement and a national curriculum statement. These are the statements of official policy in relation to the teaching of mathematics and statistics that give direction to each school's curriculum and assessment responsibilities (section 127 of the Act), teaching and learning programmes (section 164 of the Act), and monitoring and reporting of student performance (section 165 of the Act and associated Regulations). School boards must ensure that they and their principal and staff give effect to these statements.

The sections of *Mathematics and Statistics Years 0–8* that are published as a national curriculum statement are the Understand–Know–Do (UKD) progress outcomes for each phase (pages 27–29, 55–57, and 83–85). These set out what students are expected to learn over their time at school, including the desirable levels of knowledge, understanding, and skill to be achieved in mathematics and statistics.

The rest is published as a foundation curriculum policy statement. This sets out expectations for teaching, learning, and assessment that underpin the national curriculum statement and give direction for effective mathematics and statistics (or maths, including numeracy) teaching and learning programmes.

The statements come into effect on **1 January 2025**. They replace curriculum levels 1–4 of the existing mathematics and statistics national curriculum statement (learning area). The remainder of the existing mathematics and statistics national curriculum statement remains in force. Apart from those for *English Years 0–6*, other existing foundation curriculum policy statements and national curriculum statements for the New Zealand Curriculum remain in place.

Schools should choose the appropriate mathematics and statistics statements for their students' needs. This means that intermediate and secondary schools may choose to make use of the new statements for some students if they are currently working below curriculum level 5, or that primary and intermediate schools may choose to make use of the existing statements for some students if they are already working above phase 3.

### Reading, writing, and maths teaching time requirements

The teaching and learning of reading, writing,<sup>1</sup> and maths<sup>2</sup> is a priority for all schools. So that all students are getting sufficient teaching and learning time for reading, writing, and maths, each school board with students in years 0–8 must, through its principal and staff, structure their teaching and learning programmes and/or timetables to provide:

- › 10 hours per week of teaching and learning focused on supporting students' progress and achievement in reading and writing, and recognising the important contribution oral language development makes, particularly in the early phases of learning
- › 5 hours per week of teaching and learning focused on supporting students' progress and achievement in maths.

Where reading, writing, and/or maths teaching and learning time is occurring within the context of national curriculum statements other than English or mathematics and statistics, the progression of students' reading, writing, and/or maths dispositions, knowledge, and skills at the appropriate level must be explicitly and intentionally planned for and attended to.

1 While the terms reading and writing are used, these expectations are inclusive of alternative methods of communication, including New Zealand Sign Language, augmentative and alternative communication (AAC), and Braille.

2 For simplicity, 'maths' is used as an all-encompassing term to refer to the grouping of subject matter, dispositions, skills, competencies, and understandings that encompasses all aspects of numeracy, mathematics, and statistics.

## Purpose statement

*Ānō me he whare pūngāwerewere.  
Behold, it is like the web of a spider.*

This whakataukī celebrates intricacy, complexity, interconnectedness, and strength. The learning area of mathematics and statistics weaves together the effort and creativity of many cultures that over time have used mathematical and statistical ideas to understand their world.

In the mathematics and statistics learning area, students learn about and appreciate the power of symbolic representation, reasoning, and abstraction. They learn to investigate, interpret, and explain patterns and relationships in quantity, space, time, data, and uncertainty. As they achieve deep conceptual understanding and procedural fluency in the learning area, students can accurately and efficiently use mathematics and statistics as a foundation for new learning and to solve problems.

Students engage with mathematics and statistics through the exploration of problems, patterns, and trends and appreciate the everyday value of this learning in many areas of their lives, such as personal finance, health, dance, and design. Every student in New Zealand can engage in mathematics and statistics and discover personal enjoyment and curiosity in their learning.

Throughout their learning, students engage with diverse perspectives as they apply their mathematical and statistical understandings. They also learn that mathematics and statistics has an evolving history; many cultures have contributed to, and continue to contribute to, innovations that shape our current thinking.

As they move through the phases of the learning area, students come to understand the value of mathematical and statistical investigation as a lens for collective local, national, and global challenges. Mathematics and statistics allow us to engage with important societal matters, such as the robust and ethical gathering, interpretation, and communication of data, and the use of valid and reliable data to challenge misinformation and disinformation.

Learning in mathematics and statistics builds literacy by developing students' skills in oral and written communication, reasoning, and comprehension. The learning area opens pathways into a wide range of industries that rely on mathematical and statistical knowledge and reasoning. Learning how to use this knowledge purposefully and flexibly allows students to participate fully in an increasingly technology- and information-rich world of work.

# Understand-Know-Do Overview



## Understand

**Understand** describes the deep and enduring mathematical and statistical **big ideas** that students develop over phases 1-5.

### Patterns and variation | Ngā ia auau me ngā rerekētanga

The world is full of patterns and is defined by a multitude of relationships in which change and variation occur. Mathematics and statistics provide structures that are useful for noticing, exploring, and describing different types of patterns and relationships, enabling us to generate insights or make conjectures.

### Logic and reasoning | Te whakaaro arorau me te whakaaroaro

By engaging with mathematical concepts, we develop logical reasoning and critical thinking skills that enable us to evaluate information, question assumptions, and present arguments with clarity. Statistical reasoning from observation and theory allows us to differentiate what is probable from what is possible and to draw reliable conclusions about what is reasonable.

### Visualisation and application | Te whakakite me te whakatinana

The visualisation of mathematical and statistical ideas profoundly influences how we perceive, understand, and interact with abstract concepts. Application in mathematics and statistics involves creating structures and processes that help us understand complex situations, enabling better decision making and communication of ideas.

# Know

**Know** describes the meaningful and important mathematical and statistical **concepts and procedures** through which students develop understanding of the big ideas.

## Number | Mātauranga tau

*Number* focuses on the study of numerical concepts. People use numbers to represent quantities, estimate, and measure. We perform operations on numbers to calculate or compare. Throughout history, different number systems have been developed, reflecting practical and social needs.

## Algebra | Taurangi

*Algebra* focuses on making and using generalisations to reason mathematically. It allows us to identify patterns and underlying mathematical relationships. These generalisations, patterns, and relationships can be represented and communicated using diagrams, graphs, and symbols (including variables). The algebra we use today was created and refined over thousands of years.

## Measurement | Ine

*Measurement* focuses on the concepts and techniques that allow us to quantify phenomena, using appropriate units and systems of measurement. Countries around the world use both standard and non-standard units to measure tangible and intangible objects and quantities.

## Geometry | Āhuahanga

*Geometry* focuses on visualising, representing, and reasoning about the shape, position, orientation, and transformation of objects. Many cultures use tools and techniques derived from the natural world when exploring and describing objects and space.

## Statistics | Tauanga

*Statistics* focuses on tools, concepts, and systematic processes for interpreting situations, using data and its context to understand uncertainty, make conjectures, and inform decision making. Statistical practices include considering the ethics of data collection and the responsibility of safely and securely handling data in different contexts.

## Probability | Tūponotanga

*Probability* focuses on tools and concepts for quantifying chance, dealing with expectation, and using evidence to identify how likely events are to occur. People around the world have relied on and continue to rely on probabilistic thinking when making decisions.

# Do

**Do** describes the **processes** that are fundamental to all mathematical and statistical activities and that underpin students' learning of the big ideas, concepts, and procedures.

## Investigating situations | Te tūhura pūāhua

When we investigate situations using mathematics and statistics, we describe and explore them to build our understanding of them. When investigating, we need to decide which approaches, concepts, and tools to use and how to use them. We often begin with a question or focus of interest and proceed in systematic but flexible ways, using mathematical and statistical concepts and procedures to solve problems and make sense of findings in context. We conclude by evaluating the investigation, which involves reflecting on the solutions and outcomes and our approaches and choices to determine whether they were reasonable, made sense in context, and could be improved on in future investigations.

## Representing situations | Te whakaata pūāhua

When we represent situations mathematically and statistically, we use words or symbols and mental, oral, physical, digital, graphical, or diagrammatic ways to show concepts and findings. We can use representations to compare, explore, simplify, illustrate, prove, and justify, as well as to look for patterns, variations, and trends. Representing a situation in multiple ways enables a deeper and more flexible understanding and allows us to communicate with different audiences.

## Connecting situations | Te tūhono pūāhua

When we connect situations using mathematics and statistics, we recognise and make links by noticing similarities and differences. Connecting helps us to understand the relationships between concepts and procedures in mathematics and statistics. This is important because number, algebra, measurement, geometry, statistics, and probability form a web of interconnected ideas and approaches that can be easier to remember and understand if the connections between them are clear. Connecting also involves linking mathematics and statistics to other learning areas and to a range of contexts.

## Generalising findings | Te whakatauwānui i ngā kitenga

When we generalise mathematical and statistical findings, we move from specific examples to general principles. We use the patterns, regularities, and structures that we find to make conjectures that might apply to other situations. Further investigation can test and refine these conjectures and determine if they apply in all cases. In statistics, we generalise by using trends and variation in data to make inferences and conjectures and to articulate and evaluate claims about similar situations.

## Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

When we explain and justify, we use mathematical and statistical ways of communicating and reasoning to share our ideas and to respond to the ideas, reasoning, and inferences of others. Explaining is how we communicate our inferences and conjectures, build arguments, and unpack stories from data. Justifying involves describing why decisions and findings are reasonable, taking into account limitations arising from assumptions and choices and the evidence on which findings are based.

# Mathematics and statistics learning area structure

This section describes the structure of the mathematics and statistics learning area and how it changes over the five phases of learning. (See [pages 9-11](#) for the general structure of each learning area in the New Zealand curriculum.)

Each phase has:

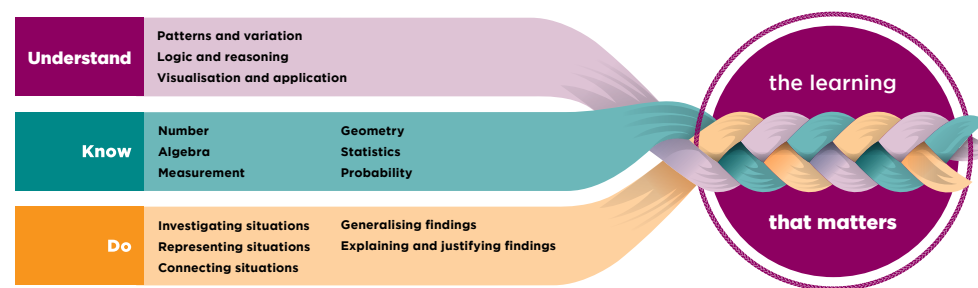
- › a progress outcome describing what students understand, know, and can do by the end of the phase
- › an introduction to the teaching sequence highlighting how to teach during this particular phase
- › a year-by-year teaching sequence highlighting what to teach in the phase, along with teaching considerations for particular aspects of content.


## Progress outcomes

The progress outcomes (one per phase) describe what students will understand, know, and be able to do by the end of the phase.

- › **Understand** describes the big ideas that students develop from learning mathematics and statistics over phases 1 to 5. They help connect school mathematics and statistics with the wider world and represent the critical big-picture concepts of mathematics and statistics.
- › **Know** describes the meaningful and important concepts and procedures in mathematics and statistics. They are broken down into six strands: number, algebra, measurement, geometry, statistics, and probability.
- › **Do** describes the processes students use to represent and work with what they know and understand in mathematics and statistics. These processes are central to how students learn and apply mathematical and statistical knowledge. While there are small progressions in the processes from phase to phase, in general the increasing sophistication of their use comes from applying them to more advanced concepts and procedures.

It is through the interweaving of Understand, Know, and Do that students develop their conceptual understandings and procedural fluency, supporting success and bringing richness and meaning to mathematics and statistics for them.





As students progress through the phases, the focus of their learning shifts. In phase 1, the focus is on developing foundational skills across all strands. In phases 2 and 3, students expand their range of representations and their reasoning to work with increasingly complex concepts across all strands.

This change in focus is seen in how the Understand, Know, and Do progress outcomes are reflected in the year-by-year teaching sequences. The descriptors of what to teach each year have the stem 'Informed by prior learning ...' in order to reinforce that teachers will use their professional judgment about what content to teach and how to teach it. They will make these judgments in response to the prior knowledge, strengths, and experiences that students bring to their learning.

### Teaching sequences

The year-by-year teaching sequences are organised in line with the strands from Know. They describe the incremental teaching required each year as students work towards the progress outcome.

Some statements in the teaching sequences are repeated across multiple years, allowing more time for progression and consolidation. Not all statements are progressed each year; some topics start and others end, reflecting what is developmentally appropriate in learning in mathematics and statistics.

Each statement in a sequence varies in the amount of teaching time it requires. The learning area is designed to enable knowledge and procedures to be connected and taught together, so individual statements in a year sequence should be combined in ways that enhance learning.

The year-by-year content can be viewed both vertically and horizontally. The vertical view helps teachers know what to plan for the next year. The horizontal view allows teachers to follow the statements for one concept across several stages. This helps them understand the prior knowledge students may bring to their learning and helps them decide how to extend this learning. The horizontal view also helps teachers plan for mixed-level classes.

The teaching sequence statements are supported by 'teaching considerations'. These describe evidence-based practices and show how teachers can integrate the processes of Do to help their students develop conceptual and procedural knowledge.

# Teaching guidance

Key characteristics of how people learn have informed the development of the mathematics and statistics learning area. These characteristics are:

- › We learn best when we experience a sense of belonging in the learning environment and feel valued and supported.
- › A new idea or concept is always interpreted through, and learned in association with, existing knowledge.
- › Establishing knowledge in a well-organised way in long-term memory reduces students' cognitive load when building on that knowledge. It also enables them to apply and transfer the knowledge.
- › Our social and emotional wellbeing directly impacts on our ability to learn new knowledge.
- › Motivation is critical for wellbeing and engagement in learning.<sup>1</sup>

All five characteristics are interconnected in a dynamic way. They are always only pieces of the whole, so it is critical to consider them all together. The dynamic and individual nature of learning explains why we see individual learners develop along different paths and at different rates.

The implications of these characteristics for teaching mathematics and statistics are described in this section, with more detail in the introduction to each phase and the 'teaching considerations' in the year-by-year teaching sequences.

The remainder of this section focuses on three key areas of teacher decision making:

- › developing a comprehensive teaching and learning programme
- › using assessment to inform teaching
- › planning.

<sup>1</sup> A description of each characteristic is found on [page 5](#).

## Developing a comprehensive teaching and learning programme

A comprehensive mathematics and statistics programme needs the following components:

- › explicit teaching
- › positive relationships with mathematics and statistics
- › rich tasks
- › communication in mathematics and statistics.

### Explicit teaching

Explicit teaching is a structured, carefully sequenced approach to teaching. The sequencing of content is thought out and broken down into manageable steps, each of which is clearly and concisely explained and modelled by the teacher. Explicit teaching requires a high level of teacher-student interaction, guided student practice, and, when proficiency is achieved, independent practice.

Explicit teaching supports cumulative learning as new knowledge is built on what students already know. Teachers provide multiple opportunities for practising, reviewing, consolidating, and using previous learning alongside new learning.

Explicit teaching takes account of cognitive overload. With sufficient practice, new learning is transferred to long-term memory. This frees up working memory, opening up opportunities for extension, enrichment, and new learning.

Explicit teaching is strongly interactive – it is not simply teacher talk. It includes rich discussions between teachers and students and amongst students, to check on understanding. Teachers adapt the pace of their teaching in response to students' progress. They engage students in creative and challenging tasks to foster motivation and engagement.

Using materials and visual representations throughout explicit teaching supports students to develop conceptual understandings as they move towards more abstract forms of representation, such as equations. Teachers can reduce students' cognitive load by carefully considering the ways in which visual and written information are presented (e.g., how working and explanations are laid out) and by removing unnecessary information to focus on the key teaching and learning points.

Explicit teaching involves:

- › connecting the current focus to previous learning
- › providing concise, step-by-step explanations, accompanied by student input and discussion
- › explaining, modelling, and demonstrating
- › regularly checking for understanding and providing feedback
- › providing opportunities for collaborative and independent practice.

## Positive relationships with mathematics and statistics

Learning is enhanced when students succeed in and feel positive about their learning. If students feel anxious, they have fewer cognitive resources available for learning.

Positive relationships with mathematics and statistics are supported by teachers through:

- › setting high expectations
- › planning experiences that are accessible to every student and provide daily opportunities for success
- › incorporating students' interests, cultures, and prior knowledge
- › planning opportunities for students to explore and think critically
- › supporting students to use mathematics and statistics to make sense of their world and address local, national, and global issues
- › providing manageable challenges that encourage students to develop perseverance, reinforcing that conceptual understanding and procedural fluency develop with consistent effort
- › increasing scaffolding and supports in response to anxiety as a result of cognitive overload
- › valuing mistakes as an important part of the learning process.

Involving families in students' learning journeys and offering opportunities for collaboration support positive relationships with mathematics and statistics. Teachers also model such relationships by showing curiosity, persistence, and enjoyment, and by engaging in mathematics and statistics themselves.

## Rich tasks

Rich tasks are meaningful problem-solving and investigation experiences, designed to invoke curiosity and engagement. They should relate both to mathematical contexts and wider contexts relevant to the communities, cultures, interests, and aspirations of students.

Rich tasks provide a motivational hook when exploring new concepts and procedures. They can also be used to consolidate concepts and procedures that have already been taught, to develop the mathematical and statistical processes of Do, and to facilitate the transfer and application of learning to new situations. These experiences often allow students to decide how to approach the task, developing their agency, confidence, and motivation.

Teachers design rich tasks that are accessible to all students and offer different levels of challenge. They ensure that students are clear about the purpose of learning, and they consider the core requirements of the task as well as the range of possible responses. As students work on rich tasks, teachers plan opportunities for discussion, collaboration, and feedback. They are actively involved in monitoring, prompting, and questioning during the task, to encourage students to ask questions, test conjectures, make generalisations, and form connections.

## Communication in mathematics and statistics

Students communicate throughout the learning process, both to develop conceptual understanding and to share their thinking and reasoning. Rich, extended interactions are pivotal to students' development of knowledge, processes, and dispositions in mathematics and statistics. Effective discussions build knowledge through sharing, active listening or attending, critiquing, questioning, and extending thinking and reasoning.

Rich interactions make students' reasoning visible. This helps teachers recognise how well students are developing mathematical and statistical processes and concepts, and it provides opportunities for teachers to identify misconceptions and correct them. These interactions also allow teachers to develop students' use of mathematical and statistical language, vocabulary, symbols, representations, and reasoning.

## Using assessment to inform teaching

Assessment that informs decisions about adapting teaching practice is moment-by-moment and ongoing. Teachers use observation, conversations, and low-stakes testing to continuously monitor students' progress in relation to their year level in the teaching sequence. They ensure that they notice and recognise the development, consolidation, and use of learning-area knowledge by students within daily lessons, and that they provide timely feedback. They respond by adapting their practice accordingly. For example, they reduce or increase scaffolding and supports, paying particular attention to anxiety caused by cognitive overload. Formative assessment information can also be collected through self and peer assessment, with students reflecting on goals and identifying next steps.

In addition to daily monitoring, teachers use purposefully designed, formative assessment tasks at different points throughout a unit or topic to highlight the concepts and reasoning students use and understand. Teachers ensure such tasks are valid by addressing barriers to learning, so that every student is able to demonstrate what they know and can do.

When planning next steps for teaching and learning, teachers consider students' strengths and responses along with potential opportunities for further consolidation. Next steps could include:

- › designing scaffolds to support students to access and enrich their learning
- › providing opportunities for students to apply new learning
- › planning lessons focused on revisiting, reteaching, or consolidating learning.

Providing timely feedback throughout the learning process and identifying and addressing misconceptions as they arise lead to the efficient and accurate development of learning-area concepts and promote further learning. Teachers can use feedback to prompt students to recall previous learning, make connections, and extend their understanding.


## Planning

This section provides guidance on what to pay attention to when planning mathematics and statistics teaching and learning programmes. In every classroom, there are many ways in which students engage in learning and show what they know and can do. Using assessment information and designing inclusive experiences, teachers plan an 'entry point' to a new concept or procedure that every student can access. Students' interests and the school culture and community shape the planning, adding richness, creativity, and meaning to the programme.

Teaching and learning plans are developed for each year, topic or unit, week, and lesson and make optimal use of instructional time. The following considerations are critical when planning and designing learning:

- › Develop plans using the teaching sequence statements for the year and knowledge of students' prior learning. Plan for all students to experience all the statements in the sequence for their year level.
- › Map out a year-long programme composed of 'units' by looking for opportunities where statements from the teaching sequence can be taught together. These may be in the same strand or across several strands (e.g., statistics and measurement; algebra and geometry). Plan to weave together learning under Know and Do across the unit to build understanding of the big ideas.

- › Order the units so that new learning builds on students' previous learning and connects over the course of the year. Consider the length of time allocated to specific strands and concepts across the year – some concepts may require more teaching time than others. Ensure the year's programme includes opportunities to retrieve, consolidate, and extend learning around previously taught concepts and processes. Regular opportunities to revisit learning within and across units and years supports students to develop procedural fluency with mathematics and statistics concepts. The shape of these opportunities will vary, depending on students' learning needs.
- › Within unit or weekly plans, break down new concepts and procedures into a series of manageable learning experiences, so that students have several opportunities to develop understanding and fluency. Teach mathematics and statistics for an hour a day. Plan for a balance of explicit teaching (to introduce and reinforce learning) and rich tasks (to investigate a concept, support consolidation of previously taught concepts or procedures, and apply learning to new situations).

- 
- › Plan for inclusive teaching and learning at all times. Consider offering multiple methods of participating to all students so that they can engage in a variety of learning experiences and have multiple ways to show their progress. Design for equitable access in all learning opportunities. Identify and reduce barriers to learning, and plan universal supports that are available to all students.
  - › Use flexible groups within a lesson, based on the learning purpose for the lesson (e.g., working as a whole class for demonstration and discussion, in smaller groups to investigate a situation or solve a problem, in pairs to explain thinking and findings). Provide opportunities for both individual and collaborative work, and enable students to determine when they need to work with others and when they need time and space to work independently.
  - › Teach students to use digital tools accurately, appropriately, and efficiently to support their purpose. Enhance teaching and learning with tools for calculating, representing graphs and shapes, and analysing data. While using digital technology is an important skill, students still need the ability to estimate, visualise, and reason, so that they can evaluate whether findings generated by a digital tool are reasonable and effective.

To support students who have not developed the prior knowledge needed for teaching sequence statements for their year or have not learnt everything they have been explicitly taught, teachers can use accelerative approaches. These are approaches that make year-level concepts and procedures accessible to students. They can include additional, targeted small-group teaching, the use of verbal and visual prompts, carefully chosen representations, and explicit teaching of problem-solving strategies.

Teachers can extend students who have developed deep conceptual understanding and procedural fluency for their year by using more challenging rich tasks and problem solving that allow the students to apply their understanding to unfamiliar situations. This also encourages the students to develop further generalisations and to strengthen their mathematical and statistical communication and reasoning.

## Dedicated mathematics and statistics lessons

Depending on the purpose of the lesson, plan to include one or more experiences in each part (Getting started, Working, and Connecting and reflecting). As students are working, take time to notice, recognise, and respond to their learning.

<b>Getting started</b>	<ul style="list-style-type: none"><li>› Recall and connect to prior learning to provide a starting point for all students to access and understand new concepts or processes.</li><li>› Introduce new concepts using a focus activity, group challenge, or task that activates prior knowledge and interests.</li></ul>
<b>Working</b>	<ul style="list-style-type: none"><li>› Provide whole-class, small-group, paired, or individual work opportunities for students to develop or apply concepts and procedures through investigations, tasks, or games.</li><li>› Explicitly teach concepts and procedures by leading interactions that include explanations, demonstrations, questioning, short tasks, and discussion. Use clear and concise language, including correct mathematical and statistical vocabulary, and clear working layouts and notation.</li><li>› Provide additional explicit teaching based on the learning needs of individual students.</li><li>› Help students organise new knowledge in ways that connect with their prior learning – for example, by discussing connections, using graphic organisers, or carefully ordering concepts and procedures in relation to prior learning.</li><li>› Support consolidation of knowledge with targeted practice and activities. For students early in the process of consolidation, these activities should be scaffolded and guided. As students develop understanding and fluency, they complete the activities with increasing independence.</li><li>› Support students to retrieve and use previously taught concepts and procedures in connected ways, such as applying them while investigating situations.</li></ul>
<b>Connecting and reflecting</b>	<ul style="list-style-type: none"><li>› Clearly summarise and connect to the purpose of the lesson.</li><li>› Review learning by discussing, sharing, and analysing the experiences of the lesson.</li><li>› Make connections with prior learning, between mathematics and statistics concepts, with other learning areas, and with situations outside of the classroom.</li><li>› Pre-teach to prepare students for the next lesson.</li><li>› Highlight progress and examples of curiosity, resilience, and persevering through challenge.</li></ul>

Phase

1

Years 0–3

## Progress outcome by the end of year 3 (Foundation)

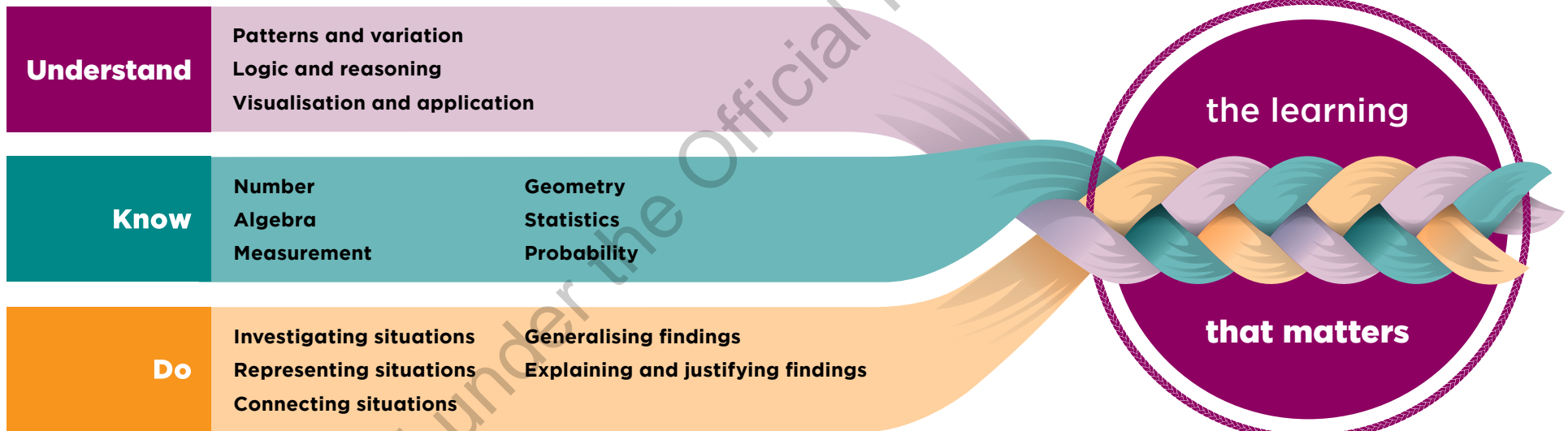
*Thriving in environments rich in literacy and maths*

*Te tupu pāhautea i te taiao ako e haumako ana i te reo matatini me te pāngarau*

The critical focus of phase 1 is for all students to thrive in environments rich in literacy and maths. In mathematics and statistics, students learn to use logic and reasoning to investigate, classify, and describe patterns and variations in quantities, shapes, and data. They begin to generalise and to understand the properties of numbers and attributes of shapes. They use materials,

number lines, and pictures to visualise these concepts, make connections between representations, and explain their reasoning.

The phase 1 progress outcome describes the understanding, knowledge, and processes that students have multiple opportunities to develop over the phase.



The phase 1 progress outcome is found on the following two pages.

## Understand

As students build knowledge through their use of the mathematical and statistical processes, they begin to understand the following.

### Patterns and variation | Ngā ia auau me ngā rerekētanga

The world is full of patterns and is defined by a multitude of relationships in which change and variation occur. Mathematics and statistics provide structures that are useful for noticing, exploring, and describing different types of patterns and relationships, enabling us to generate insights or make conjectures.

### Logic and reasoning | Te whakaaro arorau me te whakaaroaro

By engaging with mathematical concepts, we develop logical reasoning and critical thinking skills that enable us to evaluate information, question assumptions, and present arguments with clarity. Statistical reasoning from observation and theory allows us to differentiate what is probable from what is possible and to draw reliable conclusions about what is reasonable.

### Visualisation and application | Te whakakite me te whakatinana

The visualisation of mathematical and statistical ideas profoundly influences how we perceive, understand, and interact with abstract concepts. Application in mathematics and statistics involves creating structures and processes that help us understand complex situations, enabling better decision making and communication of ideas.

## Know

### Number | Mātauranga tau

By the end of this phase, students know that our number system is base 10, with ten digit symbols. The place value of a digit in a number depends on its position; as we move to the left, each column is worth ten times more, with zero used as a placeholder. Students know that they can subitise (recognise without counting) patterns to support estimations and calculations. They know that numbers can be partitioned and recombined in different ways. Addition is putting parts together to find a total or whole. Subtraction takes parts away from a whole; it is also the difference between numbers. Multiplication and division involve recognising and working with equal groups and how many are in each group, the number of groups, and the total amount.

Students come to know that fractions are numbers that can be represented using words, pictures, or symbols. When fractions are represented symbolically, the bottom number (the denominator) shows how many pieces a whole has been equally split into, and the top number (the numerator) shows how many of those parts the fraction represents. Fractions show parts of a whole region, set of objects, or measurement.

### Algebra | Taurangi

By the end of this phase, students know that patterns are made up of elements, including numeric or spatial elements, in a sequence governed by a rule. Repeating patterns have a unit of repeat; growing patterns can increase or decrease. The equal sign is relational in that it shows that the two sides of an equation represent the same quantity. Students also know that an algorithm is a set of step-by-step instructions for completing a task or solving a problem.

### Measurement | Ine

By the end of this phase, students know that systems of measurement have a history and that different cultures use different approaches to measuring. Students know that they can measure and compare various attributes, such as length, area, volume, capacity, mass (weight), temperature, time, duration, and turn, using informal or standard units. When measuring, the measurement units must remain the same and there must be no gaps or overlaps between them. Students also know that the distance around the boundary of a two-dimensional shape gives perimeter, covering a surface gives area, and filling a three-dimensional shape gives capacity or volume.

### Geometry | Āhuahanga

By the end of this phase, students know that patterns in shapes can be used to compare, classify, and predict. Two- and three-dimensional shapes have features that can be observed and described using geometric language. Shapes and objects can flip (reflect), turn (rotate), slide (translate), and be used to create patterns. Objects can be rotated in space and may appear different from other perspectives. Students know that maps are two-dimensional representations of places in the world with symbols to show locations and landmarks. The position of a location can be described relative to another location, including a known environmental feature.

## Statistics | Tauanga

By the end of this phase, students know that data is information about the world, that it comes in many forms, and that it helps them to learn about people, their lives, and their environment. They know that a statistical enquiry cycle can be used to investigate a group, using questions that they ask of the data for the group. A variable refers to an attribute or measurement of the people or objects being studied, such as colour, height, or number of children. Sorting and organising the data for variables helps to make sense of data and to answer summary investigative questions. Data visualisations are representations of all available values for one or more variables that reveal relationships or tell a story.

## Probability | Tūponotanga

By the end of this phase, students know that a chance-based situation has a set of possible outcomes that can be arranged into events. The probability of an event is the chance of it occurring.

## Do

### Investigating situations | Te tūhura pūāhua

By the end of this phase, students can work with others to pose a question for investigation, find entry points for addressing the question, and plan an investigation pathway and follow it. They can identify relationships and relevant prior experience and knowledge to support the investigation. They can describe progress on the investigation pathway and work with others to make sense of outcomes or conclusions in the light of a given situation and context.

### Representing situations | Te whakaata pūāhua

By the end of this phase, students can use representations to explore, find, and illustrate patterns. They use representations to learn new ideas and explain ideas to others, and they use visualisation to mentally represent and manipulate groups and shapes. They select or create appropriate mental, oral, physical, or virtual representations.

### Connecting situations | Te tūhono pūāhua

By the end of this phase, students can suggest connections between concepts, ideas, approaches, and representations. They connect new ideas to things they already know. They also make connections with ideas in other learning areas and with familiar local contexts.

### Generalising findings | Te whakatauwānui i ngā kitenga

By the end of this phase, students can notice and explore patterns, structure, and regularity and make conjectures about them. They identify relationships, including similarities, differences, and new connections. They represent specific instances and look for when conjectures about them might be applied in another situation or always be true. They test conjectures, using reasoning and counterexamples to decide if they are true or not. They use words and pictures to express generalisations.

### Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

By the end of this phase, students can make statements and give explanations about what they notice and wonder, and they make deductions based on prior knowledge. They ask questions to clarify and understand others' thinking and use evidence and reasoning to explain why they agree or disagree with statements. They develop collective understandings by sharing and building on ideas with others and can present basic explanations and arguments for an idea, solution, or process.

## Teaching sequence

*Thriving in environments rich in literacy and maths*

*Te tupu pāhautea i te taiao ako e haumako ana i te reo matatini me te pāngarau*

This section describes how the components of a comprehensive mathematics and statistics teaching and learning programme are used during the first phase of learning at school.

Throughout phase 1, students experience teaching that encourages curiosity and fosters success, as they explore environments and contexts rich in number and spatial elements. Active, hands-on experiences engage them in mathematics and statistics, with meaningful tasks that reflect their interests and the world outside the classroom.

Continuously monitor students' reasoning, questions, engagement, and use of representations, and respond quickly to address any misconceptions. Be mindful of providing manageable learning experiences, building on students' prior learning and leading to further challenge.

### Explicit teaching

- › Engage students in the mathematical and statistical processes of Do. Explicitly teach students to use them, and demonstrate them regularly as part of the teaching.
- › Teach connected concepts and procedures together. For example, when teaching time (within the measurement strand) connect with fractions (within number) and turns (within geometry). Point out connections within concepts (e.g., “If I know  $3 + 4$ , then I know  $4 + 3$ ”).
- › Demonstrate new concepts or procedures using clearly explained, manageable steps.
- › Think ‘aloud’. Voice decision making (e.g., about which numbers or operations to use) while demonstrating a procedure or process.
- › Ensure that every student engages in the active recall of previous learning (e.g., through games, matching activities, ‘think, pair, share’). Prompt students to make connections between previous and new learning.
- › Plan ways for students to consolidate their mathematical and statistical learning and build procedural fluency. Use a range of guided and independent practice tasks, such as working on problems that use a procedure that has been demonstrated. Use songs, games, materials, families of facts, and digital tools to build fluency and for students to practise skip counting, addition, subtraction, multiplication, and division facts.

## Positive relationships with mathematics and statistics

- › Encourage students to ‘have a go’ and take risks. Reinforce the idea that mistakes help us learn as we try new procedures or share ideas.
- › Select highly interesting contexts based on knowledge of students’ personal experiences and backgrounds. Encourage students to connect with mathematics and statistics outside school by bringing in photos, resources, books, and other artefacts from home that link to mathematics and statistics learning.

## Rich tasks

- › Use open-ended investigations with the whole class, groups, or individuals to support students to understand concepts and extend their learning. For example, plan investigations into local situations (e.g., “What should the new items on the lunch order menu cost?”) and into mathematical situations (e.g., the different ways of partitioning 24 into smaller groups).
- › Choose problems or investigations that help students notice structures and relationships (e.g., present and discuss ‘odd-one-out’ numbers or shapes).
- › Teach problem-solving and investigation strategies. Support students to read and make sense of a problem – through drawing, using materials, or trying some numbers – and to then plan how to solve it, take action to apply their plan, and check their findings.

## Communication in mathematics and statistics

- › Use numbers, materials, and pictorial representations (e.g., diagrams and pictures). Select representations that support the purpose of learning and help students to show their thinking and reasoning and to learn new ideas. Over the phase, move students towards using symbols and showing operations as equations. Number lines are a key representation in this phase for showing, ordering, and comparing numbers (including fractions) and for demonstrating operations.
- › Prompt students to visualise and identify patterns, connections, and structures. Engage them in tasks where they are sorting, grouping, partitioning, and discussing what they have noticed and are wondering about. Guide them to notice and respond to patterns, similarities, and differences.
- › Build students’ mathematical and statistical vocabulary. Use games, songs, word walls, books, and digital tools. Intentionally use vocabulary to connect students’ informal language with appropriate mathematical and statistical language. In doing so, draw on students’ first and heritage languages, so that they can use their languages as a resource to connect their thinking and learning.
- › Foster interactions that allow students to discuss, clarify, and explain their mathematical and statistical ideas. Encourage students to summarise, ask questions, and make suggestions. Help them to recall and connect mathematical and statistical learning using questions, materials, and verbal or visual prompts.

# Number

	<b>During the first 6 months</b> <i>Informed by prior learning, teach students to:</i>	<b>During the first year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the second year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the third year</b> <i>Informed by prior learning, teach students to:</i>
<b>Number structure</b>	› subitise (recognise without counting) the number of objects in a collection of up to 5	› subitise (recognise without counting) the number of objects in a collection of up to 10, including by combining two patterns of 1-5 objects	› group objects in a collection of at least 10, subitise the number of objects in each part, and find the total number in the collection using the parts	› estimate the number of objects in a collection of less than 100, using patterns and groupings
	› count forwards or backwards from any whole number between 1 and 10, and then between 1 and 20	› count forwards or backwards in 1s, 2s, and 10s from any whole number between 1 and 20, and then between 1 and 100	› count forwards or backwards in 1s, 2s, 5s, and 10s from any whole number between 1 and 100	› count forwards or backwards in 2s, 3s, 5s, and 10s from any whole number between 1 and 1,000
	› identify, read, and write whole numbers up to at least 10	› identify, read, and write whole numbers up to at least 20, and represent them using the ten-and-ones structure of teen (11-19) and -ty (multiples of 10) numbers (e.g., $17 = 10 + 7$ , $20 = 2 \times 10$ )	› identify, read, and write whole numbers up to at least 100, and represent them using base 10 structure	› identify, read, and write whole numbers up to at least 1,000, and represent them using base 10 structure
	› compare and order whole numbers up to at least 10 and ordinal numbers (e.g., 1st, 2nd, 3rd), using words	› compare and order whole numbers up to at least 20 and ordinal numbers (e.g., 1st, 2nd, 3rd), using words or numerals and suffixes	› compare and order whole numbers up to at least 100	› compare and order whole numbers up to at least 1,000

## Teaching considerations

Use a range of materials and images that **represent** structured and unstructured patterns and collections (e.g., dot patterns, 10s frames, dice, materials that can be grouped in 10 such as ice-block sticks).

Also use language that quantifies and compares pattern arrangements (e.g. more, less, the same, different, combine, separate).

**Connect** subitising to partitioning collections of objects (e.g., 6 and 2 on two dice are the same as 5 and 3 on two 10s frames).

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Use a range of materials (e.g., number lines, 100s boards, number flip charts, 1,000s books, a Slavonic abacus, ice-block stick bundles).

In general, support students to practise counting (e.g., in 2s and 5s) in short sequences (e.g., at year 3, “Count in 1s from 895 to 904; count in 2s from 90 to 110”).

**Investigate** short patterns in multiples of 2s, 3s, 5s, and 10s, using rhymes, songs, choral counting, the grouping of discrete objects, the recording of patterns, and picture books.

Have students practise finding 1, 10, or 100 more or less for a given number. Use materials to support students to identify numbers and patterns (e.g., 100s boards, 1,000s books).

**Connect** to te reo Māori to support place-value (PV) understanding (e.g., tekau mā tahi (10 and 1), toru tekau mā rua (30 and 2)).

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Have students practise saying, reading, and writing any given number within an identified number range. Use materials to support this (e.g., number flip boards, PV flip charts and houses).

**Explain** that base 10 structure is based on groups of ten (ten ones form one ten, ten tens form one hundred, ten hundreds form one thousand etc.) and that both the position and value of a digit indicate the quantity it represents (e.g., 64 has 6 tens and 4 ones,  $60 + 4 = 64$ ).

Have students **investigate** and **represent** the base 10 structure of numbers using a range of materials and digital tools (e.g., 100s boards, PV houses, PV blocks, ice-block sticks, arrow cards, number fans, words, numerals).

**Investigate** odd and even numbers and the patterns they notice.

**Connect** numerals, representations of them, and language (e.g., 652 represented with PV money: “ $652 = 600 + 50 + 2$ , 6 hundreds + 5 tens + 2 ones, six hundred and fifty two”).

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Show the sequencing of numbers using a number line (select-numbered, marked, or empty). Change the number-line orientation from horizontal to vertical if students need support with the concepts of before and after.

**Explain** and use the language of comparison when demonstrating why one number is larger or smaller than another (e.g., “63 is larger than 36, as 6 tens is larger than 3 tens”).

Show how the position of digits in the PV structure helps us to order and compare two- and three-digit numbers.

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	<b>During the first 6 months</b> <i>Informed by prior learning, teach students to:</i>	<b>During the first year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the second year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the third year</b> <i>Informed by prior learning, teach students to:</i>
<b>Number structure</b>	<ul style="list-style-type: none"> <li>› partition up to 5 objects, and then up to 10 objects, using a systematic approach and noticing patterns</li> </ul>	<ul style="list-style-type: none"> <li>› partition and regroup up to 20 objects in different ways, using a systematic approach and noticing patterns</li> </ul>	<ul style="list-style-type: none"> <li>› partition and regroup whole numbers up to at least 100, using a systematic approach and noticing patterns (e.g., <math>10 + \underline{\quad} = 70</math>, <math>20 + \underline{\quad} = 70</math>, <math>30 + \underline{\quad} = 70</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› partition and regroup whole numbers up to at least 1,000, using a systematic approach and noticing patterns (e.g., <math>400 + 300 = \underline{\quad}</math>, <math>350 + \underline{\quad} = 500</math>)</li> </ul>
<b>Operations</b>		<ul style="list-style-type: none"> <li>› use estimation to predict results and to check the reasonableness of calculations</li> </ul>	<ul style="list-style-type: none"> <li>› use estimation to predict results and to check the reasonableness of calculations</li> </ul>	<ul style="list-style-type: none"> <li>› use estimation to predict results and to check the reasonableness of calculations</li> </ul>
			<ul style="list-style-type: none"> <li>› identify the nearest ten to any whole number up to 100</li> </ul>	<ul style="list-style-type: none"> <li>› round whole numbers up to 1,000 to the nearest hundred or ten</li> </ul>
	<ul style="list-style-type: none"> <li>› join and separate groups of up to a total of 10 objects by grouping and counting</li> </ul>	<ul style="list-style-type: none"> <li>› join and separate groups of up to a total of 20 objects and find the difference between groups by grouping and counting (e.g., <math>9 + 6</math>, <math>7 + \underline{\quad} = 11</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› add and subtract numbers up to 100 without renaming (e.g., <math>53 + 21</math>, <math>55 - 32</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› add and subtract numbers up to at least 100 (e.g., <math>43 - 28</math>, <math>37 + 18</math>)</li> </ul>
		<ul style="list-style-type: none"> <li>› explore addition facts up to 10 and their corresponding subtraction facts (families of facts), including doubles and halves</li> </ul>	<ul style="list-style-type: none"> <li>› recall addition facts up to 10, and explore addition facts up to 20 and their corresponding subtraction facts (families of facts), including doubles and halves</li> </ul>	<ul style="list-style-type: none"> <li>› recall addition facts up to 20 and their corresponding subtraction facts (families of facts), including doubles and halves</li> </ul>

## Teaching considerations

**Investigate** and **represent** the partitioning of numbers using appropriate materials for the year level – for example:

- › multilink cubes, bead strings, 10s frames, and counters, at 6 months and year 1
- › a Slavonic abacus, ice-block sticks, and PV money, at year 2
- › PV money and PV blocks, at year 3.

**Connect** students' subitising with pattern understanding (at 6 months and year 1) and known groupings and facts (at years 2-3).

**Explain** and discuss how to systematically record the partitioning of numbers (e.g., using partitioning diagrams, tables, vertically-listed equations).

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**Explain** and spend time developing the concepts of:

- › estimation, using the language of 'about', 'more or less', and 'close to'
- › rounding, using 100s boards and number lines marked with the multiples of 10 or 100, progressing to unmarked number lines at year 3.

Have students **investigate** and **connect** practical estimation situations that involve quantities and measures (e.g., the number of balls in a box, the number of steps to the door, the length of a piece of string).

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**Explain** and discuss addition and subtraction using **representations**, including:

- › discrete materials (counters, blocks, context items), 10s frames, and number lines (at 6 months and year 1)
- › bundles of sticks, number disks, and number lines (at year 2)
- › PV materials (PV money, blocks, and discs) and number lines (at year 3).

**Connect** symbols and equations with problems, using correct vocabulary (e.g., 'add', 'join', and 'plus' for addition). Have students practise decoding and solving word problems.

At year 3, **explain and connect** horizontal equations and the vertical-column method for addition and subtraction.

Demonstrate making estimates or mental calculations by **connecting** to place value, partitioning, and known facts.

Use a range of problem types (e.g., result, change, start-unknown).

Use worked examples and think-alouds to **explain** the most efficient approaches when solving problems.

Have students **investigate** and **generalise** adding 0 to or subtracting 0 from a number (at year 1) and applying the commutative property of addition (e.g.,  $5 + 4 = 4 + 5$ ).

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Use materials to **investigate** addition and subtraction facts (e.g., counters, 10s frames, an abacus, multilink cubes), and use part-whole diagrams to develop subtraction facts and **connect** to addition facts.

**Explain** how to record equations and families of facts, **connecting** with the language for each operation.

Provide a range of tasks to consolidate learning and develop fluency (e.g., physical and digital games, using families of facts and, at year 3, table grids).

	<b>During the first 6 months</b> <i>Informed by prior learning, teach students to:</i>	<b>During the first year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the second year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the third year</b> <i>Informed by prior learning, teach students to:</i>
<b>Operations</b>			<ul style="list-style-type: none"> <li>› identify the relationship between skip counting and multiplication facts for 2s, 5s, and 10s</li> </ul>	<ul style="list-style-type: none"> <li>› recall multiplication and corresponding division facts for 2s, 3s, 5s, and 10s</li> </ul>
		<ul style="list-style-type: none"> <li>› multiply and divide using equal grouping or counting</li> </ul>	<ul style="list-style-type: none"> <li>› multiply and divide using equal grouping or skip counting (e.g., in 2s, 5s, and 10s)</li> </ul>	<ul style="list-style-type: none"> <li>› multiply a one- or two-digit number by a one-digit number, using skip counting or known facts (e.g., <math>4 \times 6</math>, <math>2 \times 23</math>)</li> </ul>
				<ul style="list-style-type: none"> <li>› divide whole numbers by a one-digit divisor with no remainders, using grouping (e.g., <math>24 \div 3</math>, <math>32 \div 4</math>)</li> </ul>
<b>Rational numbers</b>		<ul style="list-style-type: none"> <li>› identify and represent halves and quarters as fractions of sets and regions, using equal parts of the whole</li> </ul>	<ul style="list-style-type: none"> <li>› identify, read, write (using symbols and words), and represent halves, quarters, and eighths as fractions of sets and regions, using equal parts of the whole</li> </ul>	<ul style="list-style-type: none"> <li>› identify, read, write, and represent halves, thirds, quarters, fifths, sixths, and eighths as fractions of sets and regions, using equal parts of the whole and by positioning on a number line</li> </ul>
			<ul style="list-style-type: none"> <li>› directly compare two fractions involving halves, quarters, and eighths</li> </ul>	<ul style="list-style-type: none"> <li>› compare and order fractions involving halves, quarters, and eighths and identify when two fractions are equivalent</li> </ul>

## Teaching considerations

Use a range of materials to **represent** skip counting and multiplication and division facts (e.g., 100s boards, choral counting, games, number lines, a Slavonic abacus, families of facts, and, at year 3, table grids).

Provide a range of tasks to consolidate learning and develop fluency.

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**Represent** multiplication and division problems using discrete materials, pictures, diagrams, symbols, number lines, words, equations, digital tools, and, at year 3, arrays, PV materials, and bar models.

Use correct mathematical language when discussing multiplication and division (e.g., multiply, groups of, sets of, rows of, equal groups, divide, share equally).

Have students practise decoding and solving word problems.

**Connect** with subitising and addition and subtraction concepts when demonstrating solving multiplication and division problems.

**Explain** and **represent** division as a sharing problem (e.g., “Share 12 marbles equally among 3 friends”) or a grouping problem (e.g., “You have 12 marbles. How many groups of 3 marbles can you make?”).

Use worked examples and think-alouds to **explain** the most efficient approaches when solving multiplication and division problems.

**Investigate** and **generalise** multiplying a number by 0 or 1, dividing a number by 1, dividing a number by itself, and why we cannot divide by 0 (e.g., by trying to solve  $0 \times \_ = 5$ ).

At year 3, **explain** and use the multiplicative identity (e.g.,  $5 \times 1 = 5$ ,  $4 \div 1 = 4$ ) and commutative property (e.g.,  $3 \times 4 = 4 \times 3$ ).

Demonstrate making estimates or mental calculations by **connecting** to place value, partitioning, and known facts.

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**Represent** fractions using a range of materials – continuous (bar models, number lines), discrete (sets of objects), and digital.

**Explain** and reinforce that when fractions are represented symbolically:

- › the denominator is the bottom number and shows how many pieces a whole has been equally split into
- › the numerator is the top number and shows how many of those parts the fraction represents.

Have students practise saying, reading, and writing fractions in words and symbols.

**Explain** how to fold paper strips to create fractions of one whole. Label the parts using words and symbols, and use them to create a fraction wall for comparing and ordering fractions.

**Explain** that a fraction is a number that can be placed on a number line.

	<b>During the first 6 months</b> <i>Informed by prior learning, teach students to:</i>	<b>During the first year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the second year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the third year</b> <i>Informed by prior learning, teach students to:</i>
<b>Rational numbers</b>		<ul style="list-style-type: none"> <li>› find a half or quarter of a set using equal sharing and grouping.</li> </ul>	<ul style="list-style-type: none"> <li>› find a half and quarter of a set by identifying groups and patterns (rather than sharing by ones), and identify the whole set or shape when given a half or quarter</li> </ul>	<ul style="list-style-type: none"> <li>› find a unit fraction of a whole number (e.g., <math>\frac{1}{3}</math> of 15), and identify the whole set or amount when given a unit fraction (e.g., "<math>\frac{1}{4}</math> of the set is 3, what is the whole set?")</li> </ul>
				<ul style="list-style-type: none"> <li>› add and subtract unit fractions with the same denominator (e.g., <math>\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}</math>)</li> </ul>
<b>Financial mathematics</b>			<ul style="list-style-type: none"> <li>› recognise and order New Zealand denominations up to \$20 according to their value, make groups of 'like' denominations, and calculate their value.</li> </ul>	<ul style="list-style-type: none"> <li>› make amounts of money using one- and two-dollar coins and 5-, 10-, 20-, 50-, and 100-dollar notes.</li> </ul>

## Teaching considerations

**Investigate** a range of practical situations using a range of **representations**, including materials, drawings and diagrams, and digital tools (e.g., discrete objects, bar models, paper strips for partitioning).

Make **connections** between:

- › symbols, words, and pictures
- › counting, subitising patterns and known groupings, and skip counting to solve problems (at years 1-2)
- › skip counting and using known addition and multiplication facts to solve problems (at year 3).

Use mathematical language to develop an understanding of fractions (e.g., numerator, denominator, shared equally, divide, partition, equal parts).

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**Investigate** adding and subtracting fractions within familiar contexts (e.g., cutting apples into eighths or partitioning paper strips into six equal parts, and then **representing** addition and subtraction with these materials).

**Connect representations**, including symbols and equations, to drawings and materials (e.g., fraction walls, paper fraction strips), and show them on a number line.

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Have students use play money (coins and notes) to **represent** practical financial situations.

At year 2, compare only notes with notes or cents with cents, not a mixture of them.

At year 2, **investigate** appropriate financial situations that involve both saving and spending.

**Connect** to place value, addition and subtraction, and skip counting when calculating amounts.

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# Algebra

	<b>During the first 6 months</b> <i>Informed by prior learning, teach students to:</i>	<b>During the first year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the second year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the third year</b> <i>Informed by prior learning, teach students to:</i>
<b>Equations and relationships</b>		<ul style="list-style-type: none"> <li>› solve true or false number sentences and open number sentences involving addition and subtraction of one-digit numbers, using an understanding of the equal sign (e.g., <math>2 + 5 = 3 + \underline{\quad}</math>, <math>7 - 5 = 6 - 4</math> (T or F?))</li> </ul>	<ul style="list-style-type: none"> <li>› solve true or false number sentences and open number sentences involving addition and subtraction of one- and two-digit numbers, using an understanding of the equal sign (e.g., <math>18 + \underline{\quad} = 17 + 6</math>, <math>17 = 25</math> (T or F?))</li> </ul>	<ul style="list-style-type: none"> <li>› solve true or false number sentences and open number sentences involving addition and subtraction, using an understanding of the equal sign</li> </ul>
	<ul style="list-style-type: none"> <li>› copy, continue, create, and describe a repeating pattern with two elements.</li> </ul>	<ul style="list-style-type: none"> <li>› copy, continue, create, and describe a repeating pattern with three elements, and identify missing elements in a pattern</li> </ul>	<ul style="list-style-type: none"> <li>› recognise and describe the unit of repeat in a repeating pattern, and use it to predict further elements using the ordinal position</li> </ul>	<ul style="list-style-type: none"> <li>› recognise, continue, and create repeating and growing patterns, and describe a rule to explain a pattern</li> </ul>
<b>Algorithmic thinking</b>		<ul style="list-style-type: none"> <li>› follow step-by-step instructions to complete a simple task.</li> </ul>	<ul style="list-style-type: none"> <li>› follow and give step-by-step instructions for a simple task, identifying and correcting errors as the instructions are followed.</li> </ul>	<ul style="list-style-type: none"> <li>› create and use a set of precise, step-by-step instructions for carrying out a familiar routine or task.</li> </ul>

## Teaching considerations

**Represent** the equal sign as the 'same as' to demonstrate it is a symbol of equivalence.

**Investigate** number sentences using **representations** such as:

- › 10s frames and discrete materials (at years 1-2)
- › word problems with comparisons (at year 3).

At years 2-3, solve number sentences that have numbers beyond what students are using in operations, so that the emphasis is on the equal relationship, not operating.

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**Investigate** repeating and growing patterns in a range of contexts (e.g., cultural patterns, patterns in the local environment and on everyday objects).

Use materials, sound, movement, and digital tools to **represent** and continue repeating and growing patterns. At years 2-3, demonstrate recording the pattern in a table.

Form **generalisations** when students notice that repeating patterns constructed in different ways are similar (e.g., 'red, blue, red, blue' and 'hop, jump, hop, jump' are ABAB patterns). Help students to notice the similarities and differences between patterns by recording them.

With students at year 2, **generalise** by using the unit of repeat and ordinal position to identify further elements in a pattern.

Use mathematical language and sentence starters to support students to **explain** and **justify** how a pattern is repeating or growing and to predict further terms.

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**Represent** step-by-step instructions using drawings, words, flow diagrams, and verbal instructions that form a sequence.

With students, **investigate** sorting unfamiliar and familiar objects according to a set of instructions, directing a person or object (e.g., through an obstacle course or maze), and following and creating a set of pictorial instructions.

**Explain, justify**, and show how a set of instructions is complete or incomplete, using think-alouds and prompts.

**Connect** a series of events from a story, narrative, or daily timetable with statements in Number, Algebra, Measurement, and Geometry.

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# Measurement

	<b>During the first 6 months</b> <i>Informed by prior learning, teach students to:</i>	<b>During the first year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the second year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the third year</b> <i>Informed by prior learning, teach students to:</i>
<b>Measuring</b>			<ul style="list-style-type: none"> <li>› estimate and use an informal unit repeatedly to measure the length, mass (weight), volume, or capacity of an object</li> </ul>	<ul style="list-style-type: none"> <li>› estimate and then reliably measure length, capacity, and mass (weight) using whole-number metric units (e.g., from tools with labelled markings)</li> </ul>
	<ul style="list-style-type: none"> <li>› directly compare two objects by an attribute (e.g., length, mass (weight), capacity)</li> </ul>	<ul style="list-style-type: none"> <li>› compare the length, mass (weight), volume, or capacity of objects directly or indirectly (e.g., by comparing each of them with another object, used repeatedly)</li> </ul>	<ul style="list-style-type: none"> <li>› compare and order several objects using informal units of length, mass (weight), volume, or capacity</li> </ul>	<ul style="list-style-type: none"> <li>› compare and order objects using metric units of length, mass (weight), or capacity</li> </ul>
			<ul style="list-style-type: none"> <li>› turn, and describe how far an object or person has turned, using full, half, and quarter turns as benchmarks</li> </ul>	<ul style="list-style-type: none"> <li>› turn, and describe how far an object or person has turned, using full, half, quarter, and three-quarter turns as benchmarks</li> </ul>
	<ul style="list-style-type: none"> <li>› connect days of the week to familiar events and daily routines (e.g., the class timetable).</li> </ul>	<ul style="list-style-type: none"> <li>› identify how the passing of time is measured in years, months, weeks, days, hours</li> <li>› name and order the days of the week, and sequence events in a day using everyday language of time</li> </ul>	<ul style="list-style-type: none"> <li>› name and order the months and seasons, and describe the duration of familiar events using months, weeks, days, and hours</li> </ul>	<ul style="list-style-type: none"> <li>› identify the duration of events using years, months, weeks, days, hours, minutes, and seconds</li> </ul>

## Teaching considerations

**Explain** estimation, using the language of ‘about’, ‘more or less’, and ‘close to’ to help students reflect on what the quantity or measure might be.

**Investigate** practical estimating and measuring situations, using appropriate measuring tools (e.g., at year 2, balance scales, capacity containers, informal units; at year 3, rulers, measuring jugs and cups, scales).

At year 3, **explain** how to construct and use measurement devices, particularly rulers, measurement containers, and balance scales. Demonstrate how to accurately measure length in centimetres, mass (weight) in grams, and capacity in millilitres (at year 3).

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**Investigate** practical measuring situations to compare and order objects – for example:

- › which is longer or shorter, is heavier or lighter, or holds more or less (at 6 months)
- › comparing and ordering up to three objects (at year 1)
- › explaining how identical informal units need to be used when measuring (at year 2)
- › using tools like rulers, measurement containers, and scales (at year 3).

Use mathematical language to **explain** and **justify** comparative measurement attributes (e.g., long and short; heavy, heavier, and heaviest; the same as; full and empty; more and less; wide, wider, and widest). Include descriptive te reo Māori that makes the properties of objects and shapes clear.

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**Investigate** and **explain** situations involving angles as ‘how far an object or person has turned.’ Have students turn physical objects and themselves.

**Connect** turns with fractions (e.g., half, a quarter, three quarters).

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Use visual **representations** to support the sequencing of events (e.g., pictorial daily timetables, calendars, day-and-month cards).

Explore estimating the duration of everyday events using minutes and seconds (e.g., “How long is it until the bell rings?”). Practise recalling a sequence of events in the past and predicting future events.

Use mathematical language to **explain** and **justify** comparisons of duration and points in time (e.g., before, after, soon, later, next, today, tomorrow, yesterday, 1st, 2nd, 3rd).

**Investigate** using a calendar to work out the number of days, weeks, or months until important events (e.g., the number of days until Matariki, the number of weeks until the end of term).

Explore informal ways of measuring short periods of time to identify which events last longer.

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	<b>During the first 6 months</b> <i>Informed by prior learning, teach students to:</i>	<b>During the first year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the second year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the third year</b> <i>Informed by prior learning, teach students to:</i>
<b>Measuring</b>		<ul style="list-style-type: none"> <li>› tell the time to the hour using the language of 'o'clock'.</li> </ul>	<ul style="list-style-type: none"> <li>› tell the time to the hour and half-hour, using the language of 'past' and 'o'clock'.</li> </ul>	<ul style="list-style-type: none"> <li>› tell the time to the hour, half-hour, and quarter past and quarter to the hour</li> </ul>
<b>Perimeter, area, and volume</b>			<ul style="list-style-type: none"> <li>› visualise, estimate, and measure the perimeter and area of 2D shapes, using informal units.</li> </ul>	<ul style="list-style-type: none"> <li>› visualise, estimate, and measure: <ul style="list-style-type: none"> <li>- the perimeter of polygons using metric units</li> <li>- the area of 2D shapes using squares of identical size</li> <li>- the volume of rectangular prisms (cuboids) by filling them with identical 3D blocks.</li> </ul> </li> </ul>

## Teaching considerations

Use digital and analogue clocks to have students practise telling the time. **Connect** using visual representations on an analogue clock to skip counting in 5s and fractions (a half and quarter).

**Connect** the 'structure' of duration (minutes, hours, days) to our measures of time ("There are 30 minutes in half an hour, 60 minutes in an hour").

Identify and **investigate** the specific times of daily events and activities in and out of school.

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**Explain** and demonstrate that:

- › perimeter is the distance around the boundary of a 2D shape
- › area is the size of the surface of a 2D shape, or how many squares cover the surface
- › volume is the amount of 3D space a shape takes up, or how many cubes fill the shape.

**Investigate** familiar practical situations involving perimeter, area, and volume.

Use think-alouds to demonstrate the use of visualising to identify the appropriate attribute for a measurement task and to imagine the number of units required.

**Explain** the importance of using the same unit when measuring, and that there should be no gaps or overlaps around the outside (perimeter) and inside (area) of 2D shapes and in filled 3D shapes (volume).

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# Geometry

	<b>During the first 6 months</b> <i>Informed by prior learning, teach students to:</i>	<b>During the first year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the second year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the third year</b> <i>Informed by prior learning, teach students to:</i>
<b>Shapes</b>	› identify, sort by one feature, and describe familiar 2D shapes	› identify, describe, and sort familiar 2D and 3D shapes presented in different orientations, including triangles, circles, rectangles (including squares), cubes, cylinders, and spheres	› identify, describe, and sort 2D and 3D shapes, including ovals, semicircles, polygons (e.g., hexagons, pentagons), rectangular prisms (cuboids), pyramids, hemispheres, and cones, using the attributes of shapes	› visualise, identify, compare, and sort 2D and 3D shapes, using the attributes of shapes
				› identify right angles in shapes and objects
<b>Spatial reasoning</b>	› compose by trial and error a target shape using smaller shapes, and decompose a shape into smaller shapes	› anticipate which smaller shapes might be used to compose a target shape, and then check by making the shape	› anticipate which smaller shapes might be used to compose and decompose a target shape, and then check by making the shape	› compose and decompose 2D shapes using the attributes of shapes (e.g., lines of symmetry), other shapes, side lengths, and angles
		› flip, slide, and turn 2D shapes to make a pattern	› recognise lines of symmetry in patterns or pictures, and create or complete symmetrical pictures or patterns	› predict the result of a one-step transformation (reflection, translation, or rotation) on 2D shapes
<b>Pathways</b>	› follow instructions to move to a familiar location or locate an object.	› follow and give instructions to move to a familiar location or locate an object	› follow and give instructions to move people or objects to a different location, using direction, distances (e.g., number of steps), and half and quarter turns	› follow and create a sequence of step-by-step instructions (an algorithm) for moving people or objects to a different location
		› use pictures, diagrams, or stories to describe the positions of objects and places.	› interpret diagrams to describe the positions of objects and places in relation to other objects and places.	› interpret, draw, and use simple maps to locate objects and places relative to other objects and places.

## Teaching considerations

Make available a range of 2D and 3D shapes, including tactile shapes and materials (e.g., playdough, pipe cleaners), pictures, diagrams, and digital tools.

**Investigate** 2D and 3D shapes in the environment.

Use everyday language and mathematical language (including te reo Māori) to **explain** and **justify** the describing and sorting of shapes (e.g., size, corners, colour, texture, sides, angles, faces, edges, vertices, triangle/tapatoru, square/tapawhā rite, same/ōrite, different/rerekē).

Use **generalisations** made by students to clarify and extend understanding (e.g., “Polygons have straight sides”, “2D shapes can be identified on 3D shapes”).

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Make available a range of materials to compose and decompose 2D shapes (e.g., pattern blocks, attribute shapes, paper shapes, playdough, tangrams).

Use think-alouds to demonstrate anticipating how small shapes can fit into or make a new shape.

Use as target shapes:

- › shapes partitioned into smaller parts (at 6 months)
- › continuous whole shapes with no partitions (at years 1–3).

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**Connect** the informal vocabulary of flip, slide, and turn with the formal vocabulary of reflect, translate, and rotate.

**Investigate** practical situations (e.g., making art, paper folding, checking symmetry with mirrors) and a range of artefacts and patterns.

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**Investigate** ways of moving to different locations within the classroom and in other parts of the school, using simple maps at year 3.

Use picture books that emphasise positional language and movement (e.g., *Scatter Cat*, *Bears in the Night*, *We’re Going on a Moa Hunt*).

Use spatial language and talk frames to support giving and following instructions (e.g., near, far, next to, beside, on top, under, over, down, up, left, right, turn).

Make **connections** between:

- › estimating distance and bodily measures (e.g., the number of steps to the door)
- › half and quarter turns and fractions
- › following or creating instructions and algorithmic thinking.

# Statistics

	<b>During the first 6 months</b> <i>Informed by prior learning, teach students to:</i>	<b>During the first year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the second year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the third year</b> <i>Informed by prior learning, teach students to:</i>
<b>Problem</b>		<ul style="list-style-type: none"> <li>› pose a summary investigative question about a group for which the data will have categorical variables (e.g., colour, brand), and anticipate what the data might show</li> </ul>	<ul style="list-style-type: none"> <li>› pose a summary investigative question about a group for which the data will have categorical variables, and anticipate what the data might show (e.g., which outcomes might be more frequent than others)</li> </ul>	<ul style="list-style-type: none"> <li>› pose a summary investigative question about an everyday situation, using categorical data and discrete numerical (whole number) data, identify the variable and group of interest, and anticipate what the data might show</li> </ul>
<b>Plan</b>		<ul style="list-style-type: none"> <li>› plan to collect data by making observations or questioning others, and discuss how the data-gathering process might affect people</li> </ul>	<ul style="list-style-type: none"> <li>› plan survey and data-collection questions for collecting data, identify who and what the data will measure, and discuss how the data-gathering process might affect people</li> </ul>	
<b>Data</b>		<ul style="list-style-type: none"> <li>› collect categorical data for one variable</li> </ul>	<ul style="list-style-type: none"> <li>› collect categorical data for more than one variable</li> </ul>	<ul style="list-style-type: none"> <li>› collect, record, and sort data, or use secondary data sources provided by someone else</li> </ul>
<b>Analysis</b>		<ul style="list-style-type: none"> <li>› create and make statements about data visualisations (e.g., pictures, graphs, dot plots) for the categorical data, giving the frequency for each category</li> </ul>	<ul style="list-style-type: none"> <li>› create and make statements about data visualisations (e.g., pictures, graphs, dot plots) for the categorical data, comparing the frequencies of categories</li> </ul>	<ul style="list-style-type: none"> <li>› create and make statements about data visualisations (e.g., pictures, graphs, dot plots, bar graphs) for the categorical and discrete numerical data</li> </ul>
<b>Conclusion</b>		<ul style="list-style-type: none"> <li>› choose from given options the statements that best answer the investigative question</li> </ul>		<ul style="list-style-type: none"> <li>› choose from given options the statements that best answer the investigative question, reflect on findings, and compare them with anticipated outcomes</li> </ul>

## Teaching considerations

Show, with student input, how to:

- › pose summary **investigative** questions about an area of interest
- › identify the variable and group of interest in investigative questions.

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Pose, with student input, survey and data-collection questions that will be used to collect the data required for the investigative question.

**Explain** the distinction between primary and secondary data and the challenges that come with sensitive topics or questions.

**Investigate** how survey questions and the words within survey questions can be interpreted differently by different people.

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**Represent** data using data cards, recording sheets, and tally tables. Use data cards that **represent** multiple variables about an individual.

Explore investigative questions using secondary data sources.

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Show creating and describing data visualisations, transitioning from data cards to dot plots to bar graphs.

**Represent** data using data cards and picture graphs (for years 1–3), frequency tables and dot plots (for years 2–3), and bar graphs (for year 3).

Have students practise using ‘I notice’ statements that include the variable name and context when describing data visualisations.

**Explain** and demonstrate ‘reading the data’ and ‘reading between the data’.

**Explain** how to describe features of data visualisations (e.g., frequency, the least/most frequent category, modes or modal groups, highest and lowest values).

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Show, with student input, how to:

- › choose the best descriptive statements that answer an investigative question
- › collate, **explain**, and **justify** their findings to others.

	<b>During the first 6 months</b> <i>Informed by prior learning, teach students to:</i>	<b>During the first year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the second year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the third year</b> <i>Informed by prior learning, teach students to:</i>
<b>Statistical literacy</b>		<ul style="list-style-type: none"> <li>› agree or disagree with others' statements about simple data visualisations (e.g., pictures, graphs, dot plots).</li> </ul>	<ul style="list-style-type: none"> <li>› match statements made by others with features in simple data visualisations, and agree or disagree with the statements.</li> </ul>	<ul style="list-style-type: none"> <li>› identify relevant features in others' data visualisations, connect these to descriptive statements, agree or disagree with the statements, and suggest improvements to them.</li> </ul>

## Probability

	<b>During the first 6 months</b> <i>Informed by prior learning, teach students to:</i>	<b>During the first year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the second year</b> <i>Informed by prior learning, teach students to:</i>	<b>During the third year</b> <i>Informed by prior learning, teach students to:</i>
<b>Probability investigations</b>		<ul style="list-style-type: none"> <li>› engage in stories or games that involve chance-based situations and:               <ul style="list-style-type: none"> <li>- decide if something will happen, won't happen, or might happen</li> <li>- identify possible and impossible outcomes (e.g., for what might happen next).</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› engage in chance-based investigations about games and everyday situations to:               <ul style="list-style-type: none"> <li>- anticipate and then identify possible outcomes</li> <li>- collect and record data</li> <li>- create data visualisations for frequencies of possible outcomes (e.g., lists, pictures, graphs)</li> <li>- describe what these visualisations show</li> <li>- answer the investigative question</li> <li>- notice variations in outcomes (e.g., how often each of the numbers on a dice come up)</li> </ul> </li> </ul>	
<b>Critical thinking in probability</b>			<ul style="list-style-type: none"> <li>› agree or disagree with the statements made by others about chance-based situations.</li> </ul>	<ul style="list-style-type: none"> <li>› explain and question statements about chance-based situations, with reference to data.</li> </ul>

## Teaching considerations

Show, with student input, how to:

- › read and understand claims made by others and identify corresponding features in data visualisations
- › **explain** agreements or disagreements with a claim made by others.

## Teaching considerations

**Investigate** probability by playing games of chance using physical objects (e.g., dice, coins, spinners, pulling things out of a hat).

**Explain** and show how to:

- › list possible outcomes
- › visualise frequencies of outcomes
- › use the vocabulary that indicates the relative order of probabilities from impossible to certain (i.e., impossible, unlikely, possible, likely, certain).

Show, with student input, how to:

- › read and understand claims made by others about chance situations
- › match statements with the relevant chance situation being described
- › **explain** and **justify** why they believe a statement is true or not.

# The language of mathematics and statistics: Phase 1

	<b>At 6 months</b> <i>Students will know the following words:</i>		<b>Year 1</b> <i>Students will know the following new words:</i>		<b>Year 2</b> <i>Students will know the following new words:</i>		<b>Year 3</b> <i>Students will know the following new words:</i>	
<b>Number</b>	<ul style="list-style-type: none"> <li>› add, plus, join</li> <li>› altogether</li> <li>› biggest, smallest</li> <li>› combine, separate</li> <li>› count</li> <li>› group</li> <li>› how many</li> <li>› in between</li> <li>› more, less</li> </ul>	<ul style="list-style-type: none"> <li>› next, before, after</li> <li>› ordinal (1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> etc.)</li> <li>› takeaway, minus</li> </ul>	<ul style="list-style-type: none"> <li>› count on, count back</li> <li>› digit</li> <li>› double, halve</li> <li>› equal group</li> <li>› equal part</li> <li>› fair share</li> <li>› forwards, backwards</li> <li>› fraction</li> <li>› half, quarter</li> </ul>	<ul style="list-style-type: none"> <li>› odd, even</li> <li>› partition</li> <li>› set</li> <li>› share</li> <li>› skip count</li> <li>› subtract</li> <li>› sum, difference</li> <li>› whole set</li> </ul>	<ul style="list-style-type: none"> <li>› cent, coin, dollar, note</li> <li>› denominator</li> <li>› eighth</li> <li>› estimate, estimation</li> <li>› money</li> <li>› multiply, divide</li> <li>› numerator</li> <li>› place value</li> </ul>	<ul style="list-style-type: none"> <li>› quantity, amount</li> <li>› regroup</li> </ul>	<ul style="list-style-type: none"> <li>› operation</li> <li>› round</li> <li>› third, fifth, sixth</li> <li>› unit fraction</li> </ul>	
<b>Algebra</b>	<ul style="list-style-type: none"> <li>› continue</li> <li>› copy</li> <li>› next</li> <li>› pattern</li> <li>› repeat</li> </ul>		<ul style="list-style-type: none"> <li>› changed, unchanged</li> <li>› element</li> <li>› equal, equivalent</li> <li>› equation</li> </ul>	<ul style="list-style-type: none"> <li>› number sentence</li> <li>› repeating pattern</li> <li>› true, false</li> <li>› unit of repeat</li> <li>› zero</li> </ul>	<ul style="list-style-type: none"> <li>› error</li> <li>› predict</li> </ul>		<ul style="list-style-type: none"> <li>› complete, incomplete</li> <li>› growing pattern</li> <li>› rule</li> <li>› sequence</li> <li>› term</li> </ul>	
<b>Measurement</b>	<ul style="list-style-type: none"> <li>› comparative words (long, taller, heaviest etc.)</li> <li>› full, empty</li> <li>› heavy, light</li> <li>› height</li> <li>› length</li> </ul>	<ul style="list-style-type: none"> <li>› measure, weigh</li> <li>› same as</li> <li>› short, tall, wide, large, small, big</li> </ul>	<ul style="list-style-type: none"> <li>› capacity</li> <li>› day, week, month, year</li> <li>› days of the week</li> <li>› distance</li> <li>› earlier, later</li> <li>› hour</li> </ul>	<ul style="list-style-type: none"> <li>› morning, afternoon, evening</li> <li>› o'clock</li> <li>› starting point, end point</li> <li>› weight</li> </ul>	<ul style="list-style-type: none"> <li>› area</li> <li>› full turn, half turn, quarter turn</li> <li>› half past</li> <li>› months of the year</li> <li>› perimeter</li> <li>› seasons of the year</li> </ul>	<ul style="list-style-type: none"> <li>› surface</li> <li>› width</li> </ul>	<ul style="list-style-type: none"> <li>› gram</li> <li>› litre, millilitre</li> <li>› measuring jug or cup</li> <li>› metre, centimetre</li> <li>› metric</li> <li>› minute, second</li> </ul>	<ul style="list-style-type: none"> <li>› quarter past, quarter to</li> <li>› ruler</li> <li>› three-quarter turn</li> <li>› unit</li> <li>› volume</li> <li>› weighing scale, balance scale</li> </ul>

	<b>At 6 months</b> <i>Students will know the following words:</i>		<b>Year 1</b> <i>Students will know the following new words:</i>		<b>Year 2</b> <i>Students will know the following new words:</i>		<b>Year 3</b> <i>Students will know the following new words:</i>	
<b>Geometry</b>	<ul style="list-style-type: none"> <li>› flip</li> <li>› positional language (next to, above, below, under, up, down, on top of, inside etc.)</li> <li>› side, corner</li> <li>› size (big, small, long, short)</li> </ul>	<ul style="list-style-type: none"> <li>› square, triangle, circle</li> <li>› straight, curved, round</li> <li>› turn</li> </ul>	<ul style="list-style-type: none"> <li>› 2D shape</li> <li>› 3D or solid shape</li> <li>› cube, cylinder, sphere</li> <li>› edge, face</li> <li>› slide</li> <li>› rectangle</li> </ul>		<ul style="list-style-type: none"> <li>› direction</li> <li>› left, right</li> <li>› oval, semicircle, polygon (hexagon, pentagon), rectangular prism (cuboid), pyramid, hemisphere, cone</li> </ul>	<ul style="list-style-type: none"> <li>› position</li> <li>› symmetry, line of symmetry</li> <li>› vertex</li> </ul>	<ul style="list-style-type: none"> <li>› location</li> <li>› quadrilateral</li> <li>› reflect, reflection</li> <li>› right angle</li> <li>› rotate, rotation</li> <li>› transform, transformation</li> <li>› translate, translation</li> </ul>	
<b>Statistics</b>			<ul style="list-style-type: none"> <li>› data</li> <li>› dot plot</li> <li>› information</li> <li>› most, least</li> <li>› picture graph</li> <li>› survey</li> <li>› tally</li> </ul>		<ul style="list-style-type: none"> <li>› category</li> <li>› graph</li> <li>› notice</li> <li>› outcome</li> <li>› statement</li> <li>› table</li> <li>› title</li> </ul>		<ul style="list-style-type: none"> <li>› bar graph</li> <li>› claim</li> <li>› finding</li> <li>› frequency</li> <li>› variable</li> </ul>	
<b>Probability</b>			<ul style="list-style-type: none"> <li>› chance</li> <li>› possible, impossible</li> <li>› will happen, won't happen, might happen</li> </ul>		<ul style="list-style-type: none"> <li>› agree, disagree</li> <li>› anticipate</li> <li>› certain, uncertain</li> <li>› likely, unlikely</li> <li>› list</li> </ul>		<ul style="list-style-type: none"> <li>› probability</li> </ul>	

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Phase

2

Years 4-6

## Progress outcome by the end of year 6

*Expanding horizons of knowledge, and collaborating*

*Te whakawhānui i ngā pae o te mātauranga, me te mahi tahi*

The critical focus of phase 2 is for all students to expand their horizons of knowledge and their collaboration with others. Students use a variety of representations to model number operations and to solve word problems. They connect and extend their reasoning about whole numbers to fractions and decimals, and they visualise and classify angles, using benchmarks to justify their classifications. Students also apply their understanding

of number operations to reasoning about perimeter and area and to investigating variations in patterns, shapes, and data.

The phase 2 progress outcome describes the understanding, knowledge, and processes that students have multiple opportunities to develop over the phase.

### Understand

Patterns and variation  
Logic and reasoning  
Visualisation and application

### Know

Number  
Algebra  
Measurement  
Geometry  
Statistics  
Probability

### Do

Investigating situations  
Representing situations  
Connecting situations  
Generalising findings  
Explaining and justifying findings

the learning

that matters

The phase 2 progress outcome is found on the following two pages.

## Understand

As students build knowledge through their use of the mathematical and statistical processes, they develop their understanding of the following.

### Patterns and variation | Ngā ia auau me ngā rerekētanga

The world is full of patterns and is defined by a multitude of relationships in which change and variation occur. Mathematics and statistics provide structures that are useful for noticing, exploring, and describing different types of patterns and relationships, enabling us to generate insights or make conjectures.

### Logic and reasoning | Te whakaaro arorau me te whakaaroaro

By engaging with mathematical concepts, we develop logical reasoning and critical-thinking skills that enable us to evaluate information, question assumptions, and present arguments with clarity. Statistical reasoning from observation and theory allows us to differentiate what is probable from what is possible and to draw reliable conclusions about what is reasonable.

### Visualisation and application | Te whakakite me te whakatinana

The visualisation of mathematical and statistical ideas profoundly influences how we perceive, understand, and interact with abstract concepts. Application in mathematics and statistics involves creating structures and processes that help us understand complex situations, enabling better decision making and communication of ideas.

## Know

### Number | Mātauranga tau

By the end of this phase, students know that in our number system each place value is a power of 10, and this continues infinitely. To the right, the system continues past the ones column to create decimals (tenths, hundredths, thousandths); the decimal point marks the column immediately to the right as the tenths column. Estimation and rounding support checking the reasonableness of solutions of operations involving whole numbers, fractions, and decimals. Students know that to evaluate expressions that have more than one operation, operations inside brackets (i.e., grouped together) are done first. If there are multiplication and division, these are then done in left-to-right order; finally, addition and subtraction are also done in left-to-right order. Division can be partitive (the number of shares is known) or quotitive (the size of the shares is known).

Students come to recognise the properties of number operations. The additive identity is 0 (e.g.,  $3 + 0 = 3$ ) and the multiplicative identity is 1 (e.g.,  $5 \times 1 = 5$ ). The commutative property (e.g.,  $3 \times 5 = 5 \times 3$ ) and associative property (e.g.,  $3 \times (4 \times 6) = (3 \times 4) \times 6$ ) apply to addition and multiplication but not to subtraction and division. The distributive property (e.g.,  $2 \times (6 - 4) = 2 \times 6 - 2 \times 4$ ) applies to multiplication over addition and subtraction.

Students also come to know that fractions can result from one number divided by another (the quotient), operate on quantities, and be larger than 1. Improper fractions can also be written as a mixed number, represented as a whole number and a fraction, combined with a hidden addition. In simplified fractions, the numerator and denominator have no common factors; if the denominator of a simplified fraction is 1, then it can be written as a whole number. Decimals are fractions that have powers of 10 as their denominators, and they can be written as numbers using a decimal point. A percentage is a fraction with a denominator of 100.

### Algebra | Taurangi

By the end of this phase, students know that the equal (=) and inequality (<, >) signs show relationships, and that applying the same operation to both sides of an equation preserves the balance of the equation. Students know that in a pattern, the relationship between the ordinal position and its corresponding element can be used for finding the pattern rule. Any element can be found by knowing its position, and any position can be found from its corresponding element. Tables and XY graphs provide a way of organising the positions and elements of a pattern to reveal relationships or rules. An algorithm is an ordered list of instructions for solving a problem.

### Measurement | Ine

By the end of this phase, students know that, like our place-value number system, the metric measurement system is based on powers of 10 and that appropriate metric units are used to quantify length, area, volume, capacity, mass (weight), and temperature. Measurements can include whole units and parts of units. Different measurement tools and scales use different-sized units, and the unit must be recorded with the amount. Duration is the amount of time it takes for an event to occur. Angles are a measure of turn and can use the unit of degrees.

### Geometry | Āhuahanga

By the end of this phase, students know that two- and three-dimensional shapes have consistent properties that can be used to define, compare, classify, predict, and identify relationships between shapes. Shapes can be transformed by rotation, reflection, translation, and resizing (when they are enlarged or reduced). Lines of symmetry can be horizontal, vertical, and diagonal. Three-dimensional shapes can be composed of connected two- or three-dimensional shapes. Students

also know that position can be described using known environmental features and elements from the natural world. Maps can use grid references to specify the position of locations, scales to show distances, and connections to show pathways.

### Statistics | Tauanga

By the end of this phase, students know that data about people and the natural world must be collected, used, and stored carefully. The statistical enquiry cycle (PPDAC) can be used in summary, comparison, and time-series investigations. A comparison investigation compares similarities and differences for a variable across two or more groups, and a time-series investigation considers how a variable changes over time. Numerical variables can be counted or measured; discrete numerical variables are counted, continuous numerical variables are measured. A conjecture or assertion involves thinking about what data will show before it is collected or analysed. Data is not always accurately recorded; it needs to be checked for errors and may need correcting. Alternative data visualisations for the same data can lead to different insights.

### Probability | Tūponotanga

By the end of this phase, students know that the statistical enquiry cycle (PPDAC) can be used for chance-based investigations. Probabilities and the language of probability are associated with values between 0 or 0% (impossible) and 1 or 100% (certain). They can be used to describe situations that involve uncertainty and help make decisions. In a chance-based investigation, the probability of an outcome is the relative frequency of the outcome in a probability experiment (the probability estimate). If outcomes are believed to be equally likely, the probability of an outcome is the number of times the outcome occurs divided by the total number of outcomes, where all possible outcomes can be listed (the theoretical probability).

## Do

### Investigating situations | Te tūhura pūāhua

By the end of this phase, students can pose a question for investigation, find entry points for addressing the question, and plan an investigation pathway and follow it step by step. They can identify relevant prior knowledge, conditions, and relationships to support the investigation. They can monitor and evaluate progress, adjusting the investigation pathway if necessary, and make sense of outcomes or conclusions in light of a given situation and context.

### Representing situations | Te whakaata pūāhua

By the end of this phase, students can use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns and variations. They use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments. They select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations. They use visualisation to mentally represent and manipulate objects and ideas.

### Connecting situations | Te tūhono pūāhua

By the end of this phase, students can suggest connections between concepts, ideas, approaches, and representations. They connect new ideas to things they already know. They also make connections with ideas in other learning areas and with familiar contexts.

### Generalising findings | Te whakatauwhānui i ngā kitenga

By the end of this phase, students can notice and explore patterns, structure, and regularity and make conjectures about them. They identify relationships, including similarities, differences, and new connections. They represent specific instances and look for when conjectures about them might be applied in another situation or always be true. They test conjectures, using reasoning and counterexamples to decide if they are true or not. They use appropriate symbols to express generalisations.

### Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

By the end of this phase, students can make statements and give explanations inductively, based on observations or data. They make deductions based on knowledge, definitions, and rules. They critically reflect on others' thinking, evaluating their logic and asking questions to clarify and understand. They use evidence, reasoning, and proofs to explain why they agree or disagree with statements. They develop collective understandings by sharing and building on ideas with others. They present reasoned explanations and arguments for an idea, solution, or process.

## Teaching sequence

*Expanding horizons of knowledge, and collaborating*

*Te whakawhānui i ngā pae o te mātauranga, me te mahi tahi*

This section describes how the components of a comprehensive teaching and learning programme for the mathematics and statistics learning area are used during the second phase of learning at school.

Throughout phase 2, encourage students to see themselves as capable, confident, and competent mathematics and statistics thinkers whose ideas are valued, who treat mistakes as part of the learning process, and whose abilities in mathematics and statistics will develop over time with consistent effort. Confidence is built through experiencing success and developing competence and understanding. Over phase 2, students collaborate with others to expand their knowledge and understanding. Support this by working with the class to establish expectations and responsibilities for working together as peers, sharing thinking, and agreeing or disagreeing about mathematical and statistical learning.

Continuously monitor students' cognitive load, reasoning, questions, and use of representations, and respond quickly to address any issues and misconceptions. Ensure teaching builds on what students already understand, know, and can do.

### Explicit teaching

- › Use warm-up routines as a form of active recall that connects back to prior learning (e.g., quick challenges, curly questions, games). Plan for students to develop fluency through practice, using a range of approaches.
- › Use worked examples and break down new learning into clearly explained, manageable steps. Use mathematical and statistical symbols and notation conventions, explaining them and how they work. Teach conceptual understanding of number operations and efficient written and mental methods for them.
- › Connect mathematical and statistical learning within and across contexts. Teach connected procedures and concepts together (e.g., multiplication and division with area and volume). Make connections explicit by highlighting concepts students have applied in other learning areas.
- › Plan ways for students to consolidate their mathematical and statistical learning. Use prompts, questions, and situations that incorporate previously taught concepts and procedures, to help students retrieve and apply them. Highlight connections with new learning.

## Positive relationships with mathematics and statistics

- › Encourage curiosity through exploring mathematics and statistics in, for example, history, games, art, and puzzles.
- › Highlight to the class the mathematical thinking and approaches of individuals or groups. Display drawings or photos of students' representations and workings throughout the learning process, and use these to start conversations about mathematical and statistical learning and progress between students and with families.

## Rich tasks

- › Plan to explore rich mathematical and statistical situations and contextual tasks that are useful and meaningful to the class or community.
- › Design tasks that use different contexts or combinations of operations to encourage students to apply their reasoning and knowledge to other types of problems (e.g., using decimals in measurement situations).
- › Encourage students to generalise by using questions such as “If I change this, what happens to that?” and “Is there another way to show this?”
- › Teach problem-solving and investigation strategies. Support students to read and make sense of a problem – through drawing, using materials, or trying some numbers – and then to identify relevant knowledge, plan how to solve the problem in a sequence of steps, take action to apply their plan (recording calculations with meaningful explanations), and check their findings.
- › Give students opportunities to notice and wonder about patterns, structures, and relationships and make statements about them.

## Communication in mathematics and statistics

- › Plan for students to actively listen to, reflect, and build on each other's thinking and learning. Use discourse-based tools and a range of open questions to facilitate productive discussions. Over the phase, encourage students to use evidence and examples to justify their claims and findings.
- › Select appropriate representations to show working and reasoning. Over the phase, move students towards using pictures, diagrams, and mathematical notation such as equations and inequalities. Teach students which representations are most effective for showing different types of information (e.g., number lines are important for representing operations, differences, the comparative size of numbers, and rounding conventions).
- › Prompt students to visualise and share their thinking about quantities, patterns, shapes, measurements, and data. Support students to visualise by estimating the number of items in a group, using rounding or known benchmarks to make estimations, and by noticing and responding to how a shape has been rotated or reflected, or is composed of other shapes.
- › Use mathematical and statistical language. Demonstrate the use of correct vocabulary that connects to the learning purpose or problem. Ask students to use correct vocabulary and to explain their findings and reasoning. In doing so, draw on students' first and heritage languages, so that they can use their languages as a resource to connect their thinking and learning.
- › Dedicate time for students to record learning (e.g., in their mathematics and statistics book). Support them to organise their ideas clearly, using words, mathematical notation, and a range of representations. Provide opportunities for them to consider their goals and to reflect on their learning.

# Number

	<b>During year 4</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 5</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 6</b> <i>Informed by prior learning, teach students to:</i>
<b>Number structure</b>	› skip count from any multiple of 100, forwards or backwards in 25s and 50s		
	› identify, read, write, compare, and order whole numbers up to 10,000, and represent them using base 10 structure	› identify, read, write, compare, and order whole numbers up to 100,000, and represent them using base 10 structure	› identify, read, write, compare, and order whole numbers up to 1,000,000, and represent them using base 10 structure
		› identify factors of numbers up to 100	› identify square numbers and factors of numbers up to 125
<b>Operations</b>	› use rounding, estimation, and inverse operations to predict results and to check the reasonableness of calculations	› use rounding, estimation, and inverse operations to predict results and to check the reasonableness of calculations	› use rounding, estimation, and inverse operations to predict results and to check the reasonableness of calculations
	› round whole numbers to the nearest thousand, hundred, or ten	› round whole numbers to the nearest ten thousand, thousand, hundred, or ten, and round tenths to the nearest whole number	› round whole numbers to a specified power of 10, and round tenths and hundredths to the nearest whole number or one decimal place
	› add and subtract two- and three-digit numbers	› add and subtract whole numbers up to 10,000	› add and subtract any whole numbers

## Teaching considerations

**Investigate** patterns in multiples, using 100s boards or 1,000s books.

Record choral counting on the board, and ask students to **explain** patterns and make **generalisations** or conjectures.

Use marked number lines to order and compare numbers and place-value (PV) houses and materials to write and **represent** numbers, using base 10 structure.

Support students to:

- › practise saying, reading, and writing given numbers, including large numbers, using PV houses
- › use PV houses to **generalise** that multiplying by 10 moves each digit in a number one place to the left, and dividing by 10 moves each digit one place to the right.

**Represent** factors of numbers using arrays or ordered lists of factor pairs.

Use multiplication charts to **investigate** factors, multiples, and square numbers.

**Connect** to students' understanding of a square to **explain** and **represent** a square number and multiplication facts involving the same two numbers.

**Explain** how to round numbers to an appropriate value to make an estimate for a calculation.

**Explain** reasoning using estimation language such as 'about', 'more or less', and 'close to'.

**Connect** rounding with:

- › known benchmarks (e.g., doubles, halves, multiples of 10), to make estimations and check calculations
- › rounding to an appropriate unit in measurement situations.

Use number lines to support rounding, **explaining** how to find the midpoint between two numbers.

**Explain** and **justify** findings by connecting to estimates and other checking methods.

Use families of facts to show the **connection** between factors and multiples. **Explain** how to use families of facts to 'work backwards' (e.g.,  $7 \times 8 = 56$ , so  $56 \div 8 = 7$ ).

**Explain** and **represent** addition and subtraction using materials such as PV materials, number lines, and number discs.

**Explain** and **connect**:

- › the horizontal method and the vertical-column method of addition or subtraction
- › making estimates or mental calculations using place value, partitioning, and known facts.

Use worked examples and a range of problem types (e.g., result, change, start-unknown), using think-alouds to **explain** the most efficient approaches.

Have students practise decoding and solving word problems, **representing** them as equations.

	<b>During year 4</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 5</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 6</b> <i>Informed by prior learning, teach students to:</i>
<b>Operations</b>	› recall multiplication and corresponding division facts for 4s and 6s	› recall multiplication facts for 7s, 8s, and 9s and corresponding division facts	› recall multiplication facts to at least $10 \times 10$ and corresponding division facts
	› multiply a two-digit by one-digit number and two one-digit whole numbers (e.g., $23 \times 5$ , $7 \times 8$ )	› multiply a three-digit by one-digit number and two two-digit whole numbers (e.g., $245 \times 6$ , $34 \times 83$ )	› multiply multi-digit whole numbers (e.g., $54 \times 112$ )
	› divide up to a three-digit whole number by a one-digit divisor, with no remainder (e.g., $65 \div 5$ )	› divide up to a three-digit whole number by a one-digit divisor, with a remainder (e.g., $83 \div 5 = 16$ , remainder 3)	› divide up to a four-digit whole number by a one-digit divisor, with a remainder (e.g., $198 \div 7$ , $4154 \div 8$ )
			› use the order of operations rule with grouping, addition, subtraction, multiplication, and division
<b>Rational numbers</b>	› identify, read, write, and represent tenths as fractions and decimals	› identify, read, write, and represent tenths and hundredths as fractions and decimals	› identify, read, write, and represent fractions, decimals (to two places), and related percentages
	› compare and order tenths as fractions and decimals, and convert decimal tenths to fractions (e.g., $0.3 = \frac{3}{10}$ )	› compare and order tenths and hundredths as fractions and decimals, and convert decimal tenths and hundredths to fractions	› compare and order fractions, decimals (to two places), and percentages, and convert decimals and percentages to fractions

## Teaching considerations

Provide a range of tasks for students to practise and develop fluency in new and previously learned multiplication and division facts (e.g., families of facts, multiplication table grids, arrays, games).

**Investigate** patterns in the multiples of times tables and to **generalise** multiplication problems beyond recalled facts by looking for patterns.

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At year 4:

- › **connect** multiplication with skip counting using jumps on a number line or arrays
- › **represent** division using diagrams and equal sharing, connecting with known families of facts
- › **generalise** the distributive property of multiplication over addition (e.g.,  $7 \times 8 = 7 \times (5 + 3) = (7 \times 5) + (7 \times 3)$ ).

At years 5–6, **represent** multiplication using the area model, and make **connections** with place value (e.g.,  $34 \times 7 = 30 \times 7 + 4 \times 7$ ).

**Explain** and demonstrate:

- › the vertical-column method for division and multiplication, ensuring students understand and practise the procedure and **connect** with place value, known facts, and estimation
- › making estimates or mental calculations by connecting to place value, partitioning, and known facts.

Have students **investigate**:

- › decoding and solving word problems, **representing** them as equations
  - › multiplication and division in measurement and proportional reasoning situations
  - › multiplication to count different combinations (e.g., “If I have 4 tops and 3 pairs of shorts, how many different outfits can I make?”)
- 

Use worked examples to demonstrate a step-by-step layout with one equal sign per line.

Have students **investigate**:

- › decoding and solving word problems, deciding which operation to use and why
- › the distributive property of multiplication over addition and subtraction (e.g.,  $6 \times 18 = 6 \times (20 - 2) = (6 \times 20) - (6 \times 2)$ ).

**Explain** the commutative, associative, and identity properties, and justify which operations they work for and which they don't.

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**Represent** and compare fractions, decimals, and percentages using continuous materials (double number lines, fraction walls, 100s squares).

Have students practise saying, reading, and writing decimals using decimal PV houses.

**Explain** and **represent** decimal tenths as a fraction with the denominator as 10, and percentages and decimals (to two places) as a fraction with the denominator of 100.

**Investigate** situations where decimals are used (e.g., in measurements at a sports day).

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	<b>During year 4</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 5</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 6</b> <i>Informed by prior learning, teach students to:</i>
<b>Rational numbers</b>	› divide whole numbers by 10 to make decimals	› divide whole numbers by 10 and 100 to make decimals	› multiply and divide numbers by 10 and 100 to make decimals and whole numbers (e.g., $1.3 \times 10 = 13$ )
	› for fractions with related denominators of 2, 4, and 8, 3 and 6, or 5 and 10: - compare and order the fractions - identify when two fractions are equivalent by directly comparing them, noticing the simplest form (e.g., $\frac{3}{6} = \frac{1}{2}$ , which is the simplest form)	› for fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100: - compare and order the fractions - identify when two fractions are equivalent	› for fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100: - compare and order the fractions - identify when two fractions are equivalent - represent the fractions in their simplest form
	› convert (using number lines) between mixed numbers and improper fractions with denominators of 2, 3, 4, 5, 6, 8, and 10	› convert between mixed numbers and improper fractions with denominators of up to 10	› convert between mixed numbers and improper fractions
	› find a unit fraction of a whole number, using multiplication or division facts and where the answer is a whole number (e.g., $\frac{1}{5}$ of 40) › identify, from a unit fraction part of a set, the whole set	› find a fraction of a whole number, using multiplication and division facts and where the answer is a whole number (e.g., $\frac{2}{3}$ of 24) › identify, from a fractional part of a set, the whole set	› find a fraction or percentage of a whole number where the answer is a whole number (e.g., $\frac{3}{8}$ of 48; 30% of \$150) › identify, from a fractional part of a set, the whole set
	› add and subtract fractions with the same denominators to make up to one whole (e.g., $\frac{3}{8} + \frac{3}{8} + \frac{2}{8} = \frac{8}{8} = 1$ )	› add and subtract fractions with the same denominators, including to make more than one whole	› add and subtract fractions with the same or related denominators (e.g., $\frac{1}{4} + \frac{1}{8}$ )
	› add and subtract decimals to one decimal place (e.g., $1.3 + 0.2 = 1.5$ )	› add and subtract whole numbers and decimals to two decimal places (e.g., $32.55 - 21.21 = 11.34$ )	› add and subtract whole numbers and decimals to two decimal places (e.g., $250.11 + 135.29 = 385.4$ )
	› use doubling or halving to scale a quantity (e.g., to double or halve a recipe)	› use known multiplication facts to scale a quantity	› use known multiplication and division facts to scale a quantity

## Teaching considerations

Use decimal PV houses to **generalise** that multiplying by 10 moves each digit in a number one place to the left (increasing the place value of the digit), and dividing moves each digit one place to the right (decreasing the place value of the digit).

Use fraction walls (equivalence materials) to **represent** and **investigate** the relationship between the denominator and numerator in a fraction and how we can use this to simplify the fraction.

Make **connections** with known facts such as halving and dividing by 4.

Count forwards and backwards in fractions, and place fractions on marked and unmarked number lines.

**Represent** improper fractions using words and materials, and place them on a number line.

At years 5–6, **explain** conversion as division with a remainder (e.g.,  $\frac{11}{4} = 2 \frac{3}{4}$  (11 divided by 4 = 2 r 3) or multiplication plus a remainder (e.g.,  $1 \frac{1}{5} = \frac{6}{5}$  ( $1 \times 5 + 1$ )).

Use bar models, diagrams, or paper strips to **represent** equal parts of a whole.

At year 4, **represent** parts of a whole set using discrete materials to make equal groups.

At years 5–6, **connect** percentages and fractions of a whole to known facts and benchmarks (e.g., 25% and dividing by 4).

**Represent** the addition and subtraction of fractions using fraction walls, number lines, and equations.

At years 4–5, **explain** that, when adding and subtracting fractions with the same denominator, the numerators are added or subtracted but the denominator stays the same.

At year 6, **explain** how to use equivalent fractions to rename fractions so that they all have the same denominator. Then add or subtract the numerators.

**Explain** and demonstrate both the horizontal method for **representing** an equation and the vertical-column method for addition or subtraction.

**Investigate** and **connect** the addition and subtraction of decimals in measurement situations.

At year 4, use number lines and decimals to add and subtract tenths, **connecting** tenths as fractions with tenths as decimals.

At years 5–6, **connect** to methods of adding and subtracting whole numbers.

**Represent** multiplicative relationships using diagrams, materials, and bar models.

Use problems such as “If this recipe feeds 4 people, how much of each ingredient do we need to feed 20 people”?

	<b>During year 4</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 5</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 6</b> <i>Informed by prior learning, teach students to:</i>
<b>Financial mathematics</b>	› make amounts of money using dollars and cents (e.g., to make 3 dollars and 70 cents)	› represent money values in multiple ways using notes and coins	› solve problems involving purchases (e.g., ensuring they have enough money) › create simple financial plans (e.g., shopping lists, a family budget)
	› estimate and calculate the total cost and change for items costing whole-dollar amounts.	› estimate to the nearest dollar and calculate the total cost of items costing dollars and cents, and the change from the nearest ten dollars.	› calculate 10%, 25%, and 50% of whole-dollar amounts (e.g., 50% of \$280).

## Teaching considerations

Have students practise grouping denominations and making amounts using play money, **connecting** with place value, skip counting, and multiplication.

**Investigate** authentic financial situations and **represent** findings using equations, spreadsheets, and tables.

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**Investigate** practical situations involving calculating costs and giving change.

At year 6:

- › use bar models to **represent** percentages of whole-dollar amounts, and connect to equivalent fractions
  - › **explain** the procedure of dividing a whole by 10 to find 10%, 2 to find 50%, or 4 to find 25%.
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# Algebra

	<b>During year 4</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 5</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 6</b> <i>Informed by prior learning, teach students to:</i>
<b>Equations and relationships</b>	<ul style="list-style-type: none"> <li>› form and solve true or false number sentences and open number sentences involving multiplication and division, using an understanding of the equal sign (e.g., <math>5 \times \_ = 20</math>; <math>\_ \div 3 = 6</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› form and solve true or false number sentences and open number sentences involving all four operations (e.g., <math>674 + 56 - \_ = 671</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› form and solve true or false number sentences and open number sentences involving all four operations, using an understanding of equality or inequality (e.g., <math>8 \times 7 &lt; 8 \times 5 + 8</math> (T or F?))</li> </ul>
	<ul style="list-style-type: none"> <li>› recognise and describe the rule for a growing pattern using words, tables, and diagrams, and make conjectures about further elements in the pattern</li> </ul>	<ul style="list-style-type: none"> <li>› use tables to recognise the relationship between the ordinal position and its corresponding element in a growing pattern, develop a rule for the pattern in words, and make conjectures about further elements or terms in the pattern</li> </ul>	<ul style="list-style-type: none"> <li>› use tables, XY graphs, and diagrams to recognise relationships in a linear pattern, develop a rule for the pattern in words (i.e., that there is a constant amount of change between consecutive elements or terms), and make conjectures about further elements in the pattern</li> </ul>
<b>Algorithmic thinking</b>	<ul style="list-style-type: none"> <li>› create and use an algorithm for generating a pattern or pathway.</li> </ul>	<ul style="list-style-type: none"> <li>› create and use an algorithm for generating a pattern, procedure, or pathway.</li> </ul>	<ul style="list-style-type: none"> <li>› create and use algorithms for making decisions that involve clear choices.</li> </ul>

## Teaching considerations

**Represent** the equal sign as 'the same as' to demonstrate it is a symbol of equivalence.

**Explain** the difference between an expression (e.g.,  $4 \times 5$ ), an equation (e.g.,  $4 \times 5 = 20$ ), and an inequality (e.g.,  $4 \times 5 < 4 \times 6$ ).

Have students practise the use of equal and inequality symbols.

**Investigate** inverse operations to find missing numbers in equations.

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**Explain** vocabulary in relation to patterns (e.g., ordinal, element, term, position, rule) and how to record the position and term for each element in a pattern.

**Investigate** visual patterns (e.g., tivaevae), making block patterns and **representing** patterns using pictures and materials.

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**Represent** a procedure as a sequence of step-by-step instructions (an algorithm). Follow the sequence by 'acting it out', asking students to describe and record each step.

**Investigate** giving directions for, or describing, the most efficient pathway on a maze or map, and sorting numbers according to a set of instructions (e.g., "Sort the odd numbers ... the multiples of 5").

**Explain** and **justify** how a procedure has been broken into steps, the order of the steps, whether there were any errors or omissions, and, if so, how they were corrected.

At years 4–5, **investigate** creating a sequence of instructions (e.g., to draw a polygon or move through a maze), using digital tools or on paper. **Connect** with geometry when giving directions or describing pathways.

At year 5, **connect** algorithmic thinking to a procedure for an operation (e.g., for multiplying two numbers).

At years 5–6, **investigate** identifying the transformations used to create geometric patterns.

At year 6, **investigate** using classification diagrams to identify an object, a shape, or data based on multiple characteristics.

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# Measurement

	<b>During year 4</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 5</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 6</b> <i>Informed by prior learning, teach students to:</i>
<b>Measuring</b>	<ul style="list-style-type: none"> <li>› measure body parts (e.g., the arm) or familiar objects and use these as benchmarks to estimate and then measure length, mass (weight), capacity, and duration, using appropriate metric or time-based units</li> </ul>	<ul style="list-style-type: none"> <li>› estimate and then accurately measure length, mass (weight), capacity, temperature, and duration, using appropriate metric or time-based units or a combination of units</li> </ul>	<ul style="list-style-type: none"> <li>› estimate and then accurately measure length, mass (weight), capacity, temperature, and duration, using appropriate metric or time-based units or a combination of units</li> </ul>
	<ul style="list-style-type: none"> <li>› use appropriate units to describe length, mass (weight), capacity, and time</li> </ul>	<ul style="list-style-type: none"> <li>› use the appropriate tool for a measurement and the appropriate unit for the attribute being measured</li> </ul>	<ul style="list-style-type: none"> <li>› select and use the appropriate tool for a measurement and the appropriate unit for the attribute being measured</li> </ul>
	<ul style="list-style-type: none"> <li>› use the metric measurement system to explore relationships between units</li> </ul>	<ul style="list-style-type: none"> <li>› use the metric measurement system to explore relationships between units, including relationships represented by benchmark fractions and decimals</li> </ul>	<ul style="list-style-type: none"> <li>› convert between common metric units for length, mass (weight), and capacity, and use decimals to express parts of wholes in measurements</li> </ul>
	<ul style="list-style-type: none"> <li>› recognise that angles can be measured in degrees, using 90, 180, and 360 degrees as benchmarks</li> </ul>	<ul style="list-style-type: none"> <li>› describe angles using the terms acute, right, obtuse, straight, and reflex, comparing them with benchmarks of 90, 180, and 360 degrees</li> </ul>	<ul style="list-style-type: none"> <li>› visualise, measure, and draw (to the nearest degree) the amount of turn in angles up to 360 degrees</li> </ul>
	<ul style="list-style-type: none"> <li>› tell the time to the nearest 5 minutes, using the language of 'minutes past the hour' and 'to the hour'</li> </ul>	<ul style="list-style-type: none"> <li>› describe the differences in duration between units of time (e.g., days and weeks, months, and years), and solve duration-of-time problems involving 'am' and 'pm' notation</li> </ul>	<ul style="list-style-type: none"> <li>› convert between units of time and solve duration-of-time problems, in both 12- and 24-hour time systems</li> </ul>

## Teaching considerations

**Investigate** practical measuring situations, and have students practise the accurate use and reading of rulers, scales, timers, thermometers, and measuring jugs.

**Explain** and accurately measure:

- › at year 4, centimetres, metres, grams, kilograms, and litres, connecting with half units (e.g., 500 mL = 0.5 L)
- › at years 5–6, centimetres, metres, millimetres, grams, kilograms, litres, millilitres, and degrees Celsius.

**Connect** reading a measuring tool with rounding to the nearest given unit (e.g., 3.6 cm to the nearest cm).

Discuss the meaning of measurements in context. Explain benchmarks and prompt students to develop them (e.g., “A big step is about a metre, so roughly how long is our classroom?”)

**Explain** and **justify** the use of appropriate metric units or tools for measuring a given attribute with the precision necessary for the problem, noting that using smaller units provides more accuracy.

**Explain** measurement prefixes (e.g., milli-, centi-), how they **connect** metric units, and how they are based on powers of ten and relate to place value.

**Investigate** how measures can be partitioned and combined like other numbers, and how smaller units are created by equally partitioning larger units.

**Investigate** different angles using physical and digital tools and angles in the environment, and comparing and classifying them as acute, right, reflex, or obtuse.

Make **connections** between angles, fractions of a circle, and turns.

At year 6, **explain**, demonstrate, and have students practise estimating angles and measuring and drawing them using a protractor.

**Represent** time using:

- › digital and analogue clocks (at year 4), to practise telling the time
- › analogue and digital forms (e.g., “It’s 12:45, or a quarter to one.”)

**Investigate** using calendars, timetables, schedules, and number lines to work out the time between two events or the duration of an event. Explore solar calendars (e.g., Roman, Gregorian) and lunar calendars (e.g., maramataka Māori, Chinese).

**Explain** subtracting for duration and inclusive counting (e.g., “For the number of days between now and next Tuesday, start counting from today”).

**Explain** relationships between the units of time (e.g., 60 seconds to the minute, 60 minutes to the hour, 24 hours in a day, 365 days in a year), and use them to convert between units of time.

	<b>During year 4</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 5</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 6</b> <i>Informed by prior learning, teach students to:</i>
<b>Perimeter, area, and volume</b>	<ul style="list-style-type: none"> <li>› visualise, estimate, and measure:               <ul style="list-style-type: none"> <li>- the perimeter of polygons, using metric units (cm and m)</li> <li>- the area of shapes covered with squares or half squares</li> <li>- the volume of shapes filled with centicubes, taking note of layers and stacking.</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› visualise, estimate, and calculate:               <ul style="list-style-type: none"> <li>- the perimeter of regular polygons (in m, cm, and mm)</li> <li>- the area of shapes covered with squares or partial squares</li> <li>- the volume of rectangular prisms filled with centicubes, taking note of layers and stacking.</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› visualise, estimate, and calculate the area of rectangles and right-angled triangles (in cm<sup>2</sup> and m<sup>2</sup>) and the volume of rectangular prisms (in cm<sup>3</sup>), by applying multiplication.</li> </ul>

## Geometry

	<b>During year 4</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 5</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 6</b> <i>Informed by prior learning, teach students to:</i>
<b>Shapes</b>	<ul style="list-style-type: none"> <li>› identify, classify, and describe the attributes of polygons (including triangles and quadrilaterals) using properties of shapes, including line and rotational symmetry</li> </ul>	<ul style="list-style-type: none"> <li>› identify, classify, and describe the attributes of:               <ul style="list-style-type: none"> <li>- regular and irregular polygons, using edges, vertices, and angles</li> <li>- prisms, using cross sections, faces, edges, and vertices</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› identify, classify, and explain similarities and differences between:               <ul style="list-style-type: none"> <li>- 2D shapes, including different types of triangle</li> <li>- prisms and pyramids</li> </ul> </li> </ul>
	<ul style="list-style-type: none"> <li>› compare angles in 2D shapes, classifying them as equal to, smaller than, or larger than a right angle</li> </ul>	<ul style="list-style-type: none"> <li>› identify and describe parallel and perpendicular lines, including those forming the sides of polygons</li> </ul>	<ul style="list-style-type: none"> <li>› identify and describe the interior angles of triangles and quadrilaterals</li> </ul>
<b>Spatial reasoning</b>	<ul style="list-style-type: none"> <li>› identify the 2D shapes that compose 3D shapes (e.g., a triangular prism is made from two triangles and three rectangles)</li> </ul>	<ul style="list-style-type: none"> <li>› visualise 3D shapes and connect them with nets, 2D diagrams, verbal descriptions, and the same shapes drawn from different perspectives</li> </ul>	<ul style="list-style-type: none"> <li>› visualise and draw nets for rectangular prisms</li> </ul>
	<ul style="list-style-type: none"> <li>› visualise, predict, and identify which shape is a reflection, rotation, or translation of a given 2D shape</li> </ul>	<ul style="list-style-type: none"> <li>› resize (enlarge or reduce) a 2D shape</li> </ul>	<ul style="list-style-type: none"> <li>› visualise, create, and describe 2D geometric patterns and tessellations, using rotation, reflection, and translation and identifying the properties of shapes that do not change</li> </ul>

## Teaching considerations

**Investigate** practical measuring situations and **connect**:

- › finding area with multiplication arrays
- › finding area and volume with the commutative property of multiplication
- › how part-units can be combined using number concepts, when finding the area of a shape
- › the area of a right-angled triangle with half the area of a square.

Have students **represent** written methods for calculating, with clearly laid-out working.

## Teaching considerations

Use a range of 2D and 3D shapes, including tactile shapes, diagrams, student-made constructions, and digital shapes.

**Investigate** line and rotational symmetry using mirrors and tracing paper.

**Connect** to algorithmic thinking by making classification diagrams for classifying shapes.

**Investigate** interior angles using digital tools and paper shapes to **generalise** that the interior angles of a triangle add to  $180^\circ$  and those of a quadrilateral add to  $360^\circ$ .

**Connect** these understandings to ideas about right angles, straight lines, and full turns.

**Represent** 3D shapes using digital tools, sketches, blocks, and student-made constructions.

**Investigate** nets that will or will not fold, and match solid shapes with nets.

**Investigate** using 2D shapes, squared paper, mirrors, and tracing paper to make and test conjectures about the effects of transformations.

At year 5, use a grid to scale a shape and connect the scaling with multiplication or division.

At year 6, **generalise** the properties of shapes that do not change when transformed (e.g., “Which properties of a square stay the same when we rotate it  $90$  degrees?”)

	<b>During year 4</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 5</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 6</b> <i>Informed by prior learning, teach students to:</i>
<b>Pathways</b>	<ul style="list-style-type: none"> <li>› use grid references to identify regions and plot positions on a grid map</li> <li>› interpret and describe pathways, including those involving half and quarter turns and the distance travelled.</li> </ul>	<ul style="list-style-type: none"> <li>› interpret and create grid maps to plot positions and pathways, using grid references and directional language, including the four main compass points.</li> </ul>	<ul style="list-style-type: none"> <li>› interpret and create grid references and simple scales on maps</li> <li>› use directional language, including the four main compass points, turn (in degrees), and distance (in m, km) to locate and describe positions and pathways.</li> </ul>

## Statistics

	<b>During year 4</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 5</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 6</b> <i>Informed by prior learning, teach students to:</i>
<b>Problem</b>	<ul style="list-style-type: none"> <li>› use multivariate data to investigate summary and comparison situations with categorical and discrete numerical data, by:               <ul style="list-style-type: none"> <li>- posing an investigative question that can be answered with data</li> <li>- making conjectures or assertions about expected findings</li> </ul> </li> </ul>		<ul style="list-style-type: none"> <li>› use multivariate data to investigate summary, comparison, and time-series situations, by:               <ul style="list-style-type: none"> <li>- posing an investigative question that can be answered with data</li> <li>- making conjectures or assertions about expected findings</li> </ul> </li> </ul>
<b>Plan</b>	<ul style="list-style-type: none"> <li>› plan how to collect primary data to support answering the investigative question, including:               <ul style="list-style-type: none"> <li>- deciding on the group of interest</li> <li>- deciding on the variable or variables for which data will be collected</li> <li>- taking account of ethical practices in data collection</li> </ul> </li> </ul>		<ul style="list-style-type: none"> <li>› plan how to collect primary data or how to use provided data, including identifying the variables of interest and, for provided data:               <ul style="list-style-type: none"> <li>- identifying who the data was collected from</li> <li>- identifying the original investigator's purpose for collecting the data</li> <li>- deciding if the source is reliable (e.g., by checking if survey questions appear to be biased towards a particular point of view)</li> </ul> </li> </ul>

## Teaching considerations

**Investigate** different types of maps (e.g., schematic, topographical, and digital maps).

**Explain** pathways using directional language, including te reo Māori (e.g., whakamua/forwards, whakamuri/backwards, whakamaui/to the left, whakamatau/to the right, raki/north, tonga/south, rāwhiti/east, uru/west).

**Connect** compass points and directional language with turns and angles, and simple scales with proportional reasoning.

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## Teaching considerations

Show, with student input, how to:

- › pose summary and comparison **investigative** questions
- › pose time-series **investigative** questions (at year 6).

**Connect** questions to areas of interest and value to the students and their communities.

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Show, with student input, how to:

- › ask interrogative questions about sources and ethical practices
  - › develop and closely examine survey or data-collection questions
  - › define or establish measures for variables
  - › identify 'who, what, where, when, and how' when using secondary datasets.
-

	<b>During year 4</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 5</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 6</b> <i>Informed by prior learning, teach students to:</i>
<b>Data</b>	<ul style="list-style-type: none"> <li>› use a variety of tools to collect the data, and check for errors in it</li> </ul>	<ul style="list-style-type: none"> <li>› use a variety of tools to collect the data, check for errors in it, and correct them by re-collecting the data, if possible</li> </ul>	<ul style="list-style-type: none"> <li>› collect primary data and check for errors, and provide information about variables in secondary data (e.g., how data was collected for them and possible outcomes for them)</li> </ul>
<b>Analysis</b>	<ul style="list-style-type: none"> <li>› create and describe data visualisations to make meaning from the data, with statements including the name of the variable</li> </ul>	<ul style="list-style-type: none"> <li>› create and describe data visualisations to make meaning from the data, with statements including the names of the variable and group of interest</li> </ul>	<ul style="list-style-type: none"> <li>› create and describe a variety of data visualisations to make meaning from the data, identifying features, patterns, and trends in context, and including the variable and group of interest</li> </ul>
<b>Conclusion</b>	<ul style="list-style-type: none"> <li>› choose descriptive statements that best answer the investigative question, reflecting on findings and how they compare with initial conjectures or assertions</li> </ul>	<ul style="list-style-type: none"> <li>› answer the investigative question, comparing findings with initial conjectures or assertions and their existing knowledge of the world</li> </ul>	
<b>Statistical literacy</b>	<ul style="list-style-type: none"> <li>› check the statements that others make about data to see if they make sense, using information to clarify or correct statements where needed.</li> </ul>	<ul style="list-style-type: none"> <li>› check and, if necessary, improve the statements others make about data, including data from two or more sources.</li> </ul>	<ul style="list-style-type: none"> <li>› identify, explain, check, and, if necessary, improve features in others' data investigations (e.g., biased survey questions, misleading information or statements).</li> </ul>

## Teaching considerations

Show, with student input, how to:

- › use a range of **representations** for recording data
- › identify what errors in data look like.

**Connect** multiple variables for individuals, explaining that most datasets use a table design in which each row focuses on an individual and each column includes the data on multiple individuals for one variable.

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Show, with student input, how to:

- › **represent** and analyse data visualisations, creating them at first by hand and then with digital tools
- › identify the different features of data that the data visualisation reveals and how to describe them
- › read the data, and read 'between' the data.

**Explain** how different data visualisations have different features and how to describe them in context (e.g., in relation to frequency, modes, modal groups, patterns, trends, values for numerical variables).

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Show, with student input, how to:

- › choose the best descriptive statements that answer an investigative question
  - › prepare their findings and explain them to others.
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Show, with student input, how to:

- › identify misleading data visualisations, match others' data visualisations with their statements, and check the claims made by others
  - › interpret pie graphs (but not how to create them)
  - › **explain** and **justify** the effectiveness of data visualisations in representing others' findings, using interrogative questions.
-

# Probability

	<b>During year 4</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 5</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 6</b> <i>Informed by prior learning, teach students to:</i>
<b>Probability investigations</b>	<ul style="list-style-type: none"> <li>› engage in chance-based investigations with equally likely outcomes by:               <ul style="list-style-type: none"> <li>- posing an investigative question</li> <li>- anticipating and then identifying possible outcomes for the investigative question</li> <li>- generating all possible ways to get each outcome (a theoretical approach), or undertaking a probability experiment and recording the occurrences of each outcome</li> <li>- creating data visualisations for possible outcomes</li> <li>- describing what these visualisations show</li> <li>- finding probabilities as fractions</li> <li>- answering the investigative question</li> <li>- reflecting on anticipated outcomes</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› engage in chance-based investigations, including those with not equally likely outcomes, by:               <ul style="list-style-type: none"> <li>- posing an investigative question</li> <li>- anticipating and then identifying possible outcomes for the investigative question</li> <li>- generating all possible ways to get each outcome (a theoretical approach), or undertaking a probability experiment and recording the occurrences of each outcome</li> <li>- creating data visualisations for possible outcomes</li> <li>- describing what these visualisations show</li> <li>- finding probabilities as fractions</li> <li>- answering the investigative question</li> <li>- reflecting on anticipated outcomes</li> <li>- (at year 6) comparing findings from the probability experiment and associated theoretical probabilities, if the theoretical model exists</li> </ul> </li> </ul>	
<b>Critical thinking in probability</b>	<ul style="list-style-type: none"> <li>› agree or disagree with others' conclusions about chance-based investigations.</li> </ul>	<ul style="list-style-type: none"> <li>› evaluate others' statements about chance-based investigations, with justification.</li> </ul>	<ul style="list-style-type: none"> <li>› identify, explain, and check others' statements about chance-based investigations, referring to evidence.</li> </ul>

## Teaching considerations

**Investigate** everyday chance-based situations in order to explore and experience the chance, randomness, variation, and distribution of outcomes.

Use digital tools to conduct a large number of trials in order to see what a probability estimate and probability distributions look like.

Support students to **represent**:

- › probability outcomes (theoretical and experimental) using lists, tables, tally charts, visualisations of distributions, words, and numbers
- › the chance of an outcome occurring using fractions, decimals, and percentages.

**Connect** investigative questions to outcomes and with all possible ways to get the outcomes.

**Connect** anticipated outcomes with theoretical and experimental distributions.

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Show, with student input, how to:

- › match the results of chance-based investigations with statements, and check the claims in others' investigations
  - › **explain** and **justify** the statements made by others about chance-based investigations, using interrogative questions.
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## The language of mathematics and statistics: Phase 2

	<b>Year 4</b> <i>Students will know the following new words:</i>		<b>Year 5</b> <i>Students will know the following new words:</i>		<b>Year 6</b> <i>Students will know the following new words:</i>	
<b>Number</b>	<ul style="list-style-type: none"> <li>› addend</li> <li>› convert</li> <li>› decimal</li> <li>› decimal place</li> <li>› decimal point</li> <li>› improper fraction</li> <li>› mixed number</li> </ul>	<ul style="list-style-type: none"> <li>› rename</li> <li>› scale</li> <li>› tenth</li> </ul>	<ul style="list-style-type: none"> <li>› change</li> <li>› divisor, dividend, quotient, remainder</li> <li>› factor</li> <li>› hundredth</li> <li>› multiple</li> <li>› product</li> </ul>	<ul style="list-style-type: none"> <li>› proportion</li> </ul>	<ul style="list-style-type: none"> <li>› efficient</li> <li>› inverse operation</li> <li>› percentage</li> <li>› simplest form</li> <li>› square number</li> <li>› thousandth</li> </ul>	
<b>Algebra</b>	<ul style="list-style-type: none"> <li>› conjecture</li> <li>› relationship</li> </ul>		<ul style="list-style-type: none"> <li>› algorithm</li> <li>› corresponding element</li> <li>› procedure</li> </ul>		<ul style="list-style-type: none"> <li>› constant</li> <li>› equality, inequality</li> <li>› linear pattern</li> <li>› XY graph</li> </ul>	
<b>Measurement</b>	<ul style="list-style-type: none"> <li>› angle</li> <li>› benchmark</li> <li>› degree</li> <li>› kilogram</li> <li>› minutes past, minutes to</li> </ul>		<ul style="list-style-type: none"> <li>› a.m., p.m.</li> <li>› acute angle</li> <li>› attribute</li> <li>› degrees celsius</li> <li>› kilometre, millimetre</li> <li>› obtuse, reflex, right, or straight angle</li> </ul>	<ul style="list-style-type: none"> <li>› timetable</li> </ul>	<ul style="list-style-type: none"> <li>› cubic centimetre, cubic metre</li> <li>› protractor</li> <li>› square centimetre, square metre</li> </ul>	

	<b>Year 4</b> <i>Students will know the following new words:</i>		<b>Year 5</b> <i>Students will know the following new words:</i>		<b>Year 6</b> <i>Students will know the following new words:</i>	
<b>Geometry</b>	<ul style="list-style-type: none"> <li>› grid reference</li> <li>› rotational symmetry</li> </ul>		<ul style="list-style-type: none"> <li>› compass points</li> <li>› cross section</li> <li>› net</li> <li>› parallel or perpendicular line</li> <li>› perspective</li> </ul>	<ul style="list-style-type: none"> <li>› prism</li> <li>› regular or irregular polygon</li> <li>› resize, enlarge, reduce</li> </ul>	<ul style="list-style-type: none"> <li>› centre of rotation</li> <li>› clockwise, anticlockwise</li> <li>› interior angle</li> <li>› map scale</li> </ul>	<ul style="list-style-type: none"> <li>› right-angled, equilateral, isosceles, or scalene triangle</li> <li>› tessellation</li> </ul>
<b>Statistics</b>	<ul style="list-style-type: none"> <li>› analysis</li> <li>› assertion</li> <li>› investigative question</li> <li>› conclusion</li> </ul>		<ul style="list-style-type: none"> <li>› categorical</li> <li>› data visualisation</li> <li>› discrete numerical</li> <li>› group of interest</li> <li>› source</li> </ul>		<ul style="list-style-type: none"> <li>› comparison or summary investigative question</li> <li>› feature</li> <li>› misleading</li> <li>› mode</li> </ul>	<ul style="list-style-type: none"> <li>› primary or secondary data</li> <li>› trend</li> </ul>
<b>Probability</b>	<ul style="list-style-type: none"> <li>› chance-based investigation</li> <li>› equally likely outcome</li> <li>› probability experiment</li> </ul>		<ul style="list-style-type: none"> <li>› evaluate</li> <li>› not an equally likely outcome</li> </ul>		<ul style="list-style-type: none"> <li>› evidence</li> </ul>	

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Phase

3

Years 7–8

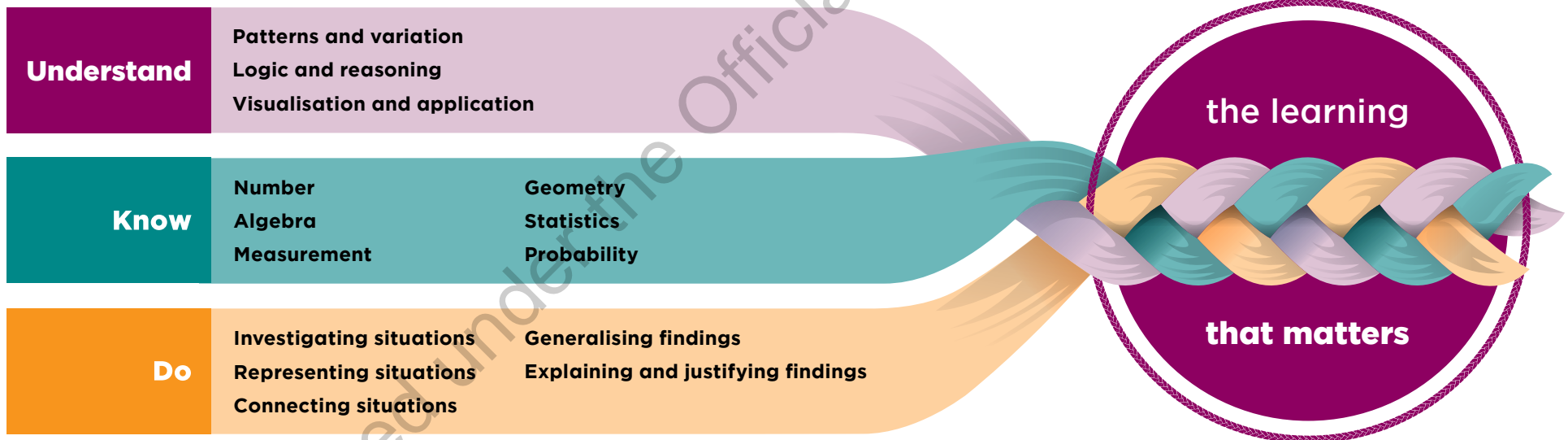
## Progress outcome by the end of year 8

*Seeing ourselves in the wider world and advocating with and for others*  
*Te aro atu ki te ao whānui me te kōkiri kaupapa hei hāpai tahi i ētahi atu*

The critical focus of phase 3 is for all students to see themselves in the wider world and to advocate with and for others. Students use logic and reasoning to identify and solve problems, make connections between mathematical and statistical concepts, and investigate patterns and variation. They communicate mathematically and statistically, using notation, conventions, and vocabulary to clearly explain and justify their approaches to solving problems. Students select, use, and adapt representations to visualise and

extend their reasoning (e.g., number lines to represent integers, equations to represent linear patterns). They make generalisations and identify unknown quantities (e.g., the size of angles) and use data visualisations to investigate claims and make conjectures.

The phase 3 progress outcome describes the understanding, knowledge, and processes that students have multiple opportunities to develop over the phase.



**The phase 2 progress outcome is found on the following two pages.**

## Understand

As students build knowledge through their use of the mathematical and statistical processes, they expand their understanding of the following.

### Patterns and variation | Ngā ia auau me ngā rerekētanga

The world is full of patterns and is defined by a multitude of relationships in which change and variation occur. Mathematics and statistics provide structures that are useful for noticing, exploring, and describing different types of patterns and relationships, enabling us to generate insights or make conjectures.

### Logic and reasoning | Te whakaaro arorau me te whakaaroaro

By engaging with mathematical concepts, we develop logical reasoning and critical thinking skills that enable us to evaluate information, question assumptions, and present arguments with clarity. Statistical reasoning from observation and theory allows us to differentiate what is probable from what is possible and to draw reliable conclusions about what is reasonable.

### Visualisation and application | Te whakakite me te whakatinana

The visualisation of mathematical and statistical ideas profoundly influences how we perceive, understand, and interact with abstract concepts. Application in mathematics and statistics involves creating structures and processes that help us understand complex situations, enabling better decision making and communication of ideas.

## Know

### Number | Mātauranga tau

By the end of this phase, students know that some numbers have special properties (e.g., primes, composites, squares, square roots, cubes). A fraction can describe a proportional relationship between two amounts. Every fraction can be represented by an infinite set of equivalent fractions that occupy the same point on the number line. Multiplying a fraction by an equivalent form of 1, such as  $\frac{3}{3}$ , results in an equivalent fraction that can be useful for comparing, adding, and subtracting fractions. Decimals continue the place-value system using negative powers of ten. They can be terminating, repeating and infinite, or non-repeating and infinite.

Students come to know that integers are positive and negative whole numbers and include zero. To compare relative magnitude, integers, fractions, and decimals can be represented on a number line. There are real-life situations described by quantities less than zero (e.g., temperature, below sea level, debt), and these quantities can be operated on.

Students also come to know that when evaluating or forming expressions, the order of operations is important. Operations inside brackets (i.e., grouped together) are done first, then powers or exponents. If there are multiplication and division, these are then done in left-to-right order; finally, addition and subtraction are also done in left-to-right order. Division can result in a remainder expressed as a whole number, fraction, or decimal.

### Algebra | Taurangi

By the end of this phase, students know that the inverse property applies to addition and multiplication. Inequalities can also include 'or equal to' ( $\leq$ ,  $\geq$ ) to show a relationship that allows for the possibility of equality. In algebra, a variable can be used to represent an unknown number, a quantity that can vary or change

(e.g.,  $y = 3x + 4$ ;  $A = bh$ ), or a specific unknown value to be solved for (e.g.,  $3a = 18$ ). In algebra, there are conventional ways of writing multiplication and division.

Students also come to know that linear patterns have a constant increase or decrease and their XY graphs are straight lines. Not all patterns are linear. Algorithms help solve problems in a systematic way. Their instructions are created, tested, and revised.

### Measurement | Ine

By the end of this phase, students know that in the metric system there are base measurements, with prefixes added to show the size of units. A measurement can be converted from smaller to bigger units, and vice versa, by multiplying or dividing by powers of 10. Length is a one-dimensional measure, area is a two-dimensional measure, and volume is a three-dimensional measure. This is apparent in the notation of units (e.g., cm, cm<sup>2</sup>, cm<sup>3</sup>). Shapes can be decomposed or recomposed to help us find their measurements (e.g., their perimeters, areas, volumes).

### Geometry | Āhuahanga

By the end of this phase, students know that the spatial properties of simple polygons and polyhedra can also apply to more complex two- and three-dimensional shapes. Properties of two- and three-dimensional shapes that do not change under a transformation are called invariant. Unknown angles can be found using the properties of angles on a straight line, angles at a point, vertically opposite angles, and interior angles in triangles and quadrilaterals. Viewing objects from different angles gives different perspectives, which can be represented in models and diagrams. Position, direction, and pathways can be described using scale, compass points, and environmental features. Coordinate systems and maps can express position, direction, and pathways.

## Statistics | Tauanga

By the end of this phase, students know that data collection and use involves a responsibility to protect the rights of people (in relation to data about them) and the ethical use and interpretation of data. People need to understand who they are giving data to and why, before they agree to contribute to a dataset. The statistical enquiry cycle (PPDAC) can be used to conduct data-based investigations about the wider community. There are different types of questions used when undertaking statistical investigations: investigative (summary, comparison, relationship, or time-series), survey, data-collection, interrogative, and analysis questions. Data visualisations show patterns, trends, and variations. Alternative visualisations of the same data can lead to different insights and communicate different information. A distribution is formed from all the possible values of a variable and their frequencies. A relationship investigation looks for a connection between paired numerical or paired categorical variables. Conjectures or assertions may not be reflected in the data, and so may need to be revised or abandoned.

## Probability | Tūponotanga

By the end of this phase, students know that a probability experiment involves repeated trials. Results from sets of repeated trials for the same experiment may vary. Some chance-based situations, such as rolling a weighted dice, can only be explored by probability experiments. Estimates of probabilities from experiments should be based on a very large number of trials (the 'law of large numbers'). The estimated probability of an event from an experiment equals the relative frequency for that event.

Students come to know that if all possible outcomes in a chance-based situation are assumed to be equally likely, the probability of an event equals the number of ways the event can happen divided by the total number of possible outcomes. The statistical enquiry cycle (PPDAC) can be used to conduct probability experiments. For a given situation, probability estimates from experiments and outcomes for theoretical probability models will differ. Probability distributions from experiments and theoretical models will also differ.

## Do

### Investigating situations | Te tūhura pūāhua

By the end of this phase, students can pose a question for investigation, find entry points for addressing the question, and plan an investigation pathway and follow it in an organised way. They can identify relevant prior knowledge, conditions, assumptions, and relationships. They can monitor and evaluate progress, adjusting the investigation pathway if necessary, and make sense of outcomes or conclusions in light of a given situation and context.

### Representing situations | Te whakaata pūāhua

By the end of this phase, students can use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns, variations, and trends. They use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments. They select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations. They use visualisation to mentally represent and manipulate relationships, objects, and ideas.

### Connecting situations | Te tūhono pūāhua

By the end of this phase, students can suggest connections between concepts, ideas, approaches, and representations. They connect new ideas to things they already know. They make connections to ideas in other learning areas and with diverse contexts.

### Generalising findings | Te whakatauwahānui i ngā kitenga

By the end of this phase, students can notice and explore patterns, structure, and regularity and make conjectures about them. They identify relationships, including similarities, differences, and new connections. They represent specific instances and look for when conjectures about them might be applied in another situation or always be true. They test conjectures, using reasoning and counterexamples to decide if they are true or not. They use appropriate symbols to express generalisations.

### Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

By the end of this phase, students can make statements and give explanations inductively based on observations or data. They make deductions based on knowledge, definitions, and rules. They critically reflect on others' thinking, distinguishing between correct and flawed logic and asking questions to clarify and understand. They use evidence, reasoning, and proofs to explain why they agree or disagree with statements. They develop collective understandings by sharing, comparing, contrasting, critiquing, and building on ideas with others. They present reasoned, coherent explanations and arguments for an idea, solution, or process.

## Teaching sequence

*Seeing ourselves in the wider world and advocating with and for others*  
*Te aro atu ki te ao whānui me te kōkiri kaupapa hei hāpai tahi i ētahi atu*

This section describes how the components of a comprehensive teaching and learning programme for the mathematics and statistics learning area are used during the third phase of learning at school.

Throughout phase 3, demonstrate, highlight, and affirm an attitude of exploration, enthusiasm, and curiosity towards mathematical and statistical endeavour and challenge, holding high expectations for every student. In this phase, students critically reflect on others' reasoning, evaluating their logic and asking questions for clarification. To promote this, facilitate ongoing discussions and reflections about established expectations for interactions in mathematical and statistical learning, reinforcing that all students will be involved. Support increasing agency for students in making decisions about investigations and problem solving (e.g., while planning their approach, selecting representations, justifying their findings).

Continuously monitor students' cognitive load, reasoning, questions, and use of representations, and respond quickly to address any issues and misconceptions. Ensure teaching builds on what students already understand, know, and can do.

### Explicit teaching

- › Use worked examples and break down new learning into clearly explained, manageable steps. Show students efficient written and mental methods. Use examples where there may be an error, misconception, or missing step, to support students to develop critical-analysis and reasoning skills.
- › Plan for students to actively recall learning, practise new procedures and processes, and make connections with prior learning. Provide regular opportunities to practise, so that students maintain their automatic recall of facts and continue to develop procedural fluency and reasoning. Following sufficient blocked practice to achieve proficiency, provide practice opportunities that interleave a mixture of operations or approaches, rather than working on only one concept or procedure in a specific way.

### Positive relationships with mathematics and statistics

- › Provide authentic tasks that reflect students' experiences, interests, and the wider world.
- › Demonstrate and teach strategies for perseverance (e.g., trying another way, drawing a diagram, talking about the task with another student).

## Rich tasks

- › Design investigations where students experience rich mathematical situations, as well as investigations where students use their findings to make decisions in their lives (e.g., making a savings plan). When planning an investigation, help students to identify appropriate questions, as well as the mathematical and statistical concepts, procedures, and representations they will need.
- › Design tasks that have multiple entry and exit points and more than one solution or pathway.
- › Teach problem-solving and investigation strategies such as:
  - making sense of the problem by drawing a diagram or considering previously solved problems to identify strategies that can be reapplied
  - trying some sequential numbers, recording the results in a table, and looking for patterns
  - identifying key information in the problem and connecting it to prior knowledge
  - translating a word problem into a linear equation, to solve for an unknown quantity
  - recording calculations in an organised way, using correct mathematical notation
  - checking the reasonableness of findings.

## Communication in mathematics and statistics

- › Set up opportunities for students to actively listen, reflect, and build on each other's thinking and learning. Use discourse-based tools and a range of open questions to facilitate productive and thought-provoking discussions. Over the phase, encourage students to convert their observations into a conjecture or claim and to use evidence to justify their claims and findings. Plan to balance 'teacher talk' with opportunities for rich, extended student interactions and discussions.
- › Encourage students to select and use representations that best support the learning purpose, including graphs, tables, and equations. Over the phase, support them to increasingly use equations to represent their reasoning and to visualise situations by drawing a diagram, which can give them a way into a problem.
- › Teach and use mathematical and statistical vocabulary and concepts. Ensure students connect the correct vocabulary to the learning purpose and problem (e.g., by using the Frayer model's four quadrants: definition, characteristics, example, non-example). Where possible, draw on students' first and heritage languages so that they can use their languages as a resource to connect their thinking and learning.
- › Prompt students to share their thinking when using visualisation to represent and manipulate relationships, shapes, quantities, and data (e.g., to predict or deduce the effect of a transformation; view a solid shape from different perspectives; use coordinate pairs and locations; identify terms in a growing pattern).

# Number

	<b>During year 7</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 8</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Number structure</b>	› identify, read, write, compare, and order whole numbers using powers of 10 (e.g., $10,000 = 10^4$ )	› identify, read, write, compare, and order whole numbers and decimals using powers of 10 (e.g., $0.01 = \frac{1}{100} = 10^{-2}$ )	<b>Represent</b> and order numbers using place-value (PV) expanders or charts and number lines.
	› find the highest common factor (HCF) of two numbers under 100, and find the least common multiple (LCM) of two numbers under 10	› use prime factorisation to represent a number and to find the HCF of two numbers	<b>Represent</b> factors using factor trees, or systematic lists. <b>Connect</b> HCFs to simplifying fractions, and LCMs to renaming fractions. <b>Generalise</b> conjectures about prime or composite numbers by investigating factors.
	› use exponents to represent repeated multiplication, and identify square roots of square numbers up to at least 100	› identify and describe the properties of prime and composite numbers up to at least 100 and cube numbers up to at least 125	<b>Investigate</b> and <b>generalise</b> divisibility tests for composite and prime numbers, and <b>connect</b> the results to square and cube numbers and square roots. <b>Investigate</b> and <b>explain</b> patterns in repeated multiplication and <b>represent</b> them using exponent notation. <b>Connect</b> prime and composite numbers with factors, and <b>represent</b> a number as a product of its prime factors (prime factorisation).
<b>Operations</b>	› use rounding and estimation to predict results and to check the reasonableness of calculations	› use rounding, estimation, and benchmarks to predict results and to check the reasonableness of calculations	<b>Explain</b> efficient methods for supporting estimation (e.g., when adding a long list of numbers, look for numbers that can be grouped and summed to roughly 10, 100, 1000). <b>Connect</b> operations to benchmarks to make estimates (e.g., 73% is roughly $\frac{3}{4}$ ). <b>Explain</b> and <b>justify</b> findings, by connecting to estimates and other checking methods such as using the inverse operation.
	› round whole numbers to any specified power of 10, and round decimals to the nearest tenth, hundredth, or whole number	› round whole numbers to any specified power of 10, and round decimals to the nearest tenth, hundredth, thousandth, or whole number	
	› recall multiplication facts to at least $10 \times 10$ and identify and describe the divisibility rules for 2, 3, 5, 9, and 10	› identify and describe the divisibility rules for 2-11	<b>Investigate</b> patterns in multiples in 100s boards and multiplication charts to <b>generalise</b> divisibility rules.

	<b>During year 7</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 8</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Operations</b>	› multiply whole numbers		<p><b>Explain</b> and demonstrate efficient methods using worked examples, including:</p> <ul style="list-style-type: none"> <li>› the vertical-column method for division and multiplication, ensuring students understand and practise the procedure and <b>connect</b> with place value, known facts, and estimation</li> <li>› making estimates or mental calculations by connecting to place value, partitioning, and known facts.</li> </ul>
	› divide whole numbers by one- or two-digit divisors (e.g., $327 \div 5 = 65.4$ or $65 \frac{2}{5}$ )	› divide whole numbers (e.g., $327 \div 15 = 21.8$ or $21 \frac{4}{5}$ )	<p><b>Investigate, explain, and justify</b> which method (including the use of digital tools) best suits a given situation.</p> <p>Have students practise decoding and solving word problems and representing them as equations.</p> <p>Represent and make sense of remainders as fractions, as decimals, and when rounded to the nearest whole number.</p>
	› use the order of operations	› use the order of operations	<p>Use worked examples to demonstrate a step-by-step layout with only one equal sign per line.</p> <p>Demonstrate how to use the mnemonic GEMA in relation to the order of operations: grouped, exponents, multiplicative (<math>\div</math> and <math>\times</math>), additive (<math>+</math> and <math>-</math>).</p>
	› order, compare, and locate integers on a number line, and explore adding and subtracting integers	› order, compare, add, and subtract integers	<p><b>Generalise</b> that a positive number has an opposite negative number, and that when they are added, the answer is zero (e.g., <math>4 + -4 = 0</math>).</p> <p><b>Explain</b> how to:</p> <ul style="list-style-type: none"> <li>› find the number of steps between two given numbers on a number line (e.g., <math>-5</math> and <math>7</math>)</li> <li>› ‘read’ equations with integers on a number line (e.g., “To solve <math>-9 + 8</math>, start at <math>-9</math> and move eight numbers in the positive direction.”)</li> <li>› use inequality symbols to compare two integers (e.g., <math>-5 &lt; -3</math>).</li> </ul> <p><b>Investigate</b> adding and subtracting integers, using number lines and two-sided counters. <i>(continued on the next page)</i></p>

	<b>During year 7</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 8</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Operations</b>			<i>(continued from the previous page)</i> <b>Explain</b> the direction of movement on a number line when adding and subtracting integers, and <b>generalise</b> that: <ul style="list-style-type: none"> <li>› adding a negative number makes the original number smaller (e.g., <math>4 + -3 = 1</math>)</li> <li>› subtracting a negative number makes the original number larger (e.g., <math>-7 - (-3) = -4</math>).</li> </ul> <b>Investigate</b> situations where negative integers are used (e.g., temperature, altitude, debt, profit and loss).
<b>Rational numbers</b>	› identify, read, write, and represent fractions, decimals (to three places), and percentages	› identify, read, write, and represent fractions, decimals, and percentages	<b>Explain and represent:</b> <ul style="list-style-type: none"> <li>› percentages using 100s squares,</li> <li>› comparing or ordering fractions, decimals, and percentages using double number lines</li> <li>› decimals or percentages as fractions with denominators of tenths or hundredths, and then renamed to their simplest form</li> <li>› fractions in equivalent forms to support comparing, ordering, and converting.</li> </ul> <b>Explain</b> and demonstrate converting a fraction to a decimal or percentage by connecting to the understanding of fractions as quotients (e.g., $\frac{5}{12} = 5 \div 12$ ). <b>Connect</b> to known benchmarks when comparing and converting (e.g., $\frac{7}{12}$ is a little more than $\frac{6}{12}$ , which is a half or 50%). <b>Represent</b> decimals using PV expanders or charts, and <b>generalise</b> that multiplying by a power of 10 moves each digit that number of places to the left, and dividing by a power of 10 moves each digit that number of places to the right. <b>Explain</b> simplifying fractions and finding equivalent fractions by using HCFs and LCMs.
	› compare, order, and convert between fractions, decimals (to three places), and percentages	› compare, order, and convert between fractions, decimals, and percentages	
	› multiply and divide numbers by 10, 100, and 1,000	› multiply and divide numbers by powers of 10	
	› find equivalent fractions, simplify fractions, and convert between improper fractions and mixed numbers	› find equivalent fractions, simplify fractions, and convert between improper fractions and mixed numbers	

	<b>During year 7</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 8</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Rational numbers</b>	› multiply fractions and decimals by whole numbers	› multiply fractions and decimals by whole numbers	<b>Explain</b> the vertical column method for multiplying decimals, making an estimate before calculating. <b>Connect</b> to the multiplicative identity to <b>generalise</b> that multiplying a whole number by a decimal less than one results in a product less than the original whole number.
	› find a percentage of a whole number, and find a whole amount, given a simple fraction or percentage (e.g., “25% is \$100, what is the total amount?”)	› find a percentage of a whole number, and find a whole amount, given a simple fraction or percentage (e.g., “75% is \$45, what is the total amount?”)	<b>Represent</b> situations involving percentages using bar models to show parts of a whole. <b>Explain</b> how to find a percentage of a whole by using the decimal equivalent to multiply the whole (e.g., 35% of 120 = $0.35 \times 120$ ) or by finding 10%, 5%, or 1% of the whole and using operations (e.g., finding 35% of 120 by finding 10%, multiplying this by 3 to get 30%, then adding half of 10% – $12 \times 3 + 6 = 42$ ).
	› add and subtract fractions with different denominators of up to a tenth, using equivalent fractions (e.g., $\frac{3}{4} + \frac{1}{3}$ )	› add and subtract fractions with different denominators, using equivalent fractions	Demonstrate and <b>explain</b> renaming fractions, using ideas about equivalence and by finding HCFs and LCMs.
	› add and subtract decimals to three decimal places, with an emphasis on estimating before calculating	› add, subtract, and multiply decimals, with an emphasis on estimating before calculating	<b>Connect</b> methods for operating on whole numbers with operating on decimals, making an estimate before calculating. <b>Investigate</b> situations where decimals are compared and the differences between them found (e.g., sporting event times and distances). Have students practise decoding and solving word problems and <b>representing</b> them as equations.
	› use proportional reasoning to explore multiplicative relationships between quantities (e.g., “If there are 3 red for every 7 blue balls, how many balls are there altogether when there are 18 red balls?”)	› use proportional reasoning to share with unequal proportions (e.g., “We have 100 stickers to share. For every 1 sticker I get, you get 3. How many do we each get?”)	<b>Investigate</b> proportional reasoning in situations such as mixing paints, cooking from recipes, and sharing resources. <b>Represent</b> situations involving proportional reasoning using diagrams and comparison bar models. <b>Connect</b> proportional reasoning to multiplicative thinking and equivalent fractions.

	<b>During year 7</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 8</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Financial mathematics</b>	› calculate total cost and change for any amount of money	› create and compare weekly, monthly, and yearly finance plans (e.g., saving plans, phone plans, budgets, and 'buy now, pay later' services)	<p><b>Explain</b> and <b>justify</b> 'best deals', considering personal priorities. Represent financial plans for practical situations using digital tools such as spreadsheets.</p> <p><b>Investigate</b> situations where there are financial percentage losses or gains (e.g., calculating discounts or profits, statistics in the media about growth or decline). <b>Connect</b> the ideas of loss and debt with integers.</p> <p><b>Explain</b>, using worked examples, finding a percentage discount by subtracting from the whole or by multiplying the whole by a decimal fraction (e.g., a 35% discount on \$180 = <math>\\$180 - (0.35 \times \\$180)</math>, or <math>0.65 \times \\$180</math>).</p>
	› apply percentage discounts to whole-dollar amounts.	› apply percentage discounts.	

## Algebra

	<b>During year 7</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 8</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Equations and relationships</b>	› form and solve one-step linear equations (e.g., $t + 7 = 12$ , $2s = 14$ )	› form and solve one- or two-step linear equations (e.g., $5s + 3 = 18$ )	<p>Have students practise writing equations to <b>represent</b> word problems. Demonstrate solving one- or two-step equations and using the inverse operation to check findings.</p> <p><b>Investigate</b> variable values in practical situations with familiar formulae (e.g., for area, volume, speed). Have students practise substituting measurements or given values into formulae.</p>
	› find the value of an expression or formula, given the values of variables (e.g., "Calculate $w + 12$ when $w = 4$ ")	› find the value of an expression or formula, given the values of variables	

	<b>During year 7</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 8</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Equations and relationships</b>	<ul style="list-style-type: none"> <li>describe and use the commutative, distributive, and associative properties of operations (e.g., <math>a \times b = b \times a</math>)</li> </ul>	<ul style="list-style-type: none"> <li>simplify algebraic expressions involving sums, products, differences, and single brackets (e.g., using the distributive property, <math>2(x + 3) + 1 = 2x + 6 + 1 = 2x + 7</math>)</li> </ul>	<p><b>Represent</b> terms in algebraic expressions using algebra tiles.</p> <p>Represent algebraic expressions and equations using the conventions of algebra (e.g., <math>3 \times b</math> or <math>b \times 3</math> is written as <math>3b</math>).</p> <p>At year 8, explain how to simplify algebraic expressions by collecting like terms together.</p> <p>At year 8, <b>investigate</b> systematic expansion approaches, including expansion tables, connecting to the distributive property.</p>
	<ul style="list-style-type: none"> <li>identify the constant increase or decrease in a linear pattern, use variables and algebraic notation to represent the rule in an equation, and use the rule to make conjectures</li> </ul>	<ul style="list-style-type: none"> <li>determine if a pattern is linear and, if it is, write the equation for the pattern and use the equation to make conjectures</li> </ul>	<p><b>Represent</b> a pattern using a table, model, or diagram, and use it to <b>generalise</b> a rule for the pattern. Use the rule and an XY graph to <b>justify</b> a conjecture for another term in the pattern.</p> <p><b>Investigate</b> the history, meaning, and structure of growing patterns (e.g., tukutuku, other well-known patterns such as the Fibonacci sequence).</p>
<b>Algorithmic thinking</b>	<ul style="list-style-type: none"> <li>create, test, and revise algorithms involving a sequence of steps and decisions.</li> </ul>	<ul style="list-style-type: none"> <li>create, test, revise, and use algorithms to identify, interpret, and explain patterns.</li> </ul>	<p><b>Connect</b> an algorithm with an operation such as the vertical-column method for multiplication or with the procedure for adding fractions.</p> <p><b>Represent</b> algorithms using flow charts, numbered step-by-step instructions, or digital tools.</p> <p>Explore algorithms <b>by investigating</b>:</p> <ul style="list-style-type: none"> <li>the formula function of a spreadsheet and the effect of changing the value of a variable in a formula (e.g., hourly wages)</li> <li>sorting and filtering multivariate data</li> <li>sorting numbers according to a set of instructions (e.g., the sieve of Eratosthenes) to find prime numbers</li> <li>situations that can be described and tested (e.g., divisibility, the use of transformations in a shape pattern, converting between units of measurement)</li> <li>creating, testing, and revising a set of instructions using a digital tool.</li> </ul>

# Measurement

	During year 7 <i>Informed by prior learning, teach students to:</i>	During year 8 <i>Informed by prior learning, teach students to:</i>	Teaching considerations
<b>Measuring</b>	<ul style="list-style-type: none"> <li>› estimate and then measure length, area, volume, capacity, mass (weight), temperature, data storage, time, and angle, using appropriate units</li> </ul>	<ul style="list-style-type: none"> <li>› estimate and then measure length, area, volume, capacity, mass (weight), temperature, data storage, time, and angle, using appropriate units</li> </ul>	<p><b>Connect</b> to benchmarks to make estimations. Have students practise the accurate use of rulers, scales, timers, protractors, thermometers, and measuring jugs in practical situations.</p> <p><b>Represent</b> all written measurements with their units. Select appropriate tools and units for a situation, and <b>explain</b> and <b>justify</b> choices.</p>
	<ul style="list-style-type: none"> <li>› select and use an appropriate base measure (e.g., metre, gram, litre) within the metric system, along with a prefix (e.g., kilo-, centi-) to show the size of units</li> </ul>	<ul style="list-style-type: none"> <li>› select and use an appropriate base measure within the metric system, along with a prefix to show the size of units</li> </ul>	
	<ul style="list-style-type: none"> <li>› convert between metric units of length, mass (weight), and capacity, using whole numbers and decimals to express parts of a unit (e.g., 724 g = 0.724 kg)</li> </ul>	<ul style="list-style-type: none"> <li>› convert between metric measurement units, including square units</li> </ul>	<p><b>Connect</b> measurement conversions with multiplying and dividing by powers of 10 (e.g., 2.05 L = 2050 mL).</p> <p><b>Investigate</b> measurement conversion situations in which all four operations are applied to whole-number and decimal measures.</p>
	<ul style="list-style-type: none"> <li>› find speed, given distance and time</li> </ul>	<ul style="list-style-type: none"> <li>› find distance, given speed and time; or time, given distance and speed</li> </ul>	<p><b>Investigate</b> the relationship between speed, distance, and time in practical situations, such as timing how long it takes to walk or run a certain distance.</p> <p>Have students practise substituting values into the speed formula.</p> <p><b>Connect</b> finding the value of variables in the speed formula with solving algebraic equations and multiplication and division operations.</p>
	<ul style="list-style-type: none"> <li>› read, interpret, and use timetables and charts that present information about duration</li> <li>› convert between units of time, and solve duration problems that involve fractions of time</li> </ul>	<ul style="list-style-type: none"> <li>› read, interpret, and use timetables, charts, and results that present information about duration</li> <li>› convert times to a common unit, such as seconds or minutes, and use decimal units of time (milliseconds)</li> </ul>	<p><b>Explain</b> how to plan journeys using timetables and charts. Draw on a range of examples, including digital tools.</p> <p><b>Explain</b> methods of calculating duration (e.g., subtracting time), using worked examples.</p> <p><b>Investigate</b> the use of decimal units (milliseconds) in situations where a more precise measurement is needed (e.g., sporting events).</p>

	<b>During year 7</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 8</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Perimeter, area, and volume</b>	<ul style="list-style-type: none"> <li>› calculate the perimeter and area of composite shapes composed of triangles and rectangles.</li> </ul>	<ul style="list-style-type: none"> <li>› calculate the volume of triangular prisms and shapes composed of rectangular prisms.</li> </ul>	<p><b>Investigate</b> perimeter, area, and volume, including finding missing lengths, in practical situations. <b>Connect</b> calculations with factors, multiples, and the commutative and associative properties.</p> <p><b>Represent</b> working for calculations using a clear layout and by sketching composite shapes to show partitioning.</p> <p><b>Generalise</b> the formulae for finding the area of triangles and volume of triangular prisms, and have students practise substituting measurement values into them. Connect the formulae with spatial representations.</p>

## Geometry

	<b>During year 7</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 8</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Shapes</b>	<ul style="list-style-type: none"> <li>› classify and name shapes based on their attributes (e.g., triangles, pyramids)</li> </ul>	<ul style="list-style-type: none"> <li>› describe triangles, quadrilaterals, and other polygons in relation to their sides, diagonals, and angles</li> </ul>	<p>Use and create a range of 2D and 3D shapes, including shapes that draw on tactile materials, diagrams, and digital tools.</p> <p><b>Investigate</b> ways of classifying shapes, including by creating algorithms and using Venn diagrams and tables.</p>
	<ul style="list-style-type: none"> <li>› identify and describe angles at a point, angles on a straight line, and vertically opposite angles</li> </ul>	<ul style="list-style-type: none"> <li>› reason about unknown angles in situations involving angles at a point, angles on a straight line, vertically opposite angles, and interior angles of triangles and quadrilaterals</li> </ul>	<p><b>Investigate</b> using digital tools and protractors to explore angles.</p> <p><b>Investigate</b> unknown angles to <b>generalise</b> the following rules:</p> <ul style="list-style-type: none"> <li>› the sum of the angles round a point is <math>360^\circ</math></li> <li>› the sum of the angles on a straight line is <math>180^\circ</math></li> <li>› vertically opposite angles are equal</li> <li>› the sum of the interior angles of a triangle is <math>180^\circ</math> and of a quadrilateral is <math>360^\circ</math>.</li> </ul> <p><b>Represent</b> the value of an unknown angle using an equation and angle notation.</p>

	<b>During year 7</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 8</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Spatial reasoning</b>	› visualise, construct, and draw plan views for front, back, left, right, and top views of 3D shapes	› visualise and draw nets for prisms with a fixed cross section	<b>Represent</b> plan views and nets, using sketches on grid paper, digital tools, and physical models (e.g., blocks, cardboard nets). <b>Connect</b> to measurement procedures when creating sketches and models.
	› transform 2D shapes, including composite shapes, by resizing by a whole number or unit fraction	› recognise the invariant properties of 2D and 3D shapes under different transformations	<b>Explain</b> and demonstrate resizing a shape using a centre of enlargement within the shape. <b>Investigate</b> transforming shapes to <b>generalise</b> which properties (angles, side lengths, area, orientation) do not change under transformation, and test the resulting generalisations using tracing paper, rulers, and protractors. <b>Investigate</b> the meaning of kōwhaiwhai patterns and other artefacts, and describe the use of transformations in them.
<b>Pathways</b>	› interpret and communicate the location of positions and pathways using coordinates, angle measures, and the 8 main and halfway compass points (e.g., NE, which is 45° E from N).	› use map scales, compass points, distance, and turn to interpret and communicate positions and pathways in coordinate systems and grid reference systems.	Use maps of familiar and unfamiliar locations to: <ul style="list-style-type: none"> <li>› explain and investigate the use of 4-digit grid references</li> <li>› calculate distances using scales</li> <li>› find efficient routes between destinations.</li> </ul> <b>Connect</b> pathways to: <ul style="list-style-type: none"> <li>› measurement procedures when finding angles and distances</li> <li>› proportional reasoning when using map scales</li> <li>› algorithms to describe routes between two points.</li> </ul> <b>Investigate</b> the navigating techniques of Māori and Pacific voyagers for locating position and finding the direction of travel.

# Statistics

	During year 7 <i>Informed by prior learning, teach students to:</i>	During year 8 <i>Informed by prior learning, teach students to:</i>	Teaching considerations
<b>Problem</b>	<ul style="list-style-type: none"> <li>› investigate, using multivariate datasets, summary, comparison, time-series, and relationship situations for paired categorical data by:               <ul style="list-style-type: none"> <li>- posing an investigative question about a local community matter</li> <li>- making conjectures or assertions about expected findings</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› investigate, using multivariate datasets, summary, comparison, time-series, and relationship situations by:               <ul style="list-style-type: none"> <li>- posing an investigative question about a local community matter</li> <li>- making conjectures or assertions about expected findings</li> </ul> </li> </ul>	<p>Show, with student input, how to pose <b>investigative</b> questions, clearly identifying the variable, the group of interest, and the intent.</p> <p><b>Connect</b> investigative questions with conjectures about expected findings.</p>
<b>Plan</b>	<ul style="list-style-type: none"> <li>› plan how to collect or source data to answer the investigative question, including:               <ul style="list-style-type: none"> <li>- determining or identifying the variables needed</li> <li>- planning how to collect data for each variable (e.g., how to measure it) or finding out how provided data was collected</li> <li>- identifying the group of interest or who the data was collected from</li> <li>- building awareness of ethical practices in data collection by strategic questioning of data-collection questions or methods</li> </ul> </li> </ul>		<p><b>Explain</b> and discuss ethical practices for the collection and use of data.</p> <p><b>Represent</b> planning using a planning tool to outline methods of data collection, 'who' and what to measure, and how.</p> <p>Show, with student input, how to pose data-collection and survey questions.</p> <p><b>Explain</b> the variables and group or groups of interest in secondary datasets.</p> <p><b>Investigate</b> how survey and data collection questions can be misinterpreted, leading to unreliable data.</p>
<b>Data</b>	<ul style="list-style-type: none"> <li>› collect primary data or gather information about variables in sourced data, create a simple informal data dictionary, and check for errors</li> </ul>	<ul style="list-style-type: none"> <li>› collect or source data, including:               <ul style="list-style-type: none"> <li>- checking for errors and following up and correcting them when possible</li> <li>- creating an informal data dictionary with information that will help others know about the context</li> </ul> </li> </ul>	<p>Show, with student input:</p> <ul style="list-style-type: none"> <li>› a range of data-collection and recording methods</li> <li>› how to identify errors in data, <b>connecting</b> to the context and explaining why they are errors</li> <li>› how to update primary data when correctable errors are found.</li> </ul> <p><b>Connect</b> multiple variables for individuals, explaining that most datasets use a table design in which each row focuses on an individual and each column includes the data on multiple individuals for one variable.</p>

	<b>During year 7</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 8</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Analysis</b>	<ul style="list-style-type: none"> <li>› create data visualisations for the investigation</li> <li>› make statements about the data, including its features and context, in descriptions of distributions</li> </ul>	<ul style="list-style-type: none"> <li>› create data visualisations for the investigation, using multiple visualisations to provide different views of the data</li> <li>› make statements about the data, including its features and context, in descriptions of distributions</li> </ul>	<p>Show, with student input, how to:</p> <ul style="list-style-type: none"> <li>› <b>represent</b> data using dot plots, bar graphs, frequency tables, time-series graphs, two-way tables or graphs, scatter plots, fractions, proportions, and percentages, creating them at first by hand and then with digital tools</li> <li>› read the data, read ‘between’ the data, and read ‘behind’ the data</li> <li>› describe what is seen in the data visualisations, recognising that data are numbers with context, and the context includes variables of interest, groups of interest, counts or proportions for categorical variables, and values and units for numerical variables</li> <li>› compare data visualisations of the same variable for different groups by looking at similarities and differences.</li> </ul> <p><b>Explain</b> how different data visualisations have different features and how to describe them in context (e.g., in relation to the middle, distributional shape, joint and conditional proportions, long-term trends).</p>
<b>Conclusion</b>	<ul style="list-style-type: none"> <li>› communicate findings in context to answer the investigative question, using evidence from analysis and comparing findings to initial conjectures or assertions and their existing knowledge of the world</li> </ul>	<ul style="list-style-type: none"> <li>› communicate findings in context to answer the investigative question, using evidence from analysis, considering possible explanations for findings, and comparing findings to initial conjectures or assertions and their existing knowledge of the world</li> </ul>	<p>Show, with student input, how to:</p> <ul style="list-style-type: none"> <li>› choose the best descriptive statements that answer an investigative question</li> <li>› explore explanations or interpretations of findings that connect to the context of the situation under investigation</li> <li>› prepare and present succinct findings</li> <li>› explain and justify whether or not findings align with initial conjectures or assertions, and if what was found makes sense given what is known about the situation.</li> </ul>
<b>Statistical literacy</b>	<ul style="list-style-type: none"> <li>› evaluate the findings of others to check if their claims or statements are supported by the data visualisations they use.</li> </ul>	<ul style="list-style-type: none"> <li>› evaluate the data-collection methods, data visualisations, and findings of others’ statistical investigations to see if their claims are reasonable.</li> </ul>	<p>Show, with student input, how to:</p> <ul style="list-style-type: none"> <li>› identify misleading data visualisations, match others’ data visualisations with their statements, and check the claims made by others</li> <li>› <b>explain</b> and <b>justify</b> others’ statements about the findings of statistical investigations and the process for collecting data, using interrogative questions.</li> </ul>

# Probability

	During year 7 <i>Informed by prior learning, teach students to:</i>	During year 8 <i>Informed by prior learning, teach students to:</i>	Teaching considerations
<b>Probability investigations</b>	<ul style="list-style-type: none"> <li>› plan and conduct probability experiments for chance-based situations, including undertaking a large number of trials using digital tools, by:               <ul style="list-style-type: none"> <li>- posing an investigative question</li> <li>- anticipating what outcomes are possible and which of them are more or less likely to occur</li> <li>- identifying and systematically listing possible answers to the investigative question</li> <li>- collecting and recording data</li> <li>- creating data visualisations for the distribution of observed outcomes</li> <li>- describing what these visualisations show</li> <li>- finding the probability estimates for the different outcomes</li> <li>- answering the investigative question</li> <li>- identifying similarities and differences between their findings and those of others</li> <li>- reflecting on anticipated outcomes</li> <li>- comparing findings from the probability experiment and associated theoretical probabilities, as appropriate</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› plan and conduct probability experiments for chance-based situations, including undertaking a large number of trials using digital tools, by:               <ul style="list-style-type: none"> <li>- posing an investigative question</li> <li>- anticipating what outcomes are possible and which of them are more or less likely to occur</li> <li>- identifying and systematically listing possible answers to the investigative question</li> <li>- collecting and recording data</li> <li>- creating data visualisations for the distribution of observed outcomes and for all possible outcomes for theoretical probability models, where they exist</li> <li>- describing what these visualisations show</li> <li>- finding the probability estimates for the different outcomes</li> <li>- proposing possible theoretical outcomes and associated probabilities, for situations where no theoretical model exists</li> <li>- answering the investigative question</li> <li>- identifying similarities and differences between their findings and those of others</li> <li>- reflecting on anticipated outcomes</li> <li>- identifying similarities and differences between findings from the probability experiment and associated theoretical probabilities, as appropriate</li> </ul> </li> </ul>	<p><b>Investigate</b>, using the statistical enquiry cycle, games of chance, other everyday chance-based situations, patterns in possible outcomes, and theoretical and experimental distributions.</p> <p><b>Represent</b> probability outcomes (theoretical and experimental) using lists, tables, tree diagrams, tally charts, visualisations of distributions, words, numbers, and technology.</p> <p><b>Explain</b> how to describe and use probability concepts (e.g., outcomes, events, trials, models; theoretical and experimental probability; with and without replacement; the law of large numbers; probability estimates, probability distributions; chance, randomness, and variation).</p> <p><b>Connect</b> anticipated outcomes with theoretical and experimental distributions.</p> <p><b>Connect</b> probabilities with proportional reasoning, fractions, and percentages, and with relative frequencies from data investigations.</p>
<b>Critical thinking in probability</b>	<ul style="list-style-type: none"> <li>› identify, explain, and check others' statements about chance-based investigations, referring to evidence.</li> </ul>		<p>Show, with student input, how to:</p> <ul style="list-style-type: none"> <li>› match the results of others' chance-based investigations with statements</li> <li>› <b>explain</b> and <b>justify</b> the statements made by others about their findings from chance-based investigations, using interrogative questions.</li> </ul>

## The language of mathematics and statistics: Phase 3

	<b>Year 7</b> <i>Students will know the following new words:</i>		<b>Year 8</b> <i>Students will know the following new words:</i>	
<b>Number</b>	<ul style="list-style-type: none"> <li>› discount</li> <li>› divisibility rule</li> <li>› exponent</li> <li>› highest common factor (HCF)</li> <li>› integer</li> <li>› lowest (least) common multiple (LCM)</li> </ul>	<ul style="list-style-type: none"> <li>› simplify</li> <li>› square root</li> </ul>	<ul style="list-style-type: none"> <li>› benchmark fraction</li> <li>› budget</li> <li>› composite number</li> <li>› cube number</li> <li>› financial plan</li> <li>› percentage increase or decrease</li> </ul>	<ul style="list-style-type: none"> <li>› powers of 10</li> <li>› prime number</li> </ul>
<b>Algebra</b>	<ul style="list-style-type: none"> <li>› coefficient</li> <li>› coordinate</li> <li>› expression</li> <li>› like term</li> <li>› line graph</li> <li>› reciprocal</li> <li>› X axis, horizontal axis</li> </ul>	<ul style="list-style-type: none"> <li>› XY plane</li> <li>› Y axis, vertical axis</li> </ul>	<ul style="list-style-type: none"> <li>› expand</li> <li>› linear relationship</li> <li>› rate of change</li> <li>› substitute</li> </ul>	
<b>Measurement</b>	<ul style="list-style-type: none"> <li>› composite shape</li> <li>› digital</li> <li>› duration</li> <li>› formula</li> <li>› rate</li> <li>› speed</li> </ul>		<ul style="list-style-type: none"> <li>› millisecond</li> <li>› square unit</li> </ul>	

	<b>Year 7</b> <i>Students will know the following new words:</i>	<b>Year 8</b> <i>Students will know the following new words:</i>
<b>Geometry</b>	<ul style="list-style-type: none"> <li>› complementary or supplementary angle</li> <li>› scale factor</li> </ul>	<ul style="list-style-type: none"> <li>› cross section</li> <li>› diagonal</li> <li>› exterior angle</li> <li>› grid reference</li> <li>› invariant property</li> </ul>
<b>Statistics</b>	<ul style="list-style-type: none"> <li>› continuous data</li> <li>› critique</li> <li>› interpret</li> <li>› measure of centre (mean, median, mode)</li> </ul>	<ul style="list-style-type: none"> <li>› distribution</li> <li>› long-term trend</li> <li>› multivariate data set</li> <li>› time series</li> </ul>
<b>Probability</b>	<ul style="list-style-type: none"> <li>› dependent, independent</li> <li>› event</li> <li>› experiment</li> <li>› experimental or theoretical probability</li> <li>› trial</li> </ul>	<ul style="list-style-type: none"> <li>› distribution</li> <li>› misconception</li> <li>› model</li> <li>› random</li> </ul>

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**Te Poutāhū**  
Curriculum Centre

**Te Mātaiaho**

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# **The New Zealand Curriculum**

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**MATHEMATICS AND  
STATISTICS YEARS 9-13**

*Mātai aho tāhūnui,  
Mātai aho tāhūroa,  
Hei takapau wānanga  
E hora nei.*

*Lay the kaupapa down  
And sustain it,  
The learning here  
Laid out before us.*



**Te Tāhuhu o  
te Mātauranga**  
Ministry of Education

**Te Kāwanatanga  
o Aotearoa**  
New Zealand Government

**DRAFT**

# Foreword

Tēnā koutou katoa,

It is my pleasure to introduce the revised mathematics and statistics learning area for years 9–13 of the New Zealand Curriculum. This marks the first time we have developed a national curriculum that is knowledge-rich, year-by-year, sequenced, and based on the science of learning. During the quality assurance process, the Education Review Office engaged with experts from around the world to give us confidence that our maths curriculum is internationally comparable and sets our students up for success. For years 11–13, the learning area is also broken out into separate subjects (mathematics and statistics) to more clearly align with NCEA achievement standards and senior secondary programmes of learning.

This new curriculum ensures clarity about what students need to understand, know, and do each year, while also providing teachers with a clear framework, resources, and supports to guide their classroom practice.

Our aim is to reduce ambiguity, empowering educators to teach with confidence and ensuring that all students have access to a high-quality education that prepares them for success in the future. This curriculum reflects the dedication of many individuals and groups working together to offer our young people the best educational opportunities, including teachers, principals and educational leaders from around the country.

In the mathematics and statistics learning area, students will develop a deep understanding of symbolic representation, reasoning, and abstraction. They will explore and explain patterns and relationships in quantity, space, time, data, and uncertainty. It is my ambition that every student in New Zealand will not only engage with mathematics and statistics but master the knowledge and skills necessary to excel, developing confidence and capability as mathematicians and a sense of personal curiosity and enjoyment that comes from making sense of the world around them. This new curriculum, alongside the years 0–8 content released last year, will form a comprehensive framework that ensures a strong, consistent foundation for mathematics education across Aotearoa New Zealand and enables us to invest in consistent, high-quality resources and professional development.

Ultimately, this new content is about raising achievement and closing the equity gap. Every learner should have the opportunity to realise their full potential and thrive in future study, work, society, and life.

I encourage you to engage with this learning area content and provide your feedback so that we can ensure your expertise as classroom teachers and experts in mathematics education is reflected in the final version of this learning area, due for release later this year. Every piece of feedback is incredibly valuable, and I look forward to working together to ensure that this curriculum is truly world-leading and sets up both our teachers and students for success.

Ngā mihi nui

**Hon Erica Stanford** (Minister of Education)

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# The New Zealand Curriculum – knowledge-rich, informed by the science of learning, and framed within the whakapapa of Te Mātaiaho

The New Zealand curriculum is a knowledge-rich curriculum that prioritises and explicitly describes the teaching that must be taught each year. It is deliberately sequenced to enable students to build knowledge, skills, and competencies systematically over time. It supports teachers to design teaching and learning programmes that bring it to life in the classroom, through local, national, and global contexts.

The curriculum builds on understandings of how we learn from the science of learning to identify five key themes:

› **We learn best when we experience a sense of belonging in our learning environments and feel valued and supported.**

Students bring with them different cultural identities, knowledge, belief systems, and experiences. They need to see that these are valued and reflected in a school environment characterised by strong relationships and mutual respect between teachers and students and sensitivity towards students' individual needs, emotions, cultures, and beliefs.

› **A new idea or concept is always interpreted through, and learned in association with, existing knowledge.**

The amount of existing knowledge a student has about a topic and the degree to which they are able to connect it to the topic influence both the quality and ease of learning. Recognising and drawing on students' prior knowledge are connected to improved learning outcomes.

› **Establishing knowledge in a well-organised way in long-term memory reduces cognitive load and allows the knowledge to be applied and transferred.**

Building a knowledge base requires active engagement and multiple opportunities to learn new knowledge and connect it to existing knowledge structures. When a learner encounters a new situation, if they have the appropriate knowledge in the form of well-structured schema, they are more likely to be able to apply this existing knowledge to the new situation. If they do not have the appropriate knowledge in long-term memory, they will be dependent on their more limited working memory and may find the new information causes cognitive overload.

› **Our social and emotional wellbeing directly impacts on our ability to engage in learning.**

Positive social wellbeing frees up cognitive capacity, enhances the process of learning new knowledge and skills, and leads to deeper, more permanent learning for students. Conversely, stress, anxiety, and negative emotions inhibit a student's ability to learn. For each student, there will be different factors that impact, positively or negatively, on their social and emotional wellbeing. The influence of these factors is dynamic, varying over time and even within the course of a day.

› **Motivation is critical for wellbeing and engagement in learning.**

Motivation develops when students feel that three basic needs are met – autonomy, a sense of competence, and connection. Success in learning helps to build motivation.

The whakapapa of Te Mātaiaho draws together these themes in a structure that is coherent and inclusive of all students.

# The New Zealand Curriculum – knowledge-rich, informed by the science of learning, and framed within the whakapapa of Te Mātaiaho

The design of this framework encompasses seven curriculum components. Te Mātaiaho as a whole weaves together these components, all of which begin with the word 'mātai', meaning to observe, examine, and deliberately consider.

## Mātaiahikā | Relationships with tangata whenua and local community

### Learning through relationships with tangata whenua and local communities

*Mātai kōrero ahiahi. | Keep the hearth occupied, maintain the stories by firelight.*

Poutama curves represent relationships with tangata whenua and the community.

## Mātaioho | National curriculum – contextualised

### The process by which schools bring the national curriculum to life through local, national, and global contexts

*Mātai oho, mātai ara, whītiki, whakatika. | Awaken, arise, and prepare for action.*

Unaunahi scales represent wealth of knowledge, purpose, and know-how.

## Mātaiaho | Learning areas

### The eight learning areas, which each include a purpose, big ideas, knowledge, and practices, year-by-year

*Mātai rangaranga te aho tū, te aho pae. | Weave the learning strands together.*

Taratara-a-kae niho notches represent diversity, resilience, and mana.

› Mathematics and statistics years 0-8

## Mātairangi | The guiding kaupapa

**The overarching kaupapa guiding the curriculum, based on the science of learning and ensuring excellent and equitable outcomes for students**

*Mātai ki te rangi, homai te kauhau wānanga ki uta, ka whiti he ora. | Look beyond the horizon, and draw near the bodies of knowledge that will take us into the future.*

The outer rings represent our guiding kaupapa.

## Mātainuku | Creating a foundation

**The curriculum principles (e.g., holding high expectations, and enabling all students to access the full scope of the curriculum)**

*Mātai ki te whenua, ka tiritiria, ka poupoua. | Ground and nurture the learning.*

The centre rings represent the foundation and calls to action.

## Mātaipū | Vision of young people

**The educational vision of young people, as conceived by young people**

*Mātaipū hei papa whenuakura. | Grow and nourish a thriving community.*

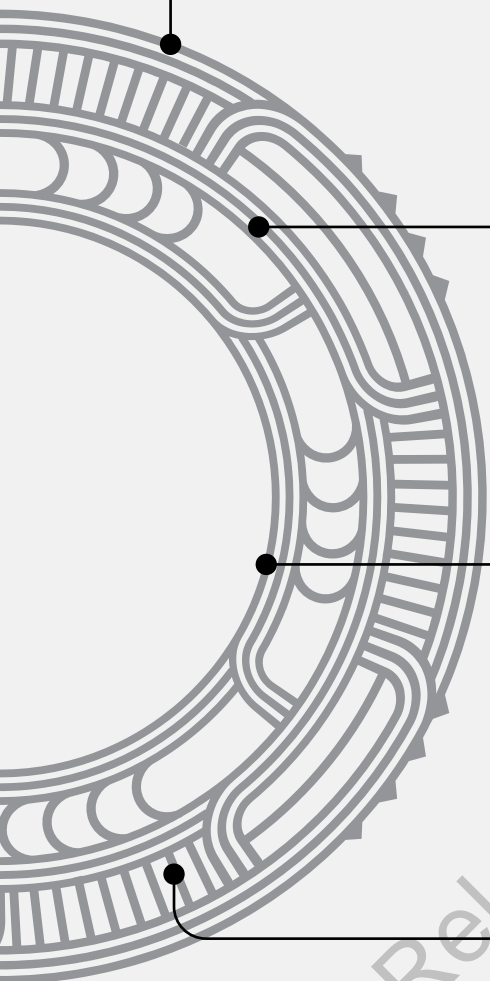
The inner rings and circular space represent the vision and students at the centre.

## Mātairea | Supporting progress

**The whole schooling pathway and the overarching focus for year-by-year learning and progress**

*Mātai ka rea, ka pihī hei māhuri. | Build and support progress.*

Niho kurī lines represent building and supporting the development of students.



# Learning areas

The curriculum has eight learning areas: English, the arts, health and physical education, learning languages, mathematics and statistics, science, social sciences, and technology. Together they provide the basis for a broad, general education for the first four phases of learning (years 0–10) and collectively lay a foundation for specialisation in phase 5 (years 11–13).

Each learning area is knowledge-rich. This knowledge has been carefully chosen to support all students in their schooling pathway and is framed using Understand, Know, and Do:

- › **Understand** – the deep and enduring big ideas and themes that students develop understanding of over the phases
- › **Know** – the meaningful and important concepts and topics at each phase that enrich students’ understanding of the big ideas and themes and that students study using the practices
- › **Do** – the practices (skills, strategies, and processes) that bring rigour to learning and support the development of the key competencies.

A **progression model** provides the structure that sequences the knowledge. It supports all students to develop greater:

- › breadth and depth of knowledge and understanding, through engaging with increasingly complex and ambiguous contexts
- › refinement and sophistication in their use of competencies, practices, strategies, processes, and skills
- › ability to connect, transfer, and apply new learning in meaningful contexts
- › knowledge and awareness of themselves as learners
- › effectiveness when working with others.

## Content of the learning areas

Knowledge and progression are reflected in how the learning areas are organised.

### Purpose statement and UKD overview

A purpose statement for each learning area describes the learning area's contribution to the lives of students. It is followed by an overview of Understand, Know, and Do, which gives a broad view of the big ideas, themes, concepts, topics, and practices that underpin the learning area.

Teachers use the purpose statement and UKD overview to develop an understanding of the learning area, so that they can share its benefits with students.

### Learning area structure

For each learning area, this section outlines its structure and the changes it undergoes over five phases of learning, particularly in the final phase, where students may opt to specialise and choose from a range of subjects.

There are five phases of learning, spanning years 0–13. Each phase covers two to three years of schooling, which reflects how most schools organise learning across year levels.

A **critical focus** for each phase establishes a sustained, strengths-based, holistic focus on the student and their social, emotional, and cognitive learning at this stage of their schooling journey. Each critical focus builds on the phase before and is reflected in the content of the learning area for the phase.

The critical focuses are:

- › **Phase 1** (years 0–3): Thriving in environments rich in literacy and maths
- › **Phase 2** (years 4–6): Expanding horizons of knowledge, and collaboration
- › **Phase 3** (years 7–8): Seeing ourselves in the wider world and advocating with and for others
- › **Phase 4** (years 9–10): Having a purpose and being empathetic and resilient
- › **Phase 5** (years 11–13): Navigating pathways and developing agency to help shape the future

Teachers use the critical focus of each phase in their selection and design of topics and activities.

## Teaching guidance

Each learning area also draws from the science of learning and wider education theory to provide a knowledge base and guidance for teachers. Teachers use this to help them make purposeful decisions about how to teach the learning area's content in ways that are inclusive of all students.

The guidance is organised under three headings:

- › Designing a comprehensive teaching and learning programme
- › Using assessment to inform teaching
- › Planning

## Progress outcomes

In each learning area, there is one progress outcome for each phase.

The progress outcomes outline expectations. They act as signposts at the end of each phase of learning and indicate what students should sufficiently understand, know, and be able to do at key points in the schooling pathway.

The content of each progress outcome is organised using the Understand-Know-Do framework and reflects the critical focus of the phase. While the Understand statements remain the same over the five phases, students' depth of understanding increases as their knowledge of the learning area's content grows and their use of the practices of Do develops.

The progress outcomes help teachers maintain an overview of what students learnt in the prior phase and the expectations of achievement for the next phase, and so are key for planning, along with the more detailed teaching sequences (described below).

Teachers should also use the progress outcomes to take an 'on-balance view' of student progress and achievement. Schools can use information from twice-yearly, standardised assessment tools to help develop an on-balance view.

The following statements describe this view:

- › Are students **using learning from the progress outcome of the previous phase to make sense of new learning in the current phase?** This demonstrates how well they can connect new learning to what they already know. It generally occurs in the first year of a phase.
- › Are students **consolidating the learning expressed in the progress outcome** in a wide range of contexts? This demonstrates how well and confidently they are using their new learning. This generally occurs in the second year of the phase.
- › Are students **secure in the learning described in the progress outcome within an increasingly complex range of contexts?** Are they showing greater depth of knowledge, understanding, and application as they use their new learning and prepare for the challenges of the next phase? This generally occurs towards the end of the final year of the phase.

On-balance views enable teachers and leaders to identify the learning to prioritise next. They can also be used to report to parents.



## Teaching sequences

Each phase has a year-by-year teaching sequence. These sequences support teachers to know what to teach and when and how to teach it as students work towards the progress outcome for the phase. They have been organised to support students to revisit ideas, knowledge, and practices in ways that deepen their learning and enable them to use it at the next phase.

There are two parts in a teaching sequence: statements of **what** to teach, and 'teaching considerations' for **how** to teach:

- › the 'what to teach' statements are preceded by the stem 'Informed by prior learning ...', which reminds teachers to use their professional judgement and assessment information when selecting what content to teach
- › the teaching considerations help teachers to know 'how to teach' this content in response to students' prior knowledge, strengths, and experiences.

The teaching sequence tables should be viewed both vertically and horizontally. Looking down the columns helps teachers know what to plan for in a year's programme. Looking across the rows at the statements for the same concept in the preceding and following years helps teachers to recognise prior learning that students may come with and to consider how they might extend this year's learning. It also helps teachers in taking an on-balance view of their students' progress (see above), and it is a strong support when planning for mixed level classes.

The approach of the year-by-year teaching sequences changes in phase 5, as the content become more discipline-focused.

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Te Mātaiaho

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# The New Zealand Curriculum

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MATHEMATICS AND  
STATISTICS YEARS 9-13

## Purpose statement

*Ānō me he whare pūngāwerewere.  
Behold, it is like the web of a spider.*

This whakataukī celebrates intricacy, complexity, interconnectedness, and strength. The learning area of mathematics and statistics weaves together the effort and creativity of many cultures that over time have used mathematical and statistical ideas to understand their world.

In the mathematics and statistics learning area, students learn about and appreciate the power of symbolic representation, reasoning, and abstraction. They learn to investigate, interpret, and explain patterns and relationships in quantity, space, time, data, and uncertainty. As they achieve deep conceptual understanding and procedural fluency in the learning area, students can accurately and efficiently use mathematics and statistics as a foundation for new learning and to solve problems.

Students engage with mathematics and statistics through the exploration of problems, patterns, and trends and appreciate the everyday value of this learning in many areas of their lives, such as personal finance, health, dance, and design. Every student in New Zealand can engage in mathematics and statistics and discover personal enjoyment and curiosity in their learning.

Throughout their learning, students engage with diverse perspectives as they apply their mathematical and statistical understandings. They also learn that mathematics and statistics has an evolving history; many cultures have contributed to, and continue to contribute to, innovations that shape our current thinking.

As they move through the phases of the learning area, students come to understand the value of mathematical and statistical investigation as a lens for collective local, national, and global challenges. Mathematics and statistics allow us to engage with important societal matters, such as the robust and ethical gathering, interpretation, and communication of data, and the use of valid and reliable data to challenge misinformation and disinformation.

Learning in mathematics and statistics builds literacy by developing students' skills in oral and written communication, reasoning, and comprehension. The learning area opens pathways into a wide range of industries that rely on mathematical and statistical knowledge and reasoning. Learning how to use this knowledge purposefully and flexibly allows students to participate fully in an increasingly technology- and information-rich world of work.

# Understand-Know-Do Overview



## Understand

**Understand** describes the deep and enduring mathematical and statistical **big ideas** that students develop over phases 1-5.

### Patterns and variation | Ngā ia auau me ngā rerekētanga

The world is full of patterns and is defined by a multitude of relationships in which change and variation occur. Mathematics and statistics provide structures that are useful for noticing, exploring, and describing different types of patterns and relationships, enabling us to generate insights or make conjectures.

### Logic and reasoning | Te whakaaro arorau me te whakaaroaro

By engaging with mathematical concepts, we develop logical reasoning and critical thinking skills that enable us to evaluate information, question assumptions, and present arguments with clarity. Statistical reasoning from observation and theory allows us to differentiate what is probable from what is possible and to draw reliable conclusions about what is reasonable.

### Visualisation and application | Te whakakite me te whakatinana

The visualisation of mathematical and statistical ideas profoundly influences how we perceive, understand, and interact with abstract concepts. Application in mathematics and statistics involves creating structures and processes that help us understand complex situations, enabling better decision making and communication of ideas.

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# Know

**Know** describes the meaningful and important mathematical and statistical **concepts and procedures** through which students develop understanding of the big ideas.

## Number | Mātauranga tau

*Number* focuses on the study of numerical concepts. People use numbers to represent quantities, estimate, and measure. We perform operations on numbers to calculate or compare. Throughout history, different number systems have been developed, reflecting practical and social needs.

## Algebra | Taurangi

*Algebra* focuses on making and using generalisations to reason mathematically. It allows us to identify patterns and underlying mathematical relationships. These generalisations, patterns, and relationships can be represented and communicated using diagrams, graphs, and symbols (including variables). The algebra we use today was created and refined over thousands of years.

## Measurement | Ine

*Measurement* focuses on the concepts and techniques that allow us to quantify phenomena, using appropriate units and systems of measurement. Countries around the world use both standard and non-standard units to measure tangible and intangible objects and quantities.

## Geometry | Āhuahanga

*Geometry* focuses on visualising, representing, and reasoning about the shape, position, orientation, and transformation of objects. Many cultures use tools and techniques derived from the natural world when exploring and describing objects and space.

## Statistics | Tauanga

*Statistics* focuses on tools, concepts, and systematic processes for interpreting situations, using data and its context to understand uncertainty, make conjectures, and inform decision making. Statistical practices include considering the ethics of data collection and the responsibility of safely and securely handling data in different contexts.

## Probability | Tūponotanga

*Probability* focuses on tools and concepts for quantifying chance, dealing with expectation, and using evidence to identify how likely events are to occur. People around the world have relied on and continue to rely on probabilistic thinking when making decisions.

# Do

**Do** describes the **processes** that are fundamental to all mathematical and statistical activities and that underpin students' learning of the big ideas, concepts, and procedures.

## Investigating situations | Te tūhura pūāhua

When we investigate situations using mathematics and statistics, we describe and explore them to build our understanding of them. When investigating, we need to decide which approaches, concepts, and tools to use and how to use them. We often begin with a question or focus of interest and proceed in systematic but flexible ways, using mathematical and statistical concepts and procedures to solve problems and make sense of findings in context. We conclude by evaluating the investigation, which involves reflecting on the solutions and outcomes and our approaches and choices to determine whether they were reasonable, made sense in context, and could be improved on in future investigations.

## Representing situations | Te whakaata pūāhua

When we represent situations mathematically and statistically, we use words or symbols and mental, oral, physical, digital, graphical, or diagrammatic ways to show concepts and findings. We can use representations to compare, explore, simplify, illustrate, prove, and justify, as well as to look for patterns, variations, and trends. Representing a situation in multiple ways enables a deeper and more flexible understanding and allows us to communicate with different audiences.

## Connecting situations | Te tūhono pūāhua

When we connect situations using mathematics and statistics, we recognise and make links by noticing similarities and differences. Connecting helps us to understand the relationships between concepts and procedures in mathematics and statistics. This is important because number, algebra, measurement, geometry, statistics, and probability form a web of interconnected ideas and approaches that can be easier to remember and understand if the connections between them are clear. Connecting also involves linking mathematics and statistics to other learning areas and to a range of contexts.

## Generalising findings | Te whakatauwāhānui i ngā kitenga

When we generalise mathematical and statistical findings, we move from specific examples to general principles. We use the patterns, regularities, and structures that we find to make conjectures that might apply to other situations. Further investigation can test and refine these conjectures and determine if they apply in all cases. In statistics, we generalise by using trends and variation in data to make inferences and conjectures and to articulate and evaluate claims about similar situations.

## Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

When we explain and justify, we use mathematical and statistical ways of communicating and reasoning to share our ideas and to respond to the ideas, reasoning, and inferences of others. Explaining is how we communicate our inferences and conjectures, build arguments, and unpack stories from data. Justifying involves describing why decisions and findings are reasonable, taking into account limitations arising from assumptions and choices and the evidence on which findings are based.

# Mathematics and statistics learning area structure

This section describes the structure of the mathematics and statistics learning area and how it changes over the five phases of learning.

Each phase has:

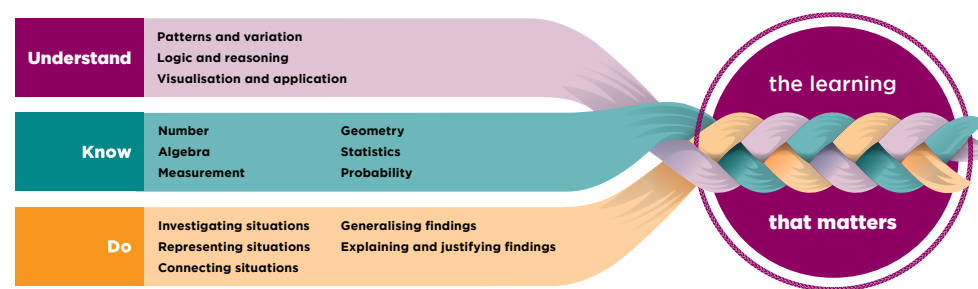
- › a progress outcome describing what students understand, know, and can do by the end of the phase
- › an introduction to the teaching sequence highlighting how to teach during this particular phase
- › a year-by-year teaching sequence highlighting what to teach in the phase, along with teaching considerations for particular aspects of content.

## Progress outcomes

The progress outcomes (one per phase) describe what students will understand, know, and be able to do by the end of the phase.

- › **Understand** describes the big ideas that students develop from learning mathematics and statistics over phases 1 to 5. They help connect school mathematics and statistics with the wider world and represent the critical big-picture concepts of mathematics and statistics.
- › **Know** describes the meaningful and important concepts and procedures in mathematics and statistics. They are broken down into six strands: number, algebra, measurement, geometry, statistics, and probability.
- › **Do** describes the processes students use to represent and work with what they know and understand in mathematics and statistics. These processes are central to how students learn and apply mathematical and statistical knowledge. While there are small progressions in the processes from phase to phase, in general the increasing sophistication of their use comes from applying them to more advanced concepts and procedures.

It is through the interweaving of Understand, Know, and Do that students develop their conceptual understandings and procedural fluency, supporting success and bringing richness and meaning to mathematics and statistics for them.



## Strands

Learning in each phase builds on learning from previous phases. In phase 1, the focus is on developing foundational concepts and procedures in all strands. In phases 2, 3, and 4, students expand their range of representations and their reasoning to work with increasingly complex concepts across all strands. In phase 5, year 11, students experience learning across the strands in a more holistic way, with measurement and geometry merged into a single strand. In years 12 and 13, the learning area allows for greater specialisation by dividing into the separate disciplines of mathematics and statistics.

Phase <b>1</b> Years 0-3	Phase <b>2</b> Years 4-6	Phase <b>3</b> Years 7-8	Phase <b>4</b> Years 9-10	Phase <b>5</b> Year 11 Years 12-13
<b>Number</b>				<b>Mathematics</b>
<b>Algebra</b>				
<b>Measurement</b>				
<b>Geometry</b>				
<b>Statistics</b>				<b>Statistics</b>
<b>Probability</b>				

## Teaching sequences

In years 1-11, the year-by-year teaching sequences are organised in line with the strands from Know. They describe the incremental teaching required each year as students work towards the progress outcome.

Some statements in the teaching sequences are repeated across multiple years, allowing more time for progression and consolidation. Not all statements are progressed each year; some topics start and others end, reflecting what is developmentally appropriate in learning in mathematics and statistics.

Each statement in a sequence varies in the amount of teaching time it requires. The learning area is designed to enable knowledge and

procedures to be connected and taught together, so individual statements in a year sequence should be combined in ways that enhance learning.

The year-by-year content can be viewed both vertically and horizontally. The vertical view helps teachers know what to plan for across each year. The horizontal view allows teachers to follow the statements for one concept across several stages. This helps them understand the prior knowledge students may bring to their learning and helps them decide how to extend this learning. The horizontal view also helps teachers plan for mixed-level classes.

The teaching sequence statements are supported by 'teaching considerations'. These describe evidence-based practices and show how teachers can integrate the processes of Do to help their students develop conceptual and procedural knowledge.

# Teaching guidance

Key characteristics of how people learn have informed the development of the mathematics and statistics learning area. These characteristics are:

- › We learn best when we experience a sense of belonging in the learning environment and feel valued and supported.
- › A new idea or concept is always interpreted through, and learned in association with, existing knowledge.
- › Establishing knowledge in a well-organised way in long-term memory reduces students' cognitive load when building on that knowledge. It also enables them to apply and transfer the knowledge.
- › Our social and emotional wellbeing directly impacts on our ability to learn new knowledge.
- › Motivation is critical for wellbeing and engagement in learning.

All five characteristics are interconnected in a dynamic way. They are always only pieces of the whole, so it is critical to consider them all together. The dynamic and individual nature of learning explains why we see individual learners develop along different paths and at different rates.

The implications of these characteristics for teaching mathematics and statistics are described in this section, with more detail in the introduction to each phase and the 'teaching considerations' in the year-by-year teaching sequences.

The remainder of this section focuses on three key areas of teacher decision making:

- › developing a comprehensive teaching and learning programme
- › using assessment to inform teaching
- › planning.

## Developing a comprehensive teaching and learning programme

A comprehensive mathematics and statistics programme needs the following components:

- › explicit teaching
- › positive relationships with mathematics and statistics
- › rich tasks
- › communication in mathematics and statistics.

### Explicit teaching

Explicit teaching is a structured, carefully sequenced approach to teaching. The sequencing of content is thought out and broken down into manageable steps, each of which is clearly and concisely explained and modelled by the teacher. Explicit teaching requires a high level of teacher-student interaction, guided student practice, and, when proficiency is achieved, independent practice.

Explicit teaching supports cumulative learning as new knowledge is built on what students already know. Teachers provide multiple opportunities for practising, reviewing, consolidating, and using previous learning alongside new learning.

Explicit teaching takes account of cognitive overload. With sufficient practice, new learning is transferred to long-term memory. This frees up working memory, opening up opportunities for extension, enrichment, and new learning.

Explicit teaching is strongly interactive – it is not simply teacher talk. It includes rich discussions between teachers and students and amongst students, to check on understanding. Teachers adapt the pace of their teaching in response to students' progress. They engage students in creative and challenging tasks to foster motivation and engagement.

Using materials and visual representations throughout explicit teaching supports students to develop conceptual understandings as they move towards more abstract forms of representation, such as equations. Teachers can reduce students' cognitive load by carefully considering the ways in which visual and written information are presented (e.g., how working and explanations are laid out) and by removing unnecessary information to focus on the key teaching and learning points.

Explicit teaching involves:

- › connecting the current focus to previous learning
- › providing concise, step-by-step explanations, accompanied by student input and discussion
- › explaining, modelling, and demonstrating
- › regularly checking for understanding and providing feedback
- › providing opportunities for collaborative and independent practice.

## Positive relationships with mathematics and statistics

Learning is enhanced when students succeed in and feel positive about their learning. If students feel anxious, they have fewer cognitive resources available for learning.

Positive relationships with mathematics and statistics are supported by teachers through:

- › setting high expectations
- › planning experiences that are accessible to every student and provide daily opportunities for success
- › incorporating students' interests, cultures, and prior knowledge
- › planning opportunities for students to explore and think critically
- › supporting students to use mathematics and statistics to make sense of their world and address local, national, and global issues
- › providing manageable challenges that encourage students to develop perseverance, reinforcing that conceptual understanding and procedural fluency develop with consistent effort
- › increasing scaffolding and supports in response to anxiety as a result of cognitive overload
- › valuing mistakes as an important part of the learning process.

Involving families in students' learning journeys and offering opportunities for collaboration support positive relationships with mathematics and statistics. Teachers also model such relationships by showing curiosity, persistence, and enjoyment, and by engaging in mathematics and statistics themselves.

## Rich tasks

Rich tasks are meaningful problem-solving and investigation experiences, designed to invoke curiosity and engagement. They should relate both to mathematical contexts and wider contexts relevant to the communities, cultures, interests, and aspirations of students.

Rich tasks provide a motivational hook when exploring new concepts and procedures. They can also be used to consolidate concepts and procedures that have already been taught, to develop the mathematical and statistical processes of Do, and to facilitate the transfer and application of learning to new situations. These experiences often allow students to decide how to approach the task, developing their agency, confidence, and motivation.

Teachers design rich tasks that are accessible to all students and offer different levels of challenge. They ensure that students are clear about the purpose of learning, and they consider the core requirements of the task as well as the range of possible responses. As students work on rich tasks, teachers plan opportunities for discussion, collaboration, and feedback. They are actively involved in monitoring, prompting, and questioning during the task, to encourage students to ask questions, test conjectures, make generalisations, and form connections.

## Communication in mathematics and statistics

Students communicate throughout the learning process, both to develop conceptual understanding and to share their thinking and reasoning. Rich, extended interactions are pivotal to students' development of knowledge, processes, and dispositions in mathematics and statistics. Effective discussions build knowledge through sharing, active listening or attending, critiquing, questioning, and extending thinking and reasoning.

Rich interactions make students' reasoning visible. This helps teachers recognise how well students are developing mathematical and statistical processes and concepts, and it provides opportunities for teachers to identify misconceptions and correct them. These interactions also allow teachers to develop students' use of mathematical and statistical language, vocabulary, symbols, representations, and reasoning.

## Using assessment to inform teaching

Assessment that informs decisions about adapting teaching practice is moment-by-moment and ongoing. Teachers use observation, conversations, and low-stakes testing to continuously monitor students' progress in relation to their year level in the teaching sequence. They ensure that they notice and recognise the development, consolidation, and use of learning-area knowledge by students within daily lessons, and that they provide timely feedback. They respond by adapting their practice accordingly. For example, they reduce or increase scaffolding and supports, paying particular attention to anxiety caused by cognitive overload. Formative assessment information can also be collected through self and peer assessment, with students reflecting on goals and identifying next steps.

In addition to daily monitoring, teachers use purposefully designed, formative assessment tasks at different points throughout a unit or topic to highlight the concepts and reasoning students use and understand. Teachers ensure such tasks are valid by addressing barriers to learning, so that every student is able to demonstrate what they know and can do.

When planning next steps for teaching and learning, teachers consider students' strengths and responses along with potential opportunities for further consolidation. Next steps could include:

- › designing scaffolds to support students to access and enrich their learning
- › providing opportunities for students to apply new learning
- › planning lessons focused on revisiting, reteaching, or consolidating learning.

Providing timely feedback throughout the learning process and identifying and addressing misconceptions as they arise lead to the efficient and accurate development of learning-area concepts and promote further learning. Teachers can use feedback to prompt students to recall previous learning, make connections, and extend their understanding.


## Planning

This section provides guidance on what to pay attention to when planning mathematics and statistics teaching and learning programmes. In every classroom, there are many ways in which students engage in learning and show what they know and can do. Using assessment information and designing inclusive experiences, teachers plan an 'entry point' to a new concept or procedure that every student can access. Students' interests and the school culture and community shape the planning, adding richness, creativity, and meaning to the programme.

Teaching and learning plans are developed for each year, topic or unit, week, and lesson and make optimal use of instructional time. The following considerations are critical when planning and designing learning:

- › Develop plans using the teaching sequence statements for the year and knowledge of students' prior learning. Plan for all students to experience all the statements in the sequence for their year level.
- › Map out a year-long programme composed of 'units' by looking for opportunities where statements from the teaching sequence can be taught together. These may be in the same strand or across several strands (e.g., statistics and measurement; algebra and geometry). Plan to weave together learning under Know and Do across the unit to build understanding of the big ideas.

- › Order the units so that new learning builds on students' previous learning and connects over the course of the year. Consider the length of time allocated to specific strands and concepts across the year – some concepts may require more teaching time than others. Ensure the year's programme includes opportunities to retrieve, consolidate, and extend learning around previously taught concepts and processes. Regular opportunities to revisit learning within and across units and years supports students to develop procedural fluency with mathematics and statistics concepts. The shape of these opportunities will vary, depending on students' learning needs.
- › Within unit or weekly plans, break down new concepts and procedures into a series of manageable learning experiences, so that students have several opportunities to develop understanding and fluency. In years 1–8, teach mathematics and statistics for an hour a day. Plan for a balance of explicit teaching (to introduce and reinforce learning) and rich tasks (to investigate a concept, support consolidation of previously taught concepts or procedures, and apply learning to new situations).

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- › Plan for inclusive teaching and learning at all times. Consider offering multiple methods of participating to all students so that they can engage in a variety of learning experiences and have multiple ways to show their progress. Design for equitable access in all learning opportunities. Identify and reduce barriers to learning, and plan universal supports that are available to all students.
  - › Use flexible groups within a lesson, based on the learning purpose for the lesson (e.g., working as a whole class for demonstration and discussion, in smaller groups to investigate a situation or solve a problem, and in pairs to explain thinking and findings). Provide opportunities for both individual and collaborative work, and enable students to determine when they need to work with others and when they need time and space to work independently.
  - › Teach students to use digital tools accurately, appropriately, and efficiently to support their purpose. Enhance teaching and learning with tools for calculating, representing graphs and shapes, and analysing data. While using digital technology is an important skill, students still need the ability to estimate, visualise, and reason, so that they can evaluate whether findings generated by a digital tool are reasonable and effective.

To support students who have not developed the prior knowledge needed for teaching sequence statements for their year or have not learnt everything they have been explicitly taught, teachers can use accelerative approaches. These are approaches that make year-level concepts and procedures accessible to students. They can include additional, targeted small-group teaching, the use of verbal and visual prompts, carefully chosen representations, and explicit teaching of problem-solving strategies.

Teachers can extend students who have developed deep conceptual understanding and procedural fluency for their year by using more challenging rich tasks and problem solving that allow the students to apply their understanding to unfamiliar situations. This also encourages the students to develop further generalisations and to strengthen their mathematical and statistical communication and reasoning.

## Dedicated mathematics and statistics lessons

Depending on the purpose of the lesson, plan to include one or more experiences in each part (Getting started, Working, and Connecting and reflecting). As students are working, take time to notice, recognise, and respond to their learning.

<b>Getting started</b>	<ul style="list-style-type: none"><li>› Recall and connect to prior learning to provide a starting point for all students to access and understand new concepts or processes.</li><li>› Introduce new concepts using a focus activity, group challenge, or task that activates prior knowledge and interests.</li></ul>
<b>Working</b>	<ul style="list-style-type: none"><li>› Provide whole-class, small-group, paired, or individual work opportunities for students to develop or apply concepts and procedures through investigations, tasks, or games.</li><li>› Explicitly teach concepts and procedures by leading interactions that include explanations, demonstrations, questioning, short tasks, and discussion. Use clear and concise language, including correct mathematical and statistical vocabulary, and clear working layouts and notation.</li><li>› Provide additional explicit teaching based on the learning needs of individual students.</li><li>› Help students organise new knowledge in ways that connect with their prior learning – for example, by discussing connections, using graphic organisers, or carefully ordering concepts and procedures in relation to prior learning.</li><li>› Support consolidation of knowledge with targeted practice and activities. For students early in the process of consolidation, these activities should be scaffolded and guided. As students develop understanding and fluency, they complete the activities with increasing independence.</li><li>› Support students to retrieve and use previously taught concepts and procedures in connected ways, such as applying them while investigating situations.</li></ul>
<b>Connecting and reflecting</b>	<ul style="list-style-type: none"><li>› Clearly summarise and connect to the purpose of the lesson.</li><li>› Review learning by discussing, sharing, and analysing the experiences of the lesson.</li><li>› Make connections with prior learning, between mathematics and statistics concepts, with other learning areas, and with situations outside of the classroom.</li><li>› Pre-teach to prepare students for the next lesson.</li><li>› Highlight progress and examples of curiosity, resilience, and persevering through challenge.</li></ul>

The progress outcomes and teaching sequences for phases 1-3 are available in the [Mathematics and Statistics Years 0-8 document](#).

Phase

4

Years 9–10

## Progress outcome by the end of year 10

*Having a purpose and being empathetic and resilient*

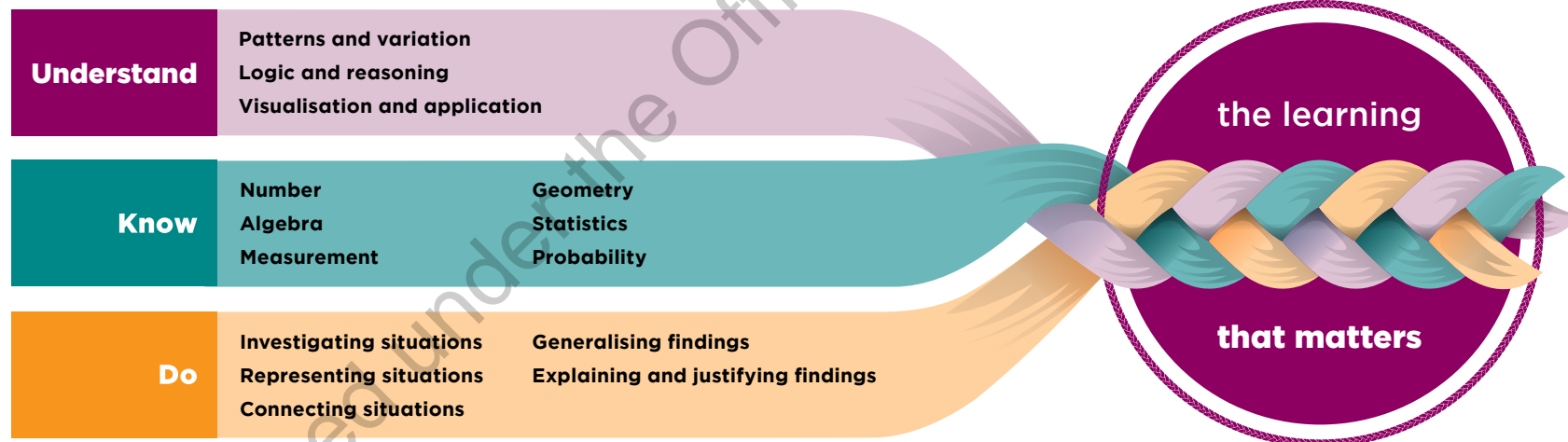
*Te whai ahunga, te manaaki i ētahi atu me te mau tonu ki te manawaroa*

In phase 4, through their learning in mathematics and statistics, students deepen their understanding of the importance and usefulness of the learning area. They apply procedures in a variety of ways to purposefully investigate simple situations, recognising and valuing other perspectives. As they learn more complex concepts they develop resilience, which they draw on when seeking mathematical and statistical solutions.

Students use proportional reasoning to transform numerical quantities, measurements, and shapes, including right-angled triangles. By investigating patterns, they begin to generalise their understanding

of tables, equations, and graphs and of how to connect the different representations. They extend their understanding of area, perimeter, and volume to work with a variety of 2D shapes, including circles, and 3D shapes, including prisms. They use data visualisations to investigate, represent, and explain patterns, trends, and variation, and they apply their knowledge to situations involving chance.

The phase 4 progress outcome describes the understanding, knowledge, and processes students have multiple opportunities to develop over the phase.



The phase 4 progress outcome is found on the following two pages.

# Understand

As students build knowledge through their use of the mathematical and statistical processes, they deepen their understanding of the following.

## Patterns and variation | Ngā ia auau me ngā rerekētanga

The world is full of patterns and is defined by a multitude of relationships in which change and variation occur. Mathematics and statistics provide structures that are useful for noticing, exploring, and describing different types of patterns and relationships, enabling us to generate insights or make conjectures.

## Logic and reasoning | Te whakaaro ararau me te whakaaroaro

By engaging with mathematical concepts, we develop logical reasoning and critical thinking skills that enable us to evaluate information, question assumptions, and present arguments with clarity. Statistical reasoning from observation and theory allows us to differentiate what is probable from what is possible and to draw reliable conclusions about what is reasonable.

## Visualisation and application | Te whakakite me te whakatinana

The visualisation of mathematical and statistical ideas profoundly influences how we perceive, understand, and interact with abstract concepts. Application in mathematics and statistics involves creating structures and processes that help us understand complex situations, enabling better decision making and communication of ideas.

# Know

## Number | Mātauranga tau

By the end of this phase, students know that the order of operations is important when evaluating or forming expressions, and that grouped operations are always done first (e.g., situations under a square root, involving the numerator of a fraction, or inside brackets). Students know that there are an infinite number of rational numbers between any two numbers, and that these can be represented by terminating decimals, recurring decimals, or fractions. Non-repeating, infinite decimals are irrational numbers, and some of these are represented by special symbols, such as  $\sqrt{2}$  and  $\pi$ . A number written in scientific notation has the form  $a \times 10^k$ , where  $1 \leq a < 10$  and  $k$  is an integer.

Students know that ratio concepts can be extended to proportionally compare two or more quantities. Ratios can show part-to-part or part-to-whole relationships and can be scaled up or down or simplified. A rate proportionally compares two quantities that have different units of measure; in rates, 'per' means 'for every' in everyday contexts.

## Algebra | Taurangi

By the end of this phase, students know that the properties of operations (commutative, distributive, associative, inverse, and identity) and the order of operations apply to numbers and variables. When operating on or writing equations with fractions, fractions bigger than 1 are usually written as improper fractions. Any positive real number has two square roots: one positive and one negative. The principal square root is the positive root. The square root operation ( $\sqrt{\quad}$ ) refers specifically to the principal square root. There are 0, 1, or 2 real-number solutions to  $x^2 = a$ , where  $a$  is a number.

## Measurement | Ine

By the end of this phase, students know that decimal measures are used for very small durations (milliseconds) but that the rest of time measurement uses a different system, based principally on 12 and 60. They know that

the number of significant figures in a measurement is the number of digits that contribute to the degree of accuracy of the measurement. Resizing a shape changes its perimeter, area, or volume proportionally according to the dimensions of the units; linear metric conversions must be squared to convert area and cubed to convert volume.

Students know that in right-angled triangles there is a fixed relationship between the lengths of the three sides given by Pythagoras' theorem. The circumference and area of a circle can be approximated using knowledge of polygons and found exactly with formulae that use pi. Students continue to strengthen their use of metric units and conversions between different-sized units, and they develop the idea of derived units, which reflect the ratio between two different measurements. For example, when multiplying lengths or dividing volume by length, the result has a derived unit.

## Geometry | Āhuahanga

By the end of this phase, students know that for all polygons, there is a generalisation for the sum of interior angles and the sum of exterior angles. In similar shapes, corresponding angles are equal and the lengths of corresponding sides are proportional. Congruent shapes are identical in shape and size. A set of points in a plane can be transformed by translation, reflection about a line, and rotation about a fixed point. When working with straight lines, angles between parallel lines and a transversal can be corresponding, co-interior, or alternate, and they have known relationships. A circle is the path traced out by a point moving in a plane and always a fixed distance (the radius) from the centre of the circle.

## Statistics | Tauanga

By the end of this phase, students know that the collection, use, and storage of data needs to respect cultural sensitivities and obey laws about data and privacy. Different countries have different laws and protocols for data and privacy. It is not always possible to get data

about everything or from everyone (the entire population), and there are ways of addressing this (e.g., by using sampling, the choosing of objects or individuals from a population to form a sample).

Students know that the statistical enquiry cycle (PPDAC) can be used to conduct data-based investigations that involve sampling from populations. When sampling from a population, the distribution for a variable varies from sample to sample. In experiments, the experimental units – where we collect the data from – can be people, animals, plants, objects, or non-physical entities. New variables can be created by combining and modifying existing variables. Uncertainty should be taken into account when making claims.

### Probability | Tūponotanga

By the end of this phase, students know that a simulation is a way of modelling a chance-based situation, generally using digital tools. The statistical enquiry cycle (PPDAC) can be used to conduct chance-based investigations involving simulations. Simulations give experimental estimates of probabilities, and these, along with theoretical-model probabilities, are approximations of the true probabilities, which are never known. These probabilities are underpinned by assumptions that are not necessarily the same; for example, we assume that a coin is fair. Some variation between experimental and theoretical-model probabilities is normal.

Students know that there are three different types of models for chance-based situations: 'good model', 'no model', and 'poor model'. A 'good model' is fit for the purpose for which it is being used (e.g., the standard theoretical model for tossing a dice). With 'no model', there is no obvious theoretical model, and so we can only estimate probabilities and probability distributions via experiments (and these estimates can be used to build a theoretical model). A 'poor model' may be unreliable – for example, with spinning a coin, it may appear that heads and tails are equally likely; but a large number of trials shows that it is not a good model and a new model is required, using estimates from experiments.

Students also know that in compound events, events can be dependent or independent, and that elements of chance affect the certainty of results from observational studies or experiments.

## Do

### Investigating situations | Te tūhura pūāhua

By the end of this phase, students can pose a question for investigation, find entry points for addressing the question, and plan an investigation pathway and follow it in a systematic and organised way. They can identify relevant prior knowledge, conditions, assumptions, constraints, and relationships. They can monitor and evaluate progress, adjusting the investigation pathway if necessary, and make sense of outcomes or conclusions in light of a given situation and context.

### Representing situations | Te whakaata pūāhua

By the end of this phase, students can use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns, variations, and trends. They use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments. They select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations. They use visualisation to mentally represent and manipulate relationships, objects, and ideas.

### Connecting situations | Te tūhono pūāhua

By the end of this phase, students can suggest connections between concepts, ideas, approaches, and representations. They connect new ideas to things they already know. They make connections to ideas in other learning areas and with diverse cultural, linguistic, and historical contexts.

### Generalising findings | Te whakatauwānui i ngā kitenga

By the end of this phase, students can notice and explore patterns, structure, and regularity and make conjectures about them. They identify relationships, including similarities, differences, and new connections. They represent specific instances and look for when conjectures about them might be applied in another situation or always be true. They test conjectures, using reasoning and counterexamples to decide if they are true or not. They use appropriate symbols to express generalisations.

### Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

By the end of this phase, students can make statements and give explanations inductively based on observations or data. They make deductions based on knowledge, definitions, and rules. They critically reflect on others' thinking, distinguishing between correct and flawed logic and asking questions to clarify and understand. They use evidence, reasoning, and proofs to explain why they agree or disagree with statements. They develop collective understandings by sharing, comparing, contrasting, critiquing, and building on ideas with others. They present reasoned, coherent explanations and arguments for an idea, solution, or process.

## Teaching sequence

*Having a purpose and being empathetic and resilient*

*Te whai ahunga, te manaaki i ētahi atu me te mau tonu ki te manawaroa*

This section describes how the components of a comprehensive teaching and learning programme for the mathematics and statistics learning area are used during the fourth phase of learning at school.

Throughout phase 4, encourage analytical thinking, perseverance, and an appreciation for the complexity of mathematical concepts. Challenge students to engage with mathematical problems and to critically reflect on both their own and others' reasoning. Encourage them to question assumptions, seek alternative approaches, and explore multiple solutions. Foster an environment of collaborative inquiry where students are expected to take an active role in discussions, supporting and encouraging each other while reflecting on one another's strategies and outcomes. Empower them to make informed decisions about their mathematical explorations, from choosing appropriate methods to justifying their conclusions. This will promote their sense of ownership of their learning journey.

### Explicit teaching

- › Use worked examples and break down new learning into clearly explained, manageable steps. Show students efficient written and mental methods. Encourage them to identify their own errors, misconceptions, and missing steps and to use recently learned techniques to correct these, with teacher guidance. Support them to develop critical analysis and reasoning skills.
- › Plan for students to actively recall learning, practise new procedures, and make connections with prior learning. Provide regular opportunities to practise, so that students maintain their automatic recall of facts and continue to develop procedural fluency and reasoning.
- › Support students to consolidate what they have learned by repeating a process or task you have demonstrated, so that they develop confidence in applying skills in unfamiliar situations.

### Positive relationships with mathematics and statistics

- › Select high-interest tasks that provide a sufficient level of challenge and connect to students' experiences in the classroom and the wider world.
- › Encourage perseverance by demonstrating the value of trying alternative approaches.

## Rich tasks

- › Design investigations where students experience rich mathematical situations, as well as investigations where they use their findings to make decisions in their lives (e.g., planning a holiday). Support students to identify appropriate questions, as well as the mathematical and statistical concepts, procedures, and representations they will need.
- › Design tasks that have multiple entry and exit points and more than one solution or pathway. Use contexts that are familiar to students and that connect with other strands or learning areas. Carefully choose questions and statements that invite students to focus on the concepts underpinning the learning purpose of the task.
- › Support students to convert their wonderings or observations into conjectures and to provide working or counterexamples to prove or disprove them. Encourage them to check if their conjectures always work, if there are times when they don't work, and if they can be extended.
- › Encourage students to identify the key components from a situation or problem. Ask students to represent them with the correct notation and to then find an efficient strategy to solve the problem, representing their working and reasoning using diagrams, materials, and digital tools as appropriate.
- › Check in at specific points throughout an investigation and support students to progress their work. Respond as appropriate with specific, targeted teaching of the mathematical or statistical skills needed to progress the investigation.

- › Encourage students to try a range of procedures and processes in their investigations. Guide them towards selecting the most appropriate process for the problem. Help them to recognise that a first approach not working is common in mathematics and statistics, and that they should then try another approach.

## Communication in mathematics and statistics

- › Set up opportunities for students to actively listen, reflect, and build on each other's thinking and learning. Initiate, shape, and sustain purposeful discussions, and model productive challenges. Over the phase, encourage students to use evidence to justify their claims and findings. Ensure that teacher talk is balanced with opportunities for students to actively participate in and lead discussions.
- › Encourage students to select and use representations that best support the learning purpose, including graphs, tables, and equations. Help them to develop confidence in using higher-order mathematical notation and vocabulary.
- › Encourage students to visualise in order to mentally represent and manipulate relationships, shapes, and quantities.
- › Encourage students to be respectful of each other's views. They should be prepared to argue their point of view or thinking and to explain their reasoning and justifications, while respecting others doing the same.

# Number

	During year 9 <i>Informed by prior learning, teach students to:</i>	During year 10 <i>Informed by prior learning, teach students to:</i>	Teaching considerations
<b>Number structure</b>	<ul style="list-style-type: none"> <li>› record, compare, and order whole and decimal numbers using scientific notation (e.g., <math>3.14 \times 10^3</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› record, compare, and order whole and decimal numbers, including those represented by negative powers in scientific notation (e.g., “Which is larger, <math>7.35 \times 10^{-3}</math> or 0.008?”)</li> </ul>	<p>Demonstrate scientific notation for numbers where the coefficient is between 1 and 10 and the powers of ten are integer exponents only.</p> <p><b>Represent</b> very large numbers and very small numbers with scientific notation.</p> <p>Explore the use of scientific notation in the mathematics strands (e.g., limits of accuracy in measurement, or significant figures in number) and in other learning areas (e.g., science).</p> <p><b>Investigate</b> contextual situations that students are familiar with (e.g., other learning areas’ use of very small or large numbers, or population growth in the news).</p>
	<ul style="list-style-type: none"> <li>› identify special properties of numbers, including cube roots of cube numbers up to 1,000</li> </ul>		<p>Demonstrate exponential growth of whole numbers, <b>explaining</b> the relationship between base, power, and root (e.g., <math>2^3 = 8</math>, <math>\sqrt[3]{8} = 2</math>).</p> <p><b>Represent</b> cube numbers and other exponential numbers in exponential form, <b>generalising</b> that the base is the root (e.g., <math>2^5 = 32</math>, <math>\sqrt[5]{32} = 2</math>).</p>
<b>Operations</b>	<ul style="list-style-type: none"> <li>› use rounding and estimation to predict results and to check the reasonableness of calculations</li> </ul>	<ul style="list-style-type: none"> <li>› use rounding and estimation to predict results and to check the reasonableness of calculations</li> </ul>	<p>Use word problems and practical situations outside the classroom, ensuring that the magnitude of numbers involved is appropriate to the year level.</p>
	<ul style="list-style-type: none"> <li>› round to the degree of precision required for the context</li> </ul>	<ul style="list-style-type: none"> <li>› round to the degree of precision required for the context</li> </ul>	<p><b>Connect</b> with measurement and other practical situations.</p>
	<ul style="list-style-type: none"> <li>› express remainders as fractions or decimals, depending on the context</li> </ul>		<p><b>Represent</b> and make sense of remainders as fractions, as decimals, and when rounded to the nearest whole number, using number lines and diagrams.</p>

	During year 9 <i>Informed by prior learning, teach students to:</i>	During year 10 <i>Informed by prior learning, teach students to:</i>	Teaching considerations
<b>Operations</b>	<ul style="list-style-type: none"> <li>› evaluate expressions with positive whole number exponents, using the order of operations</li> </ul>	<ul style="list-style-type: none"> <li>› evaluate expressions using the order of operations (e.g., <math>\frac{2-3^2}{2}</math>)</li> </ul>	<p>Demonstrate a mnemonic such as GEMA (grouped, exponents, multiplicative, additive), and common errors.</p> <p><b>Investigate:</b></p> <ul style="list-style-type: none"> <li>› the order of operations – why it is used, and what happens when it is not followed</li> <li>› situations in familiar contexts, to reinforce students' skills in practical situations (e.g., using number operation skills in measurement and algebra problems and in financial mathematics)</li> <li>› operations that involve square roots with exponents (e.g., <math>\sqrt{3^2 + 4^2} \neq 3 + 4</math>)</li> <li>› operating on numbers with whole-number exponents, including powers with negative numbers (e.g., <math>(-1)^4 = 1</math>).</li> </ul>
	<ul style="list-style-type: none"> <li>› add and subtract integers, multiply integers using repeated addition or subtraction, and divide integers by reversing a multiplication (e.g., <math>-12 \div 3 = -4</math>, since <math>3 \times -4 = -12</math>)</li> </ul>	<ul style="list-style-type: none"> <li>› add, subtract, multiply, and divide positive and negative numbers, including fractions and decimals</li> </ul>	<p><b>Represent</b> operations on number lines and with two-sided counters, connecting with practical situations.</p> <p><b>Investigate:</b></p> <ul style="list-style-type: none"> <li>› multiplicative inverses with negative numbers (e.g., <math>\frac{1}{-b} = -\frac{1}{b}</math>)</li> <li>› how multiplication and division as inverse operations extend to negative numbers (e.g., <math>3 \times -2 = -6</math>) and families of facts</li> <li>› practical situations outside the classroom that use integers (e.g., sea level or temperature)</li> <li>› why division by a fraction is equivalent to multiplication by its reciprocal.</li> </ul> <p><b>Generalise</b> relationships between positive and negative integers using the commutative, associative, and distributive properties.</p> <p><b>Explain and justify</b> the effect of dividing by numbers between 0 and 1.</p>

	<b>During year 9</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 10</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Rational numbers</b>	› identify, read, write, represent, compare, order, and convert between fractions, decimals, and percentages	› identify, read, write, represent, compare, order, and convert between fractions, decimals, and percentages	<b>Represent</b> fractions, decimals, and percentages on number lines and 100s squares. Demonstrate the use of flow diagrams in conversions. <b>Investigate:</b> › fractions, decimals, and percentages as equivalent representations of the same values › practical situations where fractions, decimals, percentages, rates, and ratios are used.
	› find equivalent fractions, simplify fractions, and convert between improper fractions and mixed numbers	› find equivalent fractions, simplify fractions, and convert between improper fractions and mixed numbers	Use factors trees to <b>investigate</b> highest common factors and lowest common multiples. <b>Explain</b> that ‘improper’ is just a technical term for ‘top-heavy’ fractions, which are a useful format for working with fractions.
	› find a fraction or percentage of a number, and increase or decrease a number by a fraction or percentage	› calculate the percentage increase or decrease between two numbers (e.g., “What is the percentage increase between 50 and 75?”)	Demonstrate how to calculate a percentage increase or decrease with a single decimal multiplier. <b>Investigate</b> common and expected uses for percentage increases and decreases outside the classroom (e.g., discounts, mark-ups, and interest).
	› find the whole amount, given a fraction or percentage (e.g., “20% of an amount is 30, what is the original amount?”)	› find the whole amount, given a fraction or percentage (e.g., “15% of an amount is 27, what is the original amount?”)	<b>Represent</b> backwards and forwards situations using flowcharts and bar models. <b>Connect</b> finding an original amount with forming and solving a linear equation (e.g., $\frac{15}{100}x = 27$ ).
	› divide fractions by whole numbers, and add, subtract, and multiply fractions		<b>Investigate</b> reciprocals, equivalent fractions, and operations using number lines.
	› divide decimals by whole numbers, and add, subtract, and multiply decimals		<b>Represent</b> decimal numbers using place-value materials. <b>Connect</b> operations with decimals to operations with whole numbers and operations with fractions. <b>Explain</b> and <b>justify</b> the effect of multiplying by numbers between 0 and 1.

	<b>During year 9</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 10</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Rational numbers</b>	<ul style="list-style-type: none"> <li>› use ratios to share in unequal proportions, and find equivalent ratios and rates by scaling up or down</li> </ul>	<ul style="list-style-type: none"> <li>› compare and use ratios and rates</li> </ul>	<p><b>Connect</b> ratios to fractions, explaining how the parts of the ratio can become the numerators of corresponding fractions, and the sum of the parts the denominator.</p> <p><b>Investigate</b> practical situations that use fractions and ratios.</p>
<b>Financial mathematics</b>	<ul style="list-style-type: none"> <li>› calculate profit, loss, and discounts as absolute quantities relative to an original amount</li> </ul>	<ul style="list-style-type: none"> <li>› convert NZ dollars into other currencies (and vice versa) and give examples of when this is useful</li> </ul>	<p><b>Connect</b> to operations that use decimals and, in year 10, operations that use rates.</p> <p><b>Investigate</b> the number of decimal places and rounding in cash versus electronic transactions.</p>
	<ul style="list-style-type: none"> <li>› calculate simple interest and GST on dollar amounts (e.g., “Find 15% GST on \$432”).</li> </ul>	<ul style="list-style-type: none"> <li>› find proportions of costs (e.g., the price of 400 g of an item when given the cost per kilogram).</li> </ul>	<p><b>Connect</b> to operations that use fractions, decimals, and percentages.</p> <p><b>Investigate</b> problems that involve accruing interest or budgeting to make a savings target.</p>

## Algebra

	<b>During year 9</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 10</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Equations and relationships</b>	<ul style="list-style-type: none"> <li>› simplify, expand, and factorise algebraic expressions involving sums, products, differences, and positive integer powers</li> </ul>	<ul style="list-style-type: none"> <li>› simplify, expand, and factorise algebraic expressions</li> </ul>	<p><b>Represent</b> operations with algebraic expressions visually, using materials such as algebra tiles, array models, and expansion tables.</p> <p><b>Investigate</b> visual patterns that lead to different ways of representing algebraic expressions, including factorised, expanded, and simplified forms, and show their equivalence.</p>
	<ul style="list-style-type: none"> <li>› form and solve linear equations with rational number solutions and linear inequalities with positive coefficients</li> </ul>	<ul style="list-style-type: none"> <li>› form and solve linear equations and inequalities with rational number solutions, giving exact and rounded solutions</li> </ul>	<p>Demonstrate algorithms for solving equations.</p> <p>Consolidate learning with puzzle squares.</p> <p><b>Explain</b> how the solution to an equation has been reached.</p>

	<b>During year 9</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 10</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Equations and relationships</b>	› find the value of an expression or formula given the values of its variables, and rearrange simple formulae	› substitute into, rearrange, and simplify expressions or formulae	Demonstrate the use of variable values in practical situations that involve familiar formulae (e.g., areas or temperature conversions). <b>Connect</b> to other learning areas that use formulae (e.g., science, technology, or social sciences).
	› create, interpret, graph, and express linear relationships in the form $y = mx + c$ , where $m$ is the constant rate of change (gradient or slope) and $c$ is the y-intercept	› create, interpret, graph, and express linear relationships in more than one form	Demonstrate different forms of linear equations using points, gradient-and-intercept, and intercept-and-intercept methods of forming equations and graphing linear relationships. <b>Investigate</b> relationships between equations and graphs, using manual and digital tools.
	› recognise the relationships between elements in a non-linear pattern, write an equation using variables and algebraic notation to represent the rule for the pattern (e.g., $y = x^2 + c$ ), and use the equation to make conjectures	› determine the effect on XY graphs of changing the coefficient of $x^2$ and the fixed value, for a range of quadratic relationships of the form $y = ax^2$ or $y = x^2 + c$ , where $a$ is a positive whole number and $c$ is an integer	<b>Investigate</b> changes to parabolas that result from changes to their equations, using manual and digital tools. <b>Generalise</b> patterns using words, formulae, and graphical representations.
<b>Algorithmic thinking</b>	› test and improve algorithms that use sequence (determining the order of steps), selection (choosing steps), and iteration (repeating steps).	› design, create, test, and refine algorithms that use sequence, selection – using logical operators, such as equals or not equals – and iteration.	Introduce students to basic coding language and associated digital tools. Use the formula function of a spreadsheet to <b>investigate</b> : › the effect of changing one or more variables on results (i.e., cell values) › the modelling of practical situations that involve measurements and values that change. <b>Connect</b> : › iterative thinking to other strands (e.g., the relationship between the perimeter and area of a rectangle in measurement) › with other learning areas to provide opportunities for students to engage with algorithmic thinking and coding. <b>Investigate</b> a range of digital tools to explore different methods of approaching algorithmic thinking digitally.

# Measurement

	<b>During year 9</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 10</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Measuring</b>	<ul style="list-style-type: none"> <li>› estimate, calculate, convert, and accurately represent measurements using significant figures</li> </ul>	<ul style="list-style-type: none"> <li>› estimate, calculate, convert, and accurately represent measurements using significant figures</li> </ul>	<p>Have students practise taking accurate measurements from diagrams or physical objects using appropriate tools (e.g., a ruler, protractor, scales, or calculator). <b>Represent</b> all written measurements with their correct units.</p> <p>Provide situations in which students are engaged in the process of collecting measurements in order to find lengths and angles or solve a practical problem.</p>
	<ul style="list-style-type: none"> <li>› select and use appropriate measurement units for a given context, if necessary converting between metric units using appropriate prefixes</li> </ul>	<ul style="list-style-type: none"> <li>› convert between metric units, and use appropriate prefixes in the metric system (e.g., tera-, giga-, mega-, kilo-, centi-, milli-, micro-, and nano-)</li> </ul>	<p>Use flow diagrams to demonstrate conversions, showing that the larger the new unit, the smaller the resulting number.</p> <p>Explore how measurements are made in different disciplines, including preferences for units, (e.g., the building industry uses mm and m, whereas fashion uses cm).</p> <p><b>Connect:</b></p> <ul style="list-style-type: none"> <li>› with how other learning areas use measurement</li> <li>› between different units, including rates.</li> </ul>
	<ul style="list-style-type: none"> <li>› use Pythagoras' theorem to find the length of an unknown side in a right-angled triangle from a given diagram</li> </ul>	<ul style="list-style-type: none"> <li>› use Pythagoras' theorem to find the length of an unknown side in a right-angled triangle</li> </ul>	<p>Demonstrate the use of formulae in practical measuring.</p> <p><b>Investigate</b> the properties of triangles using Pythagoras' theorem.</p> <p><b>Connect</b> the properties of right-angled triangles with geometric reasoning about triangles and practical problems outside the classroom.</p>
	<ul style="list-style-type: none"> <li>› find speed, distance, and time, given any two of the measurements and recognising that distance over time is a derived measure</li> </ul>	<ul style="list-style-type: none"> <li>› find speed, distance, and time, given any two of the measurements and recognising that distance over time is a derived measure</li> </ul>	<p><b>Investigate</b> the relationships between speed, distance, and time in practical situations.</p> <p><b>Connect</b> finding the value of variables in the speed formula with solving algebraic equations and multiplication and division operations.</p>

	<b>During year 9</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 10</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Measuring</b>	<ul style="list-style-type: none"> <li>› reason about duration using different units of time and fractions of units of time, including decimal fractions of milliseconds where appropriate</li> </ul>	<ul style="list-style-type: none"> <li>› reason about duration using different units of time and fractions of units of time, including decimal fractions of milliseconds where appropriate</li> </ul>	<p>Demonstrate duration using stopwatches and different ways of measuring and comparing it (e.g., elapsed time in timetables).</p> <p><b>Connect</b> with:</p> <ul style="list-style-type: none"> <li>› other metric units of measurement, exploring the differences between them and time measurement</li> <li>› other systems of time measurement (e.g., the lunar calendar).</li> </ul> <p><b>Investigate</b> ways of sorting and ordering time, including time conversions, the different divisions of time, and the history of why we measure time using a non-metric system.</p> <p>Engage students in activities about time differences by exploring different time zones and calendars.</p>
<b>Perimeter, area, and volume</b>	<ul style="list-style-type: none"> <li>› find the area of parallelograms, trapeziums, and kites, relating the formulae used to the formula for a rectangle</li> <li>› find the circumference and area of circles</li> </ul>	<ul style="list-style-type: none"> <li>› find the surface area and volume or capacity of prisms and cylinders</li> </ul>	<p>Explore relationships and similarities between formulae for area and perimeter. <b>Generalise</b> from specific shapes to more general shapes, (e.g., applying findings for a rectangular prism to a prism with any face shape).</p> <p>Demonstrate the use of formulae in practical situations.</p> <p>Engage students in practical exercises that allow them to explore and make generalisations about shapes, objects, and measurement units.</p> <p><b>Investigate</b> the connection between the number of dimensions of a shape and the exponent of the corresponding measurement unit.</p>
	<ul style="list-style-type: none"> <li>› scale a shape by a factor, and find the perimeter, area, or volume of the scaled shape.</li> </ul>	<ul style="list-style-type: none"> <li>› scale a shape by a factor, and determine the scale factor for the scaled shape's area or volume.</li> </ul>	<p><b>Investigate</b> practical situations to demonstrate how the scale factor of a shape's side relates to the scale factor for the scaled shape's area or volume.</p>

# Geometry

	During year 9 <i>Informed by prior learning, teach students to:</i>	During year 10 <i>Informed by prior learning, teach students to:</i>	Teaching considerations
<b>Shapes</b>	<ul style="list-style-type: none"> <li>› identify and describe parts of a circle (e.g., a chord; the diameter, radius, and circumference) and how they relate to each other</li> </ul>	<ul style="list-style-type: none"> <li>› use the properties of similarity in 2D shapes, including right-angled triangles, to find unknown lengths and angles</li> </ul>	<p><b>Investigate</b> the relationships between the radius, diameter, and circumference of a circle. <b>Explain</b> when and how these involve the constant <math>\pi</math>.</p> <p>Demonstrate the parts of a circle (at year 9) and the properties of similarity (at year 10) using shapes, objects, diagrams, and measurement tools (e.g., rulers or string).</p>
	<ul style="list-style-type: none"> <li>› reason about unknown angles in situations involving intersecting lines, parallel lines, and transversals</li> </ul>	<ul style="list-style-type: none"> <li>› reason about unknown angles in situations involving parallel lines and transversals and the interior and exterior angles of polygons</li> </ul>	<p>Explore unknown angles using digital tools or diagrams with measuring tools (e.g., rulers or protractors).</p> <p><b>Connect</b> with algebraic thinking when solving problems that involve unknown angles, practical measurement, and composite shapes.</p> <p><b>Generalise</b> from specific physical or digital examples of circles, polygons, and sets of lines to explore rules about unknown angles, using algebra.</p> <p>Use geometry rules to <b>explain</b> and <b>justify</b> solutions to problems, exploring where the rules have come from and why they work.</p>
<b>Spatial reasoning</b>	<ul style="list-style-type: none"> <li>› represent and construct 3D shapes, including rectangular and triangular prisms, from nets, plan views, and isometric drawings</li> </ul>		<p>Use measurement tools to create physical and digital models.</p> <p><b>Connect</b> with practical measurement and technical drawing, introducing design and visual elements.</p>
	<ul style="list-style-type: none"> <li>› transform 2D shapes in the XY plane by translation, reflection across a given mirror line, and rotation about a given point by a multiple of 90 degrees</li> </ul>	<ul style="list-style-type: none"> <li>› transform 2D shapes, including composite shapes, by resizing them by any scale factor.</li> </ul>	<p>Predict and test transformations using physical and digital drawing tools (grid paper, mirrors, tracing paper, and computer-aided design programmes).</p>
<b>Pathways</b>	<ul style="list-style-type: none"> <li>› use map scales and compass directions (e.g., N30°W) to interpret and communicate distance and direction from one location to another.</li> </ul>		<p>Use map reading, orienteering, and geocaching.</p> <p><b>Connect</b> with the practical use of maps and navigation and tools within digital maps.</p> <p><b>Investigate</b> the world outside the classroom through practical activities with maps and scales.</p>

# Statistics

	During year 9 <i>Informed by prior learning, teach students to:</i>	During year 10 <i>Informed by prior learning, teach students to:</i>	Teaching considerations
<b>Problem</b>	<ul style="list-style-type: none"> <li>› investigate, using multivariate datasets from observational studies, summary, comparison, time-series, and relationship situations by:               <ul style="list-style-type: none"> <li>- posing an investigative question about groups of interest</li> <li>- making conjectures or assertions about expected findings</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› investigate, using multivariate datasets from observational studies, summary, comparison, time-series, and relationship situations by:               <ul style="list-style-type: none"> <li>- posing an investigative question, in particular a summary or comparison question about a population, or a time-series or relationship question about a group of interest</li> <li>- making conjectures or assertions about expected findings</li> </ul> </li> </ul>	<p><b>Investigate</b> an area of interest, starting broadly and fine-tuning to arrive at a specific investigative question.</p> <p>Demonstrate how to pose an <b>investigative</b> question for different investigative situations:</p> <ul style="list-style-type: none"> <li>› summary and comparison situations, including the variable and groups of interest (at year 9) or population of interest (at year 10)</li> <li>› relationship and time-series situations, including the variable and group of interest.</li> </ul> <p><b>Connect</b> the investigative question with conjectures about expected findings.</p>
<b>Plan</b>	<ul style="list-style-type: none"> <li>› plan how to collect or source data to answer the investigative question, including:               <ul style="list-style-type: none"> <li>- identifying the variables needed to answer the question</li> <li>- planning how to make valid and reliable measures for the variables (when collecting data) or finding out how they were measured (when sourcing data)</li> <li>- identifying the group of interest or who the data was collected from</li> </ul> <p><i>(continued on the next page)</i></p> </li> </ul>	<ul style="list-style-type: none"> <li>› plan how to collect or source data to answer the investigative question, including:               <ul style="list-style-type: none"> <li>- identifying the variables needed to answer the question</li> <li>- planning how to make valid and reliable measures for the variables (when collecting data) or interrogating sourced datasets to understand what the variables measure and how they were measured</li> <li>- identifying the group or population of interest or (for sourced data) 'who' the data was collected from</li> </ul> <p><i>(continued on the next page)</i></p> </li> </ul>	<p><b>Explain</b> and <b>justify</b> methods of data collection, 'who' to measure, and what and how to measure, in order to answer the investigative question.</p> <p>Support students to pose data-collection and survey questions and <b>justify</b> how they will collect the data required to answer the investigative question.</p> <p><b>Investigate</b> secondary datasets to <b>explain</b> their variables and group(s) or population(s) (at year 10) of interest.</p> <p>Reinforce how to apply ethical practices to data collection and use, including <b>explaining</b> what informed consent means and how to use data privacy protocols.</p> <p><i>(continued on the next page)</i></p>

	<b>During year 9</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 10</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Plan</b> <i>(continued from previous page)</i>	<ul style="list-style-type: none"> <li>- using interrogative questions to check that ethical practices are considered throughout the entire enquiry cycle (including testing data collection and survey questions with peers)</li> </ul>	<ul style="list-style-type: none"> <li>- determining if the investigative question is about a group or a population</li> <li>- if a population, determining what sample size is needed to make a decision about what is happening in the population</li> <li>- using interrogative questions to check that ethical practices are considered throughout the entire enquiry cycle</li> </ul>	<b>Investigate:</b> <ul style="list-style-type: none"> <li>› sample sizes needed to make a call about what is happening in the population (year 10) (about 1000 for categorical variables and about 30 for numerical variables)</li> <li>› how to determine and define the variables needed and plan to collect valid and reliable measures for them</li> <li>› possible misinterpretations of survey and data-collection questions and how this can lead to unreliable data.</li> </ul>
<b>Data</b>	<ul style="list-style-type: none"> <li>› collect or source data, including: <ul style="list-style-type: none"> <li>- checking its validity and making simple edits if appropriate</li> <li>- creating a data dictionary (for collected data) or identifying the metadata (for sourced data)</li> <li>- recategorising or constructing new variables if needed to support answering the investigative question</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› collect or source data, including: <ul style="list-style-type: none"> <li>- checking its validity and making simple edits if appropriate</li> <li>- using random samples to collect or source data about a population (for summary and comparison situations)</li> <li>- creating a data dictionary (for collected data) or identifying the metadata (for sourced data)</li> <li>- recategorising or constructing new variables if needed to support answering the investigative question</li> </ul> </li> </ul>	<p>Support students to use a variety of methods for collecting or sourcing the data, including:</p> <ul style="list-style-type: none"> <li>› making measurements</li> <li>› observing outcomes</li> <li>› using survey forms</li> <li>› using secondary data sources</li> <li>› using random samples (at year 10).</li> </ul> <p>Demonstrate how to record the data using a variety of tools – for example:</p> <ul style="list-style-type: none"> <li>› spreadsheets</li> <li>› recording sheets</li> <li>› data dictionaries for primary data</li> <li>› metadata for secondary data.</li> </ul> <p>Support students to identify errors in the data, <b>connecting</b> to the context, <b>explaining</b> why they are errors, and, if appropriate, cleaning the data using appropriate techniques.</p> <p>Demonstrate how to recategorise variables and construct new variables.</p>

	<b>During year 9</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 10</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Analysis</b>	<ul style="list-style-type: none"> <li>› create multiple data visualisations for the investigation, providing global and local views</li> <li>› describe and reason from the data visualisations, including:               <ul style="list-style-type: none"> <li>- identifying relevant features in distributions</li> <li>- interweaving the context in descriptions of distributions</li> <li>- drawing an ‘eyeballed’ line or curve of best fit to predict possible y values (for the response variable) for given x values (for the explanatory variable) in relationship situations, where appropriate</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› create multiple data visualisations for the investigation, providing global and local views</li> <li>› describe and reason from the data visualisations, including:               <ul style="list-style-type: none"> <li>- identifying relevant features in distributions</li> <li>- interweaving the context in descriptions of distributions</li> <li>- making an informal inference about what might be happening in the population, based on visual considerations in comparison situations</li> <li>- drawing an ‘eyeballed’ line or curve of best fit to predict possible y values (for the response variable) for given x values (for the explanatory variable) in relationship situations, where appropriate</li> <li>- making informal conjectures about group membership (e.g., in relation to socio-economic status or educational attainment)</li> </ul> </li> </ul>	<p><b>Represent</b> data using dot plots, bar graphs, frequency tables, box plots, histograms, paired plots, time-series graphs, two-way tables or graphs, and scatter plots. Transition from creating these by hand to using digital tools.</p> <p>During analysis, support students to identify features, using:</p> <ul style="list-style-type: none"> <li>› fractions, proportions, and percentages</li> <li>› the mode, median, mean, and distributional shape</li> <li>› maximum and minimum values</li> <li>› the interquartile range, lower quartile, upper quartile, middle 50%, and range</li> <li>› descriptions such as more than, less than, at least, at most, between A and B</li> <li>› outliers, clusters, gaps, joint and conditional proportions</li> <li>› an eyeballed line or curve of best fit, seasonal patterns, and the long-term trend</li> <li>› decision trees.</li> </ul> <p>Check data visualisations for misleading features or information.</p> <p>Describe what is and is not seen in data visualisations, recognising that data are numbers with context and that the context includes variables of interest, groups of interest, counts or proportions for categorical variables, and values and units for numerical variables.</p> <p>When comparing data visualisations of the same variable for different groups, look at similarities and differences. In year 10, make an informal inference about what might be happening in the population, using the ‘Making the call Year 10’ decision guide.</p>

	<b>During year 9</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 10</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Conclusion</b>	<ul style="list-style-type: none"> <li>› communicate findings in context to answer the investigative question, using evidence from analysis</li> <li>› provide possible explanations for findings</li> <li>› reflect on conjectures or assertions</li> <li>› evaluate the approaches taken in the different phases of the enquiry cycle</li> </ul>	<ul style="list-style-type: none"> <li>› communicate findings in context to answer the investigative question, using evidence from analysis and with an awareness of variability</li> <li>› provide possible explanations for findings</li> <li>› reflect on conjectures or assertions</li> <li>› evaluate the approaches taken in the different phases of the enquiry cycle</li> </ul>	<p>Have students practise answering the investigative question using evidence from their analysis and with an awareness of variability.</p> <p>Explore relevant explanations or interpretations of findings connected to the context of the situation, including the group or population of interest and the variable or variables of interest.</p> <p>Support students to prepare and present succinct findings and to evaluate and recommend any changes to the processes they used. Ask them to explain whether or not their findings align with their initial conjectures or assertions, if what was found makes sense with what is known about the situation, and if there are any limitations to the findings.</p> <p>Ask students to consider, as a result of the statistical investigation, what else they could <b>investigate</b>, starting the PPDAC cycle again.</p>
<b>Statistical literacy</b>	<ul style="list-style-type: none"> <li>› evaluate the data-collection methods and findings of others' statistical investigations to see if their claims are reasonable</li> <li>› critically consider data visualisations to see if they support or misrepresent the data.</li> </ul>	<ul style="list-style-type: none"> <li>› evaluate others' statistical investigations to see if their claims are reasonable</li> <li>› critique others' findings and claims by closely examining all phases of the statistical enquiry cycle</li> <li>› critically consider data visualisations to see if they support or misrepresent the data.</li> </ul>	<p>Explore existing statistical reports, graphs, visualisations, and claims in order to determine their validity. Use readily available media (e.g., news media or social media) as sources.</p> <p><b>Investigate</b> misleading data visualisations, match data visualisations with statements made, and check the claims made by others.</p> <p>Use interrogative questions to evaluate data-collection methods that involve, for example, ethical considerations, analysis processes, visualisations, and statements of findings.</p>

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# Probability

	<b>During year 9</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 10</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Probability investigations</b>	<ul style="list-style-type: none"> <li>› plan and conduct probability experiments for chance-based situations, including undertaking a large number of trials using digital tools, by:               <ul style="list-style-type: none"> <li>- posing an investigative question</li> <li>- systematically listing outcomes for the sample space</li> <li>- constructing a model using theoretical or conjectured probabilities</li> <li>- deciding on the number of trials, the tools to be used, and the recording method</li> <li>- running simulations and recording data</li> <li>- creating data visualisations for the distribution of observed outcomes from the experiment and the distribution of possible outcomes for theoretical probability models</li> <li>- describing what these visualisations show</li> <li>- finding probability estimates for the different outcomes</li> <li>- proposing possible theoretical outcomes and associated probabilities for situations where no theoretical model exists (e.g., tossing a non-regular 3D shape)</li> <li>- identifying similarities and differences between their findings and those of others</li> <li>- reflecting on anticipated outcomes</li> <li>- identifying and giving potential reasons for similarities and differences between findings from the probability experiment and associated theoretical probabilities</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› plan and conduct probability experiments for chance-based situations, including undertaking a large number of trials using digital tools, by:               <ul style="list-style-type: none"> <li>- posing an investigative question</li> <li>- systematically listing outcomes for the sample space</li> <li>- constructing a model using theoretical probabilities, probabilities based on past data, or conjectured probabilities</li> <li>- deciding on the number of trials, the tools to be used, and the recording method</li> <li>- running simulations and recording data</li> <li>- creating data visualisations for the distribution of observed outcomes from the experiment and the distribution of possible outcomes for theoretical probability models</li> <li>- describing what these visualisations show</li> <li>- finding probability estimates for the different outcomes</li> <li>- proposing possible theoretical outcomes and associated probabilities for situations where no theoretical model exists (e.g., tossing a drawing pin)</li> <li>- identifying similarities and differences between their findings and those of others</li> <li>- reflecting on anticipated outcomes</li> <li>- identifying and giving potential reasons for similarities and differences between findings from the probability experiment and associated theoretical probabilities</li> </ul> </li> </ul>	<p><b>Investigate</b>, using the statistical enquiry cycle, everyday chance-based situations, patterns in possible outcomes, theoretical and experimental distributions, and relative frequencies from data investigations.</p> <p><b>Represent:</b></p> <ul style="list-style-type: none"> <li>› probability outcomes (theoretical and experimental) using lists, tables, tree diagrams, tally charts, visualisations of distributions, words, numbers, and digital tools</li> <li>› probability models.</li> </ul> <p><b>Explain</b> how to describe and use probability concepts – for example:</p> <ul style="list-style-type: none"> <li>› outcomes, events, and the sample space</li> <li>› trials, simulations, models, and theoretical and experimental probability</li> <li>› with and without replacement, independence and dependence, joint and conditional probabilities</li> <li>› the law of large numbers</li> <li>› probability estimates, theoretical probability, and probability distributions</li> <li>› chance, randomness, and variation.</li> </ul> <p><b>Connect:</b></p> <ul style="list-style-type: none"> <li>› anticipated distributions with theoretical distributions and experimental distributions, linking representations (e.g., two-way tables) with distributions, words, and numbers</li> <li>› observed data, models, and simulated data.</li> </ul>

	<b>During year 9</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 10</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<b>Critical thinking in probability</b>	<ul style="list-style-type: none"> <li>› identify why claims about chance-based situations might not be valid, and consider improvements</li> </ul>	<ul style="list-style-type: none"> <li>› identify why claims about chance-based situations might not be valid, and consider improvements</li> </ul>	<p>Explore existing claims about known chance-based situations and discuss where they came from, how valid they are, and what might be done to improve them.</p>
	<ul style="list-style-type: none"> <li>› evaluate the reasonableness of others' chance-based claims.</li> </ul>	<ul style="list-style-type: none"> <li>› engage in critical thinking for chance-based situations by:               <ul style="list-style-type: none"> <li>- recognising potential issues in the assumptions of a theoretical probability model that has been applied</li> <li>- evaluating the reasonableness of others' claims</li> <li>- explaining whether a claim is misleading and why and how probability may have been used in a particular way to arrive at the claim (e.g., in advertising).</li> </ul> </li> </ul>	<p>Explore known chance-based situations, using logical reasoning to identify assumptions, problems, and the validity of claims.</p> <p>Provide opportunities and support for students to match the results of chance-based investigations with statements made about them, and to check others' claims about chance-based investigations.</p> <p>Evaluate statements made by others about their findings from chance-based investigations, using interrogative questions about how their probability experiment was conducted and their analysis of results.</p>

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## The language of mathematics and statistics: Phase 4

	<b>Year 9</b> <i>Students will know the following new words:</i>	<b>Year 10</b> <i>Students will know the following new words:</i>
<b>Number</b>	<ul style="list-style-type: none"> <li>› cube root</li> <li>› GST</li> <li>› loss, profit</li> <li>› original amount</li> <li>› scientific notation</li> <li>› simple interest</li> </ul>	<ul style="list-style-type: none"> <li>› currency</li> <li>› percentage change</li> </ul>
<b>Algebra</b>	<ul style="list-style-type: none"> <li>› exponential</li> <li>› factorise</li> <li>› form</li> <li>› gradient, slope</li> <li>› intercept</li> <li>› iteration</li> <li>› linear equation or relationship</li> <li>› rate of change</li> <li>› rational number</li> <li>› selection</li> </ul>	<ul style="list-style-type: none"> <li>› operator</li> <li>› quadratic equation or relationship</li> <li>› rearrange</li> </ul>
<b>Measurement</b>	<ul style="list-style-type: none"> <li>› circumference, diameter, radius</li> <li>› derived measure</li> <li>› scale factor</li> <li>› significant figure</li> </ul>	<ul style="list-style-type: none"> <li>› surface area</li> </ul>
<b>Geometry</b>	<ul style="list-style-type: none"> <li>› alternate, co-interior, or corresponding angles</li> <li>› centre of rotation</li> <li>› chord</li> <li>› intersect</li> <li>› isometric</li> <li>› transversal</li> </ul>	<ul style="list-style-type: none"> <li>› centre of resizing or enlargement</li> <li>› similarity and congruence</li> </ul>
<b>Statistics</b>	<ul style="list-style-type: none"> <li>› decision tree</li> <li>› explanatory variable</li> <li>› line or curve of best fit</li> <li>› metadata</li> <li>› observational study</li> <li>› recategorising</li> <li>› response variable</li> <li>› variability</li> </ul>	
<b>Probability</b>	<ul style="list-style-type: none"> <li>› simulation</li> </ul>	<ul style="list-style-type: none"> <li>› assumption</li> <li>› limitation</li> </ul>

# Phase

# 5

Years 11–13

## Progress outcome by the end of year 13

*Navigating pathways and developing agency to help shape the future*

*Te whakatere ara me te whakawhanake kahawhiri hei tautoko i te tāraitanga o āpōpō*

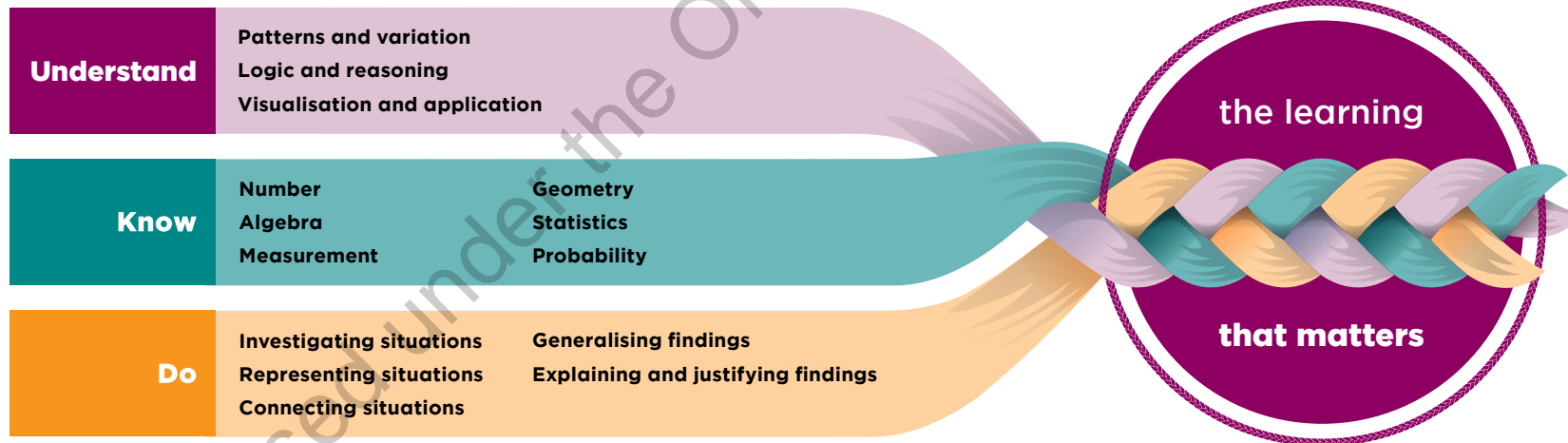
In phase 5, learning becomes more specialised over the phase as students plan their pathways beyond school.

Students explore how the mathematics and statistics they are learning are used as tools in local and global contexts. They apply increasingly specialised techniques across the stands of mathematics and statistics to visualise and investigate complex situations, including exponential growth and decay, measuring physical objects or motion, elements of chance, and multivariate data. They investigate patterns and variation in mathematical functions and statistical data. They use logic and reasoning to provide

comprehensive explanations, develop mathematical proofs, and evaluate statistical claims.

Students have opportunities to further develop and use what they have learnt in years 1–10 in a range of NCEA subjects. At years 12 and 13, the learning area mathematics and statistics separates into two separate disciplines (see page 9).

The phase 5 progress outcome describes the understanding, knowledge, and processes students have multiple opportunities to develop over the phase.



**The phase 5 progress outcome is found on the following two pages.**

# Understand

As students build knowledge through their use of the mathematical and statistical processes, they develop a deep understanding of the following.

## Patterns and variation | Ngā ia auau me ngā rerekētanga

The world is full of patterns and is defined by a multitude of relationships in which change and variation occur. Mathematics and statistics provide structures that are useful for noticing, exploring, and describing different types of patterns and relationships, enabling us to generate insights or make conjectures.

## Logic and reasoning | Te whakaaro arorau me te whakaaroaro

By engaging with mathematical concepts, we develop logical reasoning and critical-thinking skills that enable us to evaluate information, question assumptions, and present arguments with clarity. Statistical reasoning from observation and theory allows us to differentiate what is probable from what is possible and to draw reliable conclusions about what is reasonable.

## Visualisation and application | Te whakakite me te whakatinana

The visualisation of mathematical and statistical ideas profoundly influences how we perceive, understand, and interact with abstract concepts. Application in mathematics and statistics involves creating structures and processes that help us understand complex situations, enabling better decision making and communication of ideas.

# Know

## Mathematics | Pāngarau

By the end of this phase, students know that mathematical modelling involves investigating the relationships and behaviours of quantities in physical, economic, social, and everyday contexts. It is used to analyse applied situations and to make informed decisions, starting with forming assumptions. It also involves creating and solving equations and interpreting, evaluating, and refining results using the context. Digital technology plays a key role in mathematical modelling by enhancing visualisation, efficiency, and accuracy.

Students know that when calculating, they should use the appropriate form of numbers for the situation. Before and after calculating, they should use estimation to check the reasonableness of results. They give solutions with the appropriate degree of accuracy for the situation. When solving an equation, they will often rearrange it so that it equals zero. This allows for the use of the zero product property, which states that if two expressions multiply to be zero, then one or the other or both are zero.

Students know that the complex-number system is an extension of the real-number system and incorporates the square root of negative one. A complex number can be written in rectangular form, polar form, and exponential form.

By the end of this phase, students know that functions are relationships or rules in which each member of an input set maps to a single output. Functions can be represented by ordered pairs, tables, XY graphs, and algebraic equations, and they can be described in words for contextual situations. A function can undergo a variety of transformations with predictable effects on its graph. When working with functions, care needs to be taken that the distributive property is limited to expanding multiplication over addition and is not applied to expressions with powers and roots. Optimising a function enables the determination of its maximum or minimum value using various mathematical techniques (e.g., XY graphs, tables of values, linear programming, and calculus).

Students know the following key facts about specific functions:

- › There can be many equivalent equations for expressing a linear function; different forms are appropriate for different situations, including in other learning areas.
- › In a polynomial function,  $x = a$  is a root if and only if  $(x - a)$  is a factor of the polynomial.
- › Exponential and logarithmic functions are inversely related.
- › The standard unit of angular measurement is a radian, which is geometrically related to the length of the radius of a circle. A point on a unit circle at an angle of  $\theta$  in standard position is represented by the coordinates  $(\cos \theta, \sin \theta)$ . This demonstrates the periodic and symmetric nature of the sine and cosine functions, visually and algebraically.

Students know that calculus is the study of how things change. The derivative of a function at a point is the gradient of the tangent line to the curve at that point. The derivative of a function can be interpreted as its rate of change. Tangent lines are local approximations of a function. Near a specific point, the tangent line and the function have approximately the same gradient and graph. In the graph of a function, a local extremum or a change in concavity can occur at points where the gradient of the tangent line is 0 or undefined. The indefinite integral of a function is a general antiderivative that includes a constant of integration. The derivative of the indefinite integral is the original function. The definite integral gives the net signed area bounded by a curve and the x-axis over an interval. Kinematics enables us to describe the motion and direction of movement of objects within closed systems in terms of displacement, velocity, and acceleration.

## Statistics | Tauanga

By the end of this phase, students know that statistics is about learning from data and measuring, controlling, and communicating uncertainty. Sourcing data and learning from it are at the heart of evidence-based decision making. Data-based information is used to inform and influence decisions, behaviours, policies, and opinions. Data exploration requires a combination of data practices, such as transforming and visualising data. Drawing on contextual knowledge is integral to all statistical investigation and sense making. Statistical enquiry can be used to explore or understand a population, system, community, process, or issue and to inform action and advocacy.

Students learn that data can be extracted from a wide range of sources. How we collect data affects the applicability, quality, diversity, and quantity of the data, as well as the conclusions we draw from it. We need to take care about sources of bias. Data collection and use includes an ethical responsibility to obtain informed consent, to respect privacy, and to protect and benefit individuals and their environment.

Students know that, for statistical inference, data is sometimes obtained using random sampling to enable and justify sample-to-population inferences. Sometimes experiments are used to justify cause-and-effect causal inferences. Students use a range of simulation methods for different situations – for example, randomisation tests can be used with experiments to assess the strength of evidence for the existence of differences between treatments. Bootstrapping, for building confidence intervals, is a simulation method used to allow for uncertainties in estimation due to sampling error.

Students understand that there are situations in life that involve uncertainty. Probability can help us think about these situations and make decisions on them. Probability models can be developed from gathered data or from theory; both approaches make assumptions that might not be valid. Generating data from a probability model through simulation can demonstrate what outcomes are likely or unlikely under certain conditions, as well as the variability of the outcomes. Uniform, binomial, Poisson, and normal probability distributions have recognisable key features and can be used to model situations.

Students learn that findings from data are tentative and subject to revision when more evidence and insights become available. Communications with embedded statistical information should be critiqued using a statistical lens.

## Do

### Investigating situations | Te tūhura pūāhua

By the end of this phase, students can pose a question for investigation, find entry points for addressing the question, and plan an investigation pathway and follow it in a systematic and organised way. They can identify relevant prior knowledge, conditions, assumptions, constraints, relationships, and concepts. They can monitor and evaluate progress, adjusting the investigation pathway if necessary, and make sense of outcomes or conclusions in light of a given situation and context.

### Representing situations | Te whakaata pūāhua

By the end of this phase, students can use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns, variations, and trends. They use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments. They select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations. They use visualisation to mentally represent and manipulate relationships, objects, and ideas.

### Connecting situations | Te tūhono pūāhua

By the end of this phase, students can suggest connections between concepts, ideas, approaches, and representations. They connect new ideas to things they already know. They make connections to ideas in other learning areas and with diverse cultural, linguistic, and historical contexts.

### Generalising findings | Te whakatauwānui i ngā kitenga

By the end of this phase, students can notice and explore patterns, structure, and regularity and make conjectures about them. They identify relationships, including similarities, differences, and new connections. They represent specific instances and look for when conjectures about them might be applied in another situation or always be true. They test conjectures, using reasoning and counterexamples to decide if they are true or not. They use appropriate symbols to express generalisations.

### Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

By the end of this phase, students can make statements and give explanations inductively based on observations or data. They make deductions based on knowledge, definitions, and rules. They critically reflect on others' thinking, distinguishing between correct and flawed logic and asking questions to clarify and understand. They use evidence, reasoning, and proofs to explain why they agree or disagree with statements. They develop collective understandings by sharing, comparing, contrasting, critiquing, and building on ideas with others. They present reasoned, coherent explanations and arguments for an idea, solution, or process.

## Teaching sequences

*Navigating pathways and developing agency to help shape the future*

*Te whakatere ara me te whakawhanake kahawhiri hei tautoko i te tāraitanga o āpōpō*

This section describes how the components of a comprehensive teaching and learning programme for the mathematics and statistics learning area are used during the fifth phase of learning at school.

Throughout phase 5, emphasise the importance of independent thought, intellectual rigour, and the ability to engage with complex, abstract mathematical and statistical concepts. Encourage students to approach problems critically and to explore advanced mathematical and statistical theories and applications. Motivate students to extend their thinking beyond standard procedures to find creative solutions. Facilitate an environment where they are expected to not only solve problems but also to explore the underlying principles of, and connections between, different areas of mathematics and statistics. Encourage them to critique mathematical and statistical arguments, identify strengths and weaknesses, and refine their reasoning. Support students' development of sophisticated mathematical and statistical communication skills, including the ability to present and defend their ideas clearly and persuasively. Empower them to take full ownership of their learning, and reinforce the expectation that they are all capable of engaging with challenging material. Provide opportunities for them to demonstrate their understanding in diverse and creative ways.

### Explicit teaching

- › Illustrate the steps involved in working through a sophisticated problem using worked examples. Support students with appropriate scaffolds to enable them to explore mathematical and statistical connections.
- › Plan for students to actively recall learning, interleaving known concepts and procedures with new learning. Provide regular opportunities for practice to support the development of procedural fluency.
- › Ensure students are given opportunities to apply their learning to unfamiliar contexts that help them see the connections between mathematics, statistics, and the wider world.

### Positive relationships with mathematics and statistics

- › Support students to persevere with challenging tasks. Help them to recognise the value of continued practice and the growth that comes from engaging deeply with mathematics and statistics.

## Rich tasks

- › When designing tasks, consider different contexts, levels of difficulty, time requirements, procedures to be used, and multiple entry and exit points. Use contextual situations that are familiar for students, and connect the mathematics and statistics strands and disciplines as much as possible. Carefully choose questions and statements that will help develop deep conceptual understandings. Include checks during lessons to give students opportunities to refocus, summarise ideas, and ask questions to clarify next steps.
- › Support students to convert their wonderings or observations into conjectures or claims. Encourage them to check if their conjectures always work, if there are times when they don't work, and if they can be extended. Represent their working and reasoning using diagrams, materials and digital tools to engage them.
- › Check in at specific points in investigations and support students to progress their work. Respond as appropriate with specific, targeted teaching of the mathematical or statistical skills needed to progress the investigation.
- › Encourage students to try applying new skills and processes in their investigations. Support them to select the most appropriate process for solving a problem. Help them to recognise that if their first approach doesn't work, trying another approach is part of the richness of problem solving.

- › Encourage students to be respectful of others' views. As part of rich tasks, encourage them to include different perspectives – for example, what is important to others in mathematics and statistics, and appropriate ways of investigating and collecting information and data about them.

## Communication in mathematics and statistics

- › Encourage students to collaborate and build on each other's thinking, learning, and arguments.
- › Model the correct use of mathematical and statistical language, vocabulary, and notation. Support students to correctly use them, ensuring that they get multiple opportunities to practise using unfamiliar terms and notation.
- › Empower students to reason mathematically and statistically. Help them to develop confidence in critiquing their own and others' mathematical conjectures, explanations, and justifications.

# Mathematics and statistics, Year 11: Teaching sequence

*Navigating pathways and developing agency to help shape the future*

*Te whakaterere ara me te whakawhanake kahawhiri hei tautoko i te tāraitanga o āpōpō*

## Number

<b>During year 11</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› recognise and use the properties of numbers, including those of:                             <ul style="list-style-type: none"> <li>- natural numbers and integers (positive, zero, and negative)</li> <li>- prime, rational, and irrational numbers</li> <li>- reciprocals</li> </ul> </li> </ul>	<p><b>Connect</b> the properties of numbers with the use of numbers in other strands (e.g., measurement) and learning areas (e.g., science).</p>
<ul style="list-style-type: none"> <li>› calculate with numbers in scientific notation</li> </ul>	<p><b>Connect</b> with other learning areas to explore how they use and write very large and very small numbers.</p>
<ul style="list-style-type: none"> <li>› use rounding and estimation to predict outcomes and to check the reasonableness of calculations</li> </ul>	<p>Demonstrate how estimating answers and the expected size of numbers is a way of checking the results of calculations for reasonableness. Support students to <b>justify</b> estimated solutions.</p>
<ul style="list-style-type: none"> <li>› round to the degree of precision required by the context</li> </ul>	<p>Apply rounding in practical situations and explain why it is important, discussing different contexts in which it occurs.</p>
<ul style="list-style-type: none"> <li>› perform operations on positive and negative rational numbers in any form (i.e., whole numbers, fractions, and decimals), including raising them to integer powers and taking roots of them, where defined</li> </ul>	<p>Prompt students to use known benchmarks, inverse operations, and knowledge of place value. <b>Represent</b> solutions using the most appropriate number form (e.g., a percentage, a ratio, scientific notation, or an exponent).</p>
<ul style="list-style-type: none"> <li>› use fractions, decimals, and percentages, choosing whichever is most appropriate</li> </ul>	<p><b>Investigate</b> numerical solutions in practical situations, (e.g., financial situations that involve rates).</p>
<ul style="list-style-type: none"> <li>› apply rates and ratios, using proportional and inverse proportional reasoning when appropriate</li> </ul>	<p>Explore common misconceptions and errors. For example, compare the effect of increasing a quantity by a percentage and decreasing the increased quantity by the same percentage.</p>
<ul style="list-style-type: none"> <li>› perform operations with percentages, including increasing or decreasing a quantity using a single multiplier</li> </ul>	

<b>During year 11</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› calculate compound interest over a fixed amount of time, compounding annually, quarterly, monthly, or daily.</li> </ul>	Use practical examples of interest rates to <b>investigate</b> the outcomes of different investments or loans over time.

## Algebra

<b>During year 11</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› find solutions that maximise or minimise a quantity while meeting the constraints of a situation, by making lists, tables, and graphs and comparing values</li> </ul>	Use contextual situations, with an emphasis on checking that answers are appropriate for each situation. <b>Investigate</b> optimal solutions for maximising or minimising quantities.
<ul style="list-style-type: none"> <li>› operate on numeric and algebraic expressions that have integer exponents, applying exponent rules</li> </ul>	Explore common misconceptions and errors (e.g., the misconception that the distributive property works with powers and roots).
<ul style="list-style-type: none"> <li>› add, subtract, multiply, and divide algebraic fractions with numeric denominators</li> </ul>	<b>Connect</b> with fraction skills in the number strand.
<ul style="list-style-type: none"> <li>› simplify, expand, and factorise algebraic expressions</li> </ul>	Demonstrate the area method for expanding, and <b>connect</b> it with other approaches. <b>Generalise</b> number operations using algebraic representations for the commutative, associative, and distributive properties of numbers. <b>Investigate</b> the relationships between number properties and algebraic rules, to develop an understanding of why the rules work.
<ul style="list-style-type: none"> <li>› substitute into, rearrange, and simplify algebra expressions or formulae</li> </ul>	<b>Connect</b> with measurement through the use of formulae for perimeter, area, and volume.
<ul style="list-style-type: none"> <li>› interpret, graph, and express linear relationships in any form</li> </ul>	<b>Connect</b> a range of algebraic skills (e.g., forming, graphing, and solving) and explore how they work together. Use appropriate digital tools to explore the connections.
<ul style="list-style-type: none"> <li>› form and solve linear equations and inequalities</li> </ul>	
<ul style="list-style-type: none"> <li>› form and solve pairs of simultaneous linear equations with two variables, and give geometric and contextual interpretations</li> </ul>	Use a series of mathematical statements to <b>explain</b> how the solutions to equations or inequalities are reached.

<b>During year 11</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› recognise the relationships between elements of a linear or quadratic pattern, write an equation to represent the rule for the pattern, and use the equation to make conjectures</li> </ul>	<p><b>Connect</b> with number skills and properties of graphs to help students understand the connection between rules, equations, and the properties.</p> <p><b>Investigate</b> the relationships between the elements of patterns using words, formulae, and graphical <b>representations</b>.</p>
<ul style="list-style-type: none"> <li>› form and solve quadratic equations</li> </ul>	<p><b>Explain</b> why there are different numbers of solutions for quadratic equations and the link between the solutions of quadratic equations and the graph of a parabola. Consider everyday situations in which a quadratic model could be useful.</p> <p><b>Explain</b> the zero product property and the benefit of rearranging a quadratic equation to equal zero in order to find solutions.</p>
<ul style="list-style-type: none"> <li>› for quadratic and exponential functions with a positive integer base:               <ul style="list-style-type: none"> <li>- make a table and graph, given an equation</li> <li>- interpret key information and features of a graph.</li> </ul> </li> </ul>	<p><b>Connect</b> the graphing of quadratic and exponential functions, and their key features, with linear equations and graphs, noting similarities and differences.</p> <p><b>Connect</b> the key features and graphing of quadratic and exponential functions with linear equations and graphs, noting similarities and differences.</p> <p><b>Represent</b> exponential growth and decay using appropriate graphical forms.</p>

## Measurement and Geometry

<b>During year 11</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› estimate, calculate, and represent measurements accurately</li> </ul>	<p>Use a variety of analogue and digital measurement tools.</p> <p>Consolidate the skills of measuring using practical situations and examples in which students are doing physical measurements and working with non-theoretical situations.</p> <p><b>Represent</b> measurements accurately, when both estimating and taking measurements, and use them in calculations.</p> <p><b>Connect</b> the practical collection of data in measurement problems with the use of appropriate units, including those for rates.</p>

<b>During year 11</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› connect significant figures and limits of accuracy with measurements</li> </ul>	<p><b>Investigate</b> limits of accuracy for maximum and minimum solutions in problems from practical situations that round measurements to significant figures.</p> <p><b>Justify</b> appropriate rounding and units in measurement solutions using significant figures and limits of accuracy.</p>
<ul style="list-style-type: none"> <li>› convert between metric units, including for volume, capacity, mass (weight), and derived units such as rates and ratios</li> </ul>	<p><b>Connect</b> the units and conversions needed for different numbers of dimensions (e.g., <math>m^3</math> to <math>mm^3</math> will be <math>\times 1000^3</math>).</p>
<ul style="list-style-type: none"> <li>› use and apply Pythagoras' theorem in 2D and 3D situations</li> <li>› apply trigonometric ratios (sine, cosine, and tangent) in right-angled triangles to find unknowns, in 2D and 3D situations</li> </ul>	<p>Use materials and diagrams to <b>represent</b> situations and <b>visualise</b> problems (e.g., signalling with colours to identify different parts of a diagram or problem).</p> <p><b>Connect:</b></p> <ul style="list-style-type: none"> <li>› the properties of 2D right-angled triangles illustrated by Pythagoras' theorem to investigations of right-angled triangles in 3D shapes</li> <li>› algebraic thinking to the solving of problems using trigonometric ratios in right-angled triangles.</li> </ul>
<ul style="list-style-type: none"> <li>› find the surface area and volume of 3D shapes composed of cylinders, pyramids, cones, spheres, or non-rectangular prisms</li> </ul>	<p>Use materials and diagrams to <b>represent:</b></p> <ul style="list-style-type: none"> <li>› nets for the surface areas of 3D shapes</li> <li>› breakdowns of sections for composite shapes, to find their volumes.</li> </ul> <p><b>Connect</b> with:</p> <ul style="list-style-type: none"> <li>› other strands (e.g., algebra, when using formulae for area and volume)</li> <li>› contextual situations, with an emphasis on checking that answers are appropriate for each situation.</li> </ul>
<ul style="list-style-type: none"> <li>› resize a shape by a factor and find the perimeter, area, or volume of the resulting shape, relating this to the scale factor</li> </ul>	<p><b>Investigate</b> the effect of changes in dimension (e.g., from length to area) on scale factors, using practical measuring activities.</p>
<ul style="list-style-type: none"> <li>› apply transformations to lines and parabolas in the XY plane, relating changes in a graph to changes in its equation.</li> </ul>	<p><b>Investigate</b> transformations of lines and parabolas in the XY plane.</p> <p><b>Connect</b> the different forms of linear and quadratic equations to the key features of linear and parabolic graphs and to how the features change, depending on transformations of the graphs.</p>

# Statistics

<b>During year 11</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<p><b>Problem</b></p> <ul style="list-style-type: none"> <li>› investigate, using multivariate datasets from observational studies, summary, comparison, time-series, and relationship situations by:               <ul style="list-style-type: none"> <li>– posing an investigative question, in particular a summary or comparison question about a population of interest, or a time-series or relationship question about a group of interest</li> <li>– making conjectures or assertions about expected findings</li> </ul> </li> </ul>	<p><b>Connect</b> with students’ interests and backgrounds and with other subjects to find contexts for meaningful investigations.</p> <p>Demonstrate how to pose an <b>investigative</b> question for situations involving multivariate datasets:</p> <ul style="list-style-type: none"> <li>› summary and comparison situations, including the variable and population of interest</li> <li>› relationship and times-series situations, including the variable and group of interest.</li> </ul> <p>Support students to discuss, <b>explain</b>, and <b>justify</b> their conjectures or assertions.</p>
<p><b>Plan</b></p> <ul style="list-style-type: none"> <li>› plan how to collect or source data to answer the investigative question, including:               <ul style="list-style-type: none"> <li>– identifying and justifying the variables needed to answer the question</li> <li>– planning how to make and justifying valid and reliable measures for the variables (when collecting data) or interrogating sourced datasets to understand what the variables measure and how they were measured</li> <li>– identifying the group or population of interest or (for sourced data) ‘who’ the data was collected from</li> <li>– determining if the investigative question is about a group or a population</li> <li>– if a population, determining what sample size is needed to make a decision about what is happening in the population</li> <li>– using interrogative questions to check that ethical practices are considered throughout the entire enquiry cycle</li> </ul> </li> </ul>	<p><b>Explain</b> and <b>justify</b> methods of data collection, ‘who’ to measure, and what and how to measure, in order to answer the investigative question.</p> <p><b>Investigate:</b></p> <ul style="list-style-type: none"> <li>› the impact of increasing the sample size on whether or not a call can be made about what is happening in the population (in comparison situations)</li> <li>› how people’s different interpretations of instructions or use of devices can affect the quality of collected data.</li> </ul> <p>Closely examine secondary datasets to <b>explain</b> their variables and group(s) or population(s) of interest and the original investigator’s plan.</p> <p><b>Explain</b> how to apply ethical practices to data collection and use, including viewing data as taonga.</p>

<b>During year 11</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<p><b>Data</b></p> <ul style="list-style-type: none"> <li>› collect or source data, including:           <ul style="list-style-type: none"> <li>– using random samples to collect or source data about a population or populations (for summary and comparison situations)</li> <li>– creating a data dictionary (for collected data) or identifying the metadata (for sourced data)</li> <li>– managing the data collection process, including considering possible errors and ethical concerns</li> <li>– storing the data, ensuring it is kept safe and confidential</li> <li>– sorting the data in preparation for analysis</li> <li>– checking the validity of data and, if appropriate, making simple edits and imputations (i.e., cleaning it)</li> <li>– recategorising or constructing new variables if needed to support answering the investigative question</li> </ul> </li> </ul>	<p>Support students to use a variety of methods for collecting or sourcing the data, including:</p> <ul style="list-style-type: none"> <li>› making measurements</li> <li>› observing outcomes</li> <li>› using survey forms</li> <li>› using secondary data sources</li> <li>› using random samples.</li> </ul> <p><b>Explain and justify:</b></p> <ul style="list-style-type: none"> <li>› methods for recording and storing data</li> <li>› the use of recategorised variables and why new variables may have been constructed.</li> </ul> <p><b>Investigate</b> how measurers, collection methods, and processing the data can introduce errors into the data.</p> <p>Support students to identify errors in the data, <b>connecting</b> to the context, <b>explaining</b> why they are errors, and, if appropriate, cleaning the data using appropriate techniques.</p>
<p><b>Analysis</b></p> <ul style="list-style-type: none"> <li>› create multiple data visualisations for the investigation, providing global and local views</li> <li>› describe and reason from the data visualisations, including:           <ul style="list-style-type: none"> <li>– identifying relevant features in distributions</li> <li>– interweaving the context in descriptions of distributions</li> <li>– making an informal inference about what might be happening in the population, based on visual considerations in comparison situations</li> <li>– drawing a line or curve of best fit to make conjectures about or predict possible y values (for the response variable) for given x values (for the explanatory variable) in relationship situations, and, where appropriate, indicating a possible (informal) interval for predictions, using visual methods</li> <li>– making informal predictions about group membership (e.g., in relation to socio-economic status or educational attainment)</li> </ul> </li> </ul>	<p><b>Represent</b> data using dot plots, bar graphs, frequency tables, box plots, histograms, paired plots, time-series graphs, two-way tables or graphs, and scatter plots.</p> <p>Check data visualisations for misleading features or information. Read the data, read 'between' the data, read 'beyond' the data and read 'behind' the data.</p> <p><b>Explain</b> how different data visualisations have different features and how to describe them in context, using, for example:</p> <ul style="list-style-type: none"> <li>› fractions, proportions, and percentages</li> <li>› the mode, median, mean, and distributional shape</li> <li>› maximum and minimum values</li> <li>› the interquartile range, lower quartile, upper quartile, middle 50%, and range</li> <li>› descriptions such as more than, less than, at least, at most, between A and B</li> <li>› outliers, clusters, gaps, joint and conditional proportions</li> <li>› an eyeballed line or curve of best fit, seasonal patterns, and the long-term trend.</li> </ul> <p><i>(continued on the next page)</i></p>

<b>During year 11</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<p><b>Analysis</b></p> <p><i>(continued from the previous page)</i></p>	<p>Describe what is seen and is not seen in data visualisations, recognising that data are numbers with context and that the context includes variables of interest, groups of interest, counts or proportions for categorical variables, and values and units for numerical variables.</p> <p>Compare data visualisations of the same variable for different groups, looking at similarities and differences. In comparison situations, make an informal inference about what might be happening in the population, using the 'Making the call Year 11' decision guide.</p>
<p><b>Conclusion</b></p> <ul style="list-style-type: none"> <li>› answer investigative question and justify and communicate findings by: <ul style="list-style-type: none"> <li>- using evidence from analysis, with an awareness of variability</li> <li>- generalising beyond the sample to the population (for summary and comparison situations), providing evidence and accounting for uncertainty</li> <li>- providing explanations for observed patterns in the data</li> <li>- reflecting on conjectures or assertions</li> <li>- critically evaluating their investigation throughout all phases of the enquiry cycle</li> <li>- structuring their evidence and findings into a coherent whole that effectively communicates the entire statistical enquiry to a non-specialist</li> </ul> </li> </ul>	<p>Have students practise answering the investigative question succinctly and in a way that a non-specialist would understand, using evidence from analysis and informal inferences or predictions, with an awareness of variability.</p> <p>Explore relevant explanations or interpretations of findings connected to the context of the situation, including the group or population of interest and the variable or variables of interest.</p> <p>Support students to evaluate and recommend any changes to the processes they used. Ask them to <b>explain</b> whether or not their findings align with their initial conjectures or assertions, if what was found makes sense with what is known about the situation, and if there are any limitations to the findings.</p> <p>Ask students to consider, as a result of the statistical investigation, what else they could <b>investigate</b>, starting the PPDAC cycle again.</p>
<p><b>Statistical literacy</b></p> <ul style="list-style-type: none"> <li>› evaluate others' statistical findings, reports, and data visualisations to see if the claims in them are reasonable and support or misrepresent the data or situation.</li> </ul>	<p>Explore existing statistical reports, graphs, visualisations, and claims in order to determine their validity. Use readily available media (e.g., news media or social media) as sources.</p> <p>Use interrogative questions to evaluate data-collection methods that involve, for example, ethical considerations, analysis processes, visualisations, and statements of findings.</p>

# Probability

<b>During year 11</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› plan and conduct probability experiments for chance-based situations, including undertaking a large number of trials using digital tools, by:               <ul style="list-style-type: none"> <li>– posing an investigative question</li> <li>– making conjectures about the distribution of outcomes</li> <li>– recognising the underlying structure (e.g., with or without replacement)</li> <li>– constructing a model using theoretical probabilities, probabilities based on past data, or conjectured probabilities</li> <li>– identifying model assumptions and limitations</li> <li>– deciding on and justifying the number of trials, the tools to be used, and the recording method</li> <li>– collecting and recording data, including by running simulations</li> <li>– creating data visualisations for the distribution of observed outcomes from the experiment and for all possible outcomes for theoretical probability models, where they exist</li> <li>– describing what these visualisations show</li> <li>– finding probability estimates for the different outcomes</li> <li>– proposing possible theoretical outcomes and associated probabilities for situations where no theoretical model exists</li> <li>– identifying similarities and differences between their findings and those of others</li> <li>– reflecting on conjectured outcomes</li> <li>– identifying similarities and differences between findings from the probability experiment and associated theoretical probabilities, as appropriate</li> </ul> </li> </ul>	<p>Conduct practical experiments in class involving materials and digital tools, collectively critiquing and improving plans and proposals.</p> <p>Compare theoretical probabilities with experimental probabilities and explore reasons for differences.</p> <p><b>Investigate</b> every day chance-based situations, patterns in possible outcomes, theoretical and experimental distributions, and relative frequencies from data investigations. This could be using the statistical enquiry cycle.</p> <p><b>Represent:</b></p> <ul style="list-style-type: none"> <li>› probability outcomes (theoretical and experimental) using lists, tables, tree diagrams, tally charts, visualisations of distributions, words, numbers, and digital tools</li> <li>› probability models.</li> </ul> <p><b>Explain</b> how to describe and use probability concepts – for example:</p> <ul style="list-style-type: none"> <li>› outcomes, events, and the sample space</li> <li>› trials, simulations, models, and theoretical and experimental probability</li> <li>› with and without replacement, independence and dependence, joint and conditional probabilities</li> <li>› the law of large numbers</li> <li>› probability estimates, theoretical probability, and probability distributions</li> <li>› chance, randomness, and variation.</li> </ul> <p><b>Connect:</b></p> <ul style="list-style-type: none"> <li>› anticipated distributions with theoretical distributions and experimental distributions, linking <b>representations</b> (e.g., two-way tables) with distributions, words, and numbers</li> <li>› observed data, models, and simulated data.</li> </ul>
<ul style="list-style-type: none"> <li>› identify why claims about probability experiments might not be valid, and consider improvements</li> </ul>	<p>Support students with critical evaluation and reasoned conclusions.</p>

<b>During year 11</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› engage in critical thinking for chance-based situations by:               <ul style="list-style-type: none"> <li>– recognising potential issues in the assumptions of a theoretical probability model that has been applied</li> <li>– evaluating the reasonableness of others' claims</li> <li>– explaining whether a claim is misleading and why and how probability may have been used in a particular way to arrive at the claim (e.g., in advertising).</li> </ul> </li> </ul>	<p><b>Connect</b> with the use and analysis of statistics in the media, to gain an overview of how statistics and probability are shared outside academia.</p> <p>Explore different interpretations of and reactions to the same experimental results.</p> <p>Use interrogative questions to evaluate others' findings, including their model assumptions, numbers of trials, visualisations, and statements.</p>

## The language of mathematics and statistics: Phase 5

<b>Year 11</b> <i>Students will know the following new words:</i>			
<b>Number</b>	<ul style="list-style-type: none"> <li>› compound interest</li> <li>› irrational number</li> </ul>	<b>Statistics</b>	<ul style="list-style-type: none"> <li>› informal inference</li> </ul>
<b>Algebra</b>	<ul style="list-style-type: none"> <li>› exponential growth or decay</li> <li>› function</li> <li>› maximise</li> <li>› minimise</li> <li>› optimise</li> <li>› simultaneous equations</li> </ul>	<b>Probability</b>	<ul style="list-style-type: none"> <li>› two-way table</li> </ul>
<b>Measurement and Geometry</b>	<ul style="list-style-type: none"> <li>› limits of accuracy</li> <li>› non-rectangular prism</li> </ul>		

# Mathematics, Years 12–13: Teaching sequence

*Navigating pathways and developing agency to help shape the future*

*Te whakatere ara me te whakawhanake kahawhiri hei tautoko i te tāraitanga o āpōpō*

<b>During year 12</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 13</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› manipulate algebraic expressions, including algebraic fractions, by:               <ul style="list-style-type: none"> <li>– simplifying, rearranging, expanding, and factorising</li> <li>– adding, subtracting, multiplying, and dividing</li> <li>– raising them to integer and unit fraction exponents</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› manipulate algebraic expressions, including those involving roots, by:               <ul style="list-style-type: none"> <li>– simplifying, rearranging, expanding, and factorising</li> <li>– adding, subtracting, multiplying, and dividing</li> <li>– rationalising the denominator</li> <li>– raising them to rational exponents</li> </ul> </li> </ul>	<p><b>Investigate</b> how combining manipulations can simplify a task (e.g., how factorising can enable the reduction of an algebraic fraction).</p> <p><b>Connect</b> with logarithms and operations.</p> <p>Consider the connections between <b>generalising</b> number operations and algebraic manipulation.</p> <p><b>Investigate</b> situations such as:</p> <ul style="list-style-type: none"> <li>› different ways of expressing the same expression or equation, and deciding which is most appropriate for a given scenario</li> <li>› the use of algebraic proofs.</li> </ul>
<ul style="list-style-type: none"> <li>› perform operations on logarithms of base <math>b</math> (<math>b &gt; 0</math>, <math>b \neq 1</math>), including the common logarithm <math>b = 10</math>:               <ul style="list-style-type: none"> <li>– simplify logarithms using number properties (e.g., <math>\log_5 125 = 3</math>, since <math>5^3 = 125</math>)</li> <li>– convert between logarithmic and exponential forms</li> <li>– use log rules for products, quotients, and powers</li> <li>– given data for <math>x</math> and <math>y</math>, linearise it by estimating parameters in relationships of the form <math>y = ax^n</math> and <math>y = kb^x</math></li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› perform operations on logarithms of base <math>b</math> (<math>b &gt; 0</math>, <math>b \neq 1</math>), including the natural logarithm <math>b = e</math>:               <ul style="list-style-type: none"> <li>– simplify logarithms using number properties</li> <li>– convert between logarithmic and exponential forms</li> <li>– develop and use log rules, including the change-of-base rule</li> <li>– linearise exponential data</li> </ul> </li> </ul>	<p><b>Connect</b> logarithms to exponential functions, emphasising the inverse relationship. Use graphical <b>representations</b> to explore these functions.</p> <p><b>Investigate</b> situations such as:</p> <ul style="list-style-type: none"> <li>› how changes to functions affect their graphs</li> <li>› how the same change on an exponential graph and its inverse log graph affects the two graphs (e.g., what is the effect of +2 on each?)</li> <li>› how various scales work (e.g., for measuring the magnitude of earthquakes, or for measuring sound levels)</li> <li>› how different logarithmic scales can be used with graphs to linearise an exponential function.</li> </ul>

<b>During year 12</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 13</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› form, solve, and graph quadratic relationships, equations, and inequalities with rational coefficients, including by completing the square and using the discriminant</li> </ul>	<ul style="list-style-type: none"> <li>› form, solve, and graph polynomial functions, equations, and inequalities with rational coefficients, including those functions with complex roots</li> </ul>	<p>Support students to use paper and digital tools to explore graphing functions and solving systems of equations.</p> <p><b>Connect</b> features of graphs to the solutions of equations and inequations.</p> <p><b>Investigate</b> the inverses of given functions.</p> <p><b>Investigate</b> situations such as:</p>
<ul style="list-style-type: none"> <li>› form, solve, and graph exponential and logarithmic functions and equations, not including those with base <math>e</math></li> </ul>	<ul style="list-style-type: none"> <li>› form, solve, and graph exponential and logarithmic functions and equations, including those with base <math>e</math></li> </ul>	<ul style="list-style-type: none"> <li>› compound interest, calculated daily</li> <li>› completing the square to <b>explain</b> the development of the quadratic formula</li> <li>› the relationship between the nature of the discriminant and the number of roots in a quadratic function</li> </ul>
<ul style="list-style-type: none"> <li>› form, solve, and graph systems of two simultaneous equations, one of which may be linear, in two dimensions, and interpret solutions</li> </ul>	<ul style="list-style-type: none"> <li>› find the optimal solutions of a system of linear inequalities (i.e., linear programming)</li> </ul>	<ul style="list-style-type: none"> <li>› Kirschhoff's laws in physics.</li> </ul>
<ul style="list-style-type: none"> <li>› use algebra on the XY plane (i.e., coordinate geometry) to find measurements and develop geometric facts related to points, lines, and circles, including the equation of a circle</li> </ul>	<ul style="list-style-type: none"> <li>› use parametric equations to represent curves and circles</li> </ul>	<p>At year 12, <b>connect</b> algebra on the XY plane to other learning – for example, to calculate:</p> <ul style="list-style-type: none"> <li>› the distance between points, using Pythagoras' theorem</li> <li>› the midpoint of two points as the mean of the coordinates</li> <li>› intersections between lines and curves, using systems of equations</li> <li>› the approximate slope of a tangent line, using secant lines (i.e., using two points to find the slope).</li> </ul> <p>At year 12, <b>investigate</b> situations such as:</p> <ul style="list-style-type: none"> <li>› deciding on the nature of a triangle using distances between points and gradients</li> <li>› showing the relationship between the distance formula and the equation of a circle</li> <li>› using computer-aided design (CAD) for architecture and design, referenced from the point (0,0) on an XY graph.</li> </ul> <p>At year 13, <b>investigate</b> time in <b>connection</b> with parametric equations (e.g., how graphical representations change with changes to time).</p>

<b>During year 12</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 13</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› use the binomial expansion of <math>(a + b)^n</math> where <math>n</math> is a positive integer, including finding the binomial coefficients using the combinations 'n choose r' and Pascal's triangle</li> </ul>	<ul style="list-style-type: none"> <li>› perform operations on complex numbers in the Cartesian form <math>z = a + bi</math>:               <ul style="list-style-type: none"> <li>- find the real part, imaginary part, conjugate, modulus, and argument</li> <li>- calculate sums, differences, products, and quotients</li> <li>- use and draw Argand diagrams</li> </ul> </li> </ul>	<p>At year 12, <b>investigate</b> situations such as:</p> <ul style="list-style-type: none"> <li>› the multiplication principle and how this leads to combinations and Pascal's triangle</li> <li>› how using the binomial expansion for expanding powers of expressions simplifies the process.</li> </ul> <p>At year 13, <b>investigate</b> generating fractals with complex numbers (e.g., the Mandelbrot set).</p>
<ul style="list-style-type: none"> <li>› work with sequences, including those that are arithmetic, geometric, or defined recursively, and series, with and without the use of sigma notation</li> </ul>	<ul style="list-style-type: none"> <li>› manipulate and apply complex numbers:               <ul style="list-style-type: none"> <li>- find the modulus, <math>r</math>, and argument, <math>\theta</math></li> <li>- convert between Cartesian form, the polar form <math>z = r(\cos\theta + i \sin\theta)</math>, and the exponential form <math>z = r e^{i\theta}</math></li> <li>- calculate products, quotients, roots, and integer powers in polar and exponential forms</li> <li>- geometrically interpret addition, subtraction, and multiplication</li> </ul> </li> </ul>	<p>At year 12, <b>connect</b> sequences and series with linear and quadratic patterns, and develop formulae so students are able to <b>explain</b> and <b>justify</b> the formulae for different sequences and series.</p> <p>At year 12, <b>represent</b> and generate sequences using spreadsheets and graphing applications.</p> <p>At year 12, <b>investigate</b> situations such as:</p> <ul style="list-style-type: none"> <li>› the spread of disease</li> <li>› salary increases and decreases</li> <li>› population growth</li> <li>› charging and discharging capacitors in physics.</li> </ul> <p>At year 13, <b>investigate</b> situations that use complex numbers – for example:</p> <ul style="list-style-type: none"> <li>› solving the same equation in Cartesian, polar, and exponential forms, especially in situations involving powers of complex numbers</li> <li>› electromagnetic waves and electric currents</li> <li>› option pricing models in finance</li> <li>› medical imaging such as MRI (magnetic resonance imaging).</li> </ul>

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<b>During year 12</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 13</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› use the properties of circles to:               <ul style="list-style-type: none"> <li>– identify angles and convert between radians and degrees</li> <li>– calculate the area of a sector and length of an arc</li> <li>– identify that <math>\cos^2 \theta + \sin^2 \theta = 1</math> for all values of <math>\theta</math></li> <li>– develop, using a unit circle, the sine and cosine functions for all real numbers and their graphs, noting symmetries and lengths of periods</li> <li>– use <math>\tan \theta = \frac{\sin \theta}{\cos \theta}</math> to graph the tangent function</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› form, solve, and graph sine, cosine, and tangent functions and equations</li> </ul>	<p><b>Connect</b> <math>\cos^2 \theta + \sin^2 \theta = 1</math> with Pythagoras' theorem and the equation of a circle.</p> <p><b>Investigate</b> situations such as:</p> <ul style="list-style-type: none"> <li>› how the graphs of <math>f(x) = \sin(x)</math> and <math>f(x) = \cos(x)</math> can be constructed from the unit circle</li> <li>› real-life cyclical relationships (e.g., the movement of a Ferris wheel, the motion of tides, or simple harmonic motion).</li> </ul>
<ul style="list-style-type: none"> <li>› develop and use the sine rule, cosine rule, and area formula <math>\frac{1}{2} ab \sin C</math> to find the side lengths, angles, and area of a triangle</li> </ul>	<ul style="list-style-type: none"> <li>› use trigonometric identities to simplify and rearrange trigonometric expressions</li> </ul>	<p>At year 12, <b>investigate</b> situations such as:</p> <ul style="list-style-type: none"> <li>› the area of a non-rectangular section of land</li> <li>› the volume of a mud slide, calculated using cross sections</li> <li>› the ambiguous case of the sine rule.</li> </ul> <p>At year 13, <b>investigate</b> expressions involving compound angles, double angles, products, and sums in situations that involve calculations (e.g., finding exact values or integrals).</p>

<b>During year 12</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 13</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› perform operations on polynomial (constant, linear, quadratic, and cubic), power (where the power is an integer), square root, exponential, common log, absolute value, and (in degrees) sine, cosine, and tangent functions:               <ul style="list-style-type: none"> <li>- identify the domain and range</li> <li>- identify key features of graphs</li> <li>- use function notation</li> <li>- transform the graph of <math>y = f(x)</math> to <math>y = Af(x - B) + C</math></li> <li>- interpret algebraic solutions of equations graphically</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› perform operations on power, exponential, natural log, absolute value, sine, cosine, tangent, and piecewise functions:               <ul style="list-style-type: none"> <li>- identify the domain and range</li> <li>- identify key features of graphs</li> <li>- use function notation</li> <li>- transform the graph of <math>y = f(x)</math> to <math>y = Af(Bx - C) + D</math>, where <math>B = 1</math> or <math>C = 0</math></li> <li>- interpret algebraic solutions of equations graphically</li> <li>- form and use composite functions and notation</li> <li>- find inverse functions, restricting the domain if needed</li> </ul> </li> </ul>	<p><b>Investigate</b> power functions of the form <math>ax^n</math> where <math>n</math> is an integer (at year 12) or a rational number, including square roots (at year 13).</p> <p>Demonstrate how naturally occurring curves can be modelled and how the models can be used to explore aspects of practical situations, including making conjectures.</p> <p><b>Generalise</b> transformations of graphs to enable comparisons across different functions.</p> <p><b>Investigate</b> situations such as:</p> <ul style="list-style-type: none"> <li>› finding the best function to model a given natural curve, exploring how domain and range can influence the choice of the function</li> <li>› how functions can be used as models in applications such as computer-aided design (CAD)</li> <li>› supply and demand curves in economics</li> <li>› height from the ground and speed in skydiving, before and after opening one's parachute</li> <li>› real-life cyclical relationships that use trigonometric relationships to solve contextual problems.</li> </ul>
	<ul style="list-style-type: none"> <li>› determine where a function is continuous and differentiable, identify discontinuities, and estimate limits from a table or graph</li> </ul>	<p><b>Connect</b> the continuity of a function with its graph and equation.</p> <p><b>Investigate</b> situations such as:</p> <ul style="list-style-type: none"> <li>› traffic flow in towns or cities</li> <li>› changes in the stock market.</li> </ul>
<ul style="list-style-type: none"> <li>› for polynomial and integer power functions:               <ul style="list-style-type: none"> <li>- sketch the graph of the gradient function from the graph of the original function</li> <li>- sketch the graph of a function from the graph of its gradient function</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› sketch a graph of the derivative function from the graph of a function that is differentiable over the given domain, and sketch the graph of a function from the graph of the derivative function</li> </ul>	<p>Use paper and digital tools to explore key features of the graphs of a function and its derivative function.</p> <p>Demonstrate through the use of appropriate tools the connection between the procedural (polynomial differentiation rule) and conceptual approaches for finding the gradient at a point on a curve.</p> <p><b>Investigate</b> situations such as:</p> <ul style="list-style-type: none"> <li>› the family of possible functions for a given gradient function</li> <li>› the relationship between the order of a polynomial function and the order of its gradient function</li> <li>› how the key features of a gradient function relate to the key features of its original function, and vice versa.</li> </ul>

<b>During year 12</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 13</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› apply features of constant, linear, quadratic, cubic, and integer power functions:               <ul style="list-style-type: none"> <li>- find the gradients and equations of secant lines</li> <li>- use the relationship between the gradient of a graph and tangents at points as an approach to differentiation at a point</li> <li>- identify significant features by using gradients, including intervals of increase and decrease, stationary points, maxima, and minima</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› differentiate from first principles constant, linear, and quadratic relationships</li> </ul>	<p>Draw tangent lines to explore features of graphs.</p> <p><b>Investigate</b> situations such as the rates of change of the gradients for a range of functions and the relationships between the features of these functions.</p>
<ul style="list-style-type: none"> <li>› perform operations on polynomial and integer power functions to:               <ul style="list-style-type: none"> <li>- find the derivative function</li> <li>- find the gradient and equations of tangent lines at a point</li> <li>- find points where the gradient has a particular value</li> <li>- identify significant features by using the derivative, including intervals of increase and decrease, stationary points, and local and absolute extrema</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› perform operations on constant, rational power, exponential (base e), natural log, sine, cosine, and tangent functions – and functions created from them by sum, difference, product, quotient, and composition – to:               <ul style="list-style-type: none"> <li>- find the derivative function</li> <li>- find the gradient and equations of tangent and normal lines at a point</li> <li>- find points where the gradient has a particular value</li> <li>- identify significant features by using the derivative, including intervals of increase and decrease, stationary points, and local and absolute extrema</li> </ul> </li> </ul>	<p>Demonstrate systematic approaches to evaluating the characteristics of functions (e.g., their gradients).</p> <p><b>Investigate</b> situations such as:</p> <ul style="list-style-type: none"> <li>› the use of kinematics in practical situations</li> <li>› the use of calculus in economics (e.g., in profit, marginal cost analysis, and price elasticity (tangent and normal) calculations)</li> <li>› the efficient use of materials (e.g., in packaging).</li> </ul>
	<ul style="list-style-type: none"> <li>› find and use higher-order derivatives, including using the second derivative to determine concavity, local extrema, and points of inflection of the graph of the function</li> </ul>	

<b>During year 12</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 13</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
	<ul style="list-style-type: none"> <li>› calculate indefinite integrals for:               <ul style="list-style-type: none"> <li>- rational power, exponential base e, sine, and cosine functions</li> <li>- functions of the form <math>f(g(x)) g'(x)</math></li> <li>- functions created from those above by sums, differences, and constant multiples</li> </ul> </li> </ul>	Demonstrate and <b>explain</b> systematic approaches to using numerical methods. <b>Investigate</b> situations such as: <ul style="list-style-type: none"> <li>› areas of irregular shape in nature that can be modelled using functions</li> <li>› the <b>connections</b> between anti-derivatives, definite integrals, and area.</li> </ul>
<ul style="list-style-type: none"> <li>› use numerical methods on <math>f(x) &gt; 0</math> to find the area between a curve and the x-axis, using rectangles.</li> </ul>	<ul style="list-style-type: none"> <li>› calculate definite integrals:               <ul style="list-style-type: none"> <li>- using numerical methods, including Riemann sums</li> <li>- using the fundamental theorem of calculus</li> </ul> </li> </ul>	
	<ul style="list-style-type: none"> <li>› form and solve simple differential equations (e.g., with a boundary condition to determine the constant term).</li> </ul>	<b>Investigate</b> situations such as: <ul style="list-style-type: none"> <li>› population growth</li> <li>› radioactive decay</li> <li>› Newton's law of cooling.</li> </ul>
<ul style="list-style-type: none"> <li>› use mathematical modelling to approximate applied situations</li> </ul>	<ul style="list-style-type: none"> <li>› use mathematical modelling to approximate applied situations</li> </ul>	<b>Connect</b> year-level mathematical concepts, skills, and processes with meaningful contexts. <b>Explain</b> that modelling is open-ended and messy and involves connections between many different skills. When students are modelling, they must be making genuine choices. <b>Explain</b> and <b>justify</b> limitations with models, including why some values may not work. <b>Investigate</b> situations such as: <ul style="list-style-type: none"> <li>› estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed</li> <li>› planning a table tennis tournament for 7 players at a club with 4 tables and in which each player will play against every other player</li> <li>› designing the layout of the stalls at a school fair so as to raise as much money as possible</li> <li>› analysing stopping distances for a car</li> <li>› modelling a changing savings account balance, investment growth, bacterial colony growth, or medication absorption.</li> </ul>

## The language of mathematics: Phase 5

<b>Year 12</b> <i>Students will know the following new words:</i>	<b>Year 13</b> <i>Students will know the following new words:</i>
<ul style="list-style-type: none"> <li>› absolute or local extrema</li> <li>› absolute value</li> <li>› arc</li> <li>› arithmetic, geometric</li> <li>› binomial</li> <li>› derivative function</li> <li>› differentiation</li> <li>› discriminant</li> <li>› domain, range</li> <li>› linearise</li> <li>› logarithm</li> <li>› period or periodic</li> <li>› polynomial function</li> <li>› radians</li> <li>› recursive</li> <li>› secant line</li> <li>› series</li> <li>› sigma notation</li> <li>› sine, cosine, tangent</li> <li>› stationary point</li> <li>› tangent line</li> <li>› trigonometry, trigonometric</li> </ul>	<ul style="list-style-type: none"> <li>› Argand diagram</li> <li>› argument</li> <li>› Cartesian, polar, or exponential form</li> <li>› complex root</li> <li>› complex, real, or imaginary number</li> <li>› concavity</li> <li>› conjugate</li> <li>› continuous, continuity, discontinuous, discontinuity</li> <li>› definite or indefinite integral</li> <li>› differentiable</li> <li>› differential equations</li> <li>› modulus</li> <li>› natural log</li> <li>› normal line</li> <li>› parametric equation</li> <li>› piecewise function</li> <li>› point of inflection</li> <li>› rationalise</li> <li>› recurrence interval</li> <li>› trigonometric identity</li> </ul>

# Statistics, Years 12–13: Teaching sequence

*Navigating pathways and developing agency to help shape the future*

*Te whakaterere ara me te whakawhanake kahawhiri hei tautoko i te tāraitanga o āpōpō*

<b>During year 12</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 13</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› identify opportunities for using and exploring data to understand and learn about situations</li> </ul>	<ul style="list-style-type: none"> <li>› identify opportunities for using and exploring data to understand and learn about situations, and the possible impact of doing so</li> </ul>	<p>Demonstrate how to explore data-generating situations and how to identify where data can be used to solve a problem.</p> <p><b>Investigate</b> statistical or chance-based situations in order to deepen understanding of them.</p> <p>Consider:</p> <ul style="list-style-type: none"> <li>› the possible impacts of a situation and whether or not data would help to understand and explain them</li> <li>› the possible impacts and benefits of potential findings or some research work</li> <li>› the <b>connection</b> between personal interest and the wider impacts of a situation.</li> </ul> <p><b>Explain</b> and <b>justify</b> how thinking about ‘what if ...’ and ‘why should anyone care ...’ can inform a situation.</p> <p>Integrate relevant contextual knowledge and alternative perspectives.</p>
<ul style="list-style-type: none"> <li>› construct data from sources such as text, images, sounds, and movements (including in space and time)</li> </ul>	<ul style="list-style-type: none"> <li>› construct, manipulate, and restructure data from sources such as text, images, sounds, and movements (including in space and time)</li> </ul>	<p>Use data technologies to:</p> <ul style="list-style-type: none"> <li>› construct data from a variety of non-traditional sources</li> <li>› <b>connect</b> representations</li> <li>› manipulate and restructure data.</li> </ul> <p>Define, <b>explain</b>, and <b>justify</b> variables and how they will be measured (e.g., perceptions of things such as pain levels).</p>

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<ul style="list-style-type: none"> <li>› design, pilot, and refine data-collection instruments (e.g., questionnaires, sensors, and experiments) for the purpose of exploring or understanding a system, community, process, or issue</li> </ul>	<ul style="list-style-type: none"> <li>› design, using experimental design principles, and carry out an experiment for the purpose of identifying whether a causal relationship exists</li> </ul>	<p>Demonstrate how to design, pilot, and (at year 13) refine questionnaires for collecting data.</p> <p>Explore survey or data-collection questions, considering how question design impacts on the data that is collected.</p> <p>Define, <b>explain</b>, and <b>justify</b> measures for variables to ensure they will capture the data required.</p> <p>At year 12, show how to design and pilot experiments for collecting data.</p> <p><b>Investigate</b> methods and purposes for data collection – for example:</p> <ul style="list-style-type: none"> <li>› selecting appropriate instruments for collecting data, such as sensors</li> <li>› designing and conducting experiments to collect data (at year 12)</li> <li>› undertaking experiments involving a comparison of two independent groups to identify a causal relationship (at year 13).</li> </ul>
<ul style="list-style-type: none"> <li>› use ethical and responsible data practices and consider how decisions on collecting and generating data (including the questions asked and how measures are defined) will affect the applicability, quality, diversity, and quantity of the data</li> </ul>	<ul style="list-style-type: none"> <li>› use ethical and responsible data practices and consider how decisions on collecting and generating data will affect the applicability, quality, diversity, and quantity of the data</li> </ul>	<p><b>Investigate</b> data-collection tools and how they are designed to ensure the privacy and safety of participants and do no harm.</p> <p><b>Explain</b> and <b>justify</b> ethical decisions about data-collection processes – for example, those that involve:</p> <ul style="list-style-type: none"> <li>› investigative questions</li> <li>› survey questions</li> <li>› variables to be measured</li> <li>› a population.</li> </ul> <p>Give examples of how to:</p> <ul style="list-style-type: none"> <li>› check for bias in data-collection processes</li> <li>› evaluate others’ data practices</li> <li>› explore different methods of sampling and the impacts they may have.</li> </ul>

<b>During year 12</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 13</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› as part of exploring data:               <ul style="list-style-type: none"> <li>– develop new variables</li> <li>– create a range of data visualisations for different variables or combinations of variables</li> <li>– communicate what has been learned, drawing on key features, patterns, or trends in the data</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› as part of exploring data:               <ul style="list-style-type: none"> <li>– merge data sources</li> <li>– develop new variables</li> <li>– create a wide range of data visualisations for different variables or combinations of variables</li> <li>– communicate what has been learned, drawing on key features, patterns, or trends in the data</li> </ul> </li> </ul>	<p><b>Investigate</b> how to explore and manipulate data – for example:</p> <ul style="list-style-type: none"> <li>› using different methods for cleaning data</li> <li>› developing new variables from existing information in the dataset</li> <li>› merging data tables (e.g., from different sources) (at year 13)</li> <li>› exploring visualising the data using a variety of tools, including manual and digital tools.</li> </ul> <p><b>Investigate</b> patterns and trends, connecting to the data context and undertaking further exploration.</p> <p>Explain how to record what is seen during exploration, communicating what has been learned about the data context.</p>
<ul style="list-style-type: none"> <li>› calculate and interpret absolute risk and relative risk for hazardous situations, using two-way tables of counts and performing calculations to verify claims based on risks</li> </ul>	<ul style="list-style-type: none"> <li>› calculate risks for hazardous situations and make and evaluate communications on matters such as absolute risk, relative risk, increased or decreased risk, and recurrence intervals</li> </ul>	<p>Show how to use tools such as two-way tables to calculate absolute and relative risks.</p> <p><b>Investigate</b> and evaluate absolute and relative risk statements in media and other communications.</p> <p><b>Connect</b> to other ways in which the word ‘risk’ is used, especially in the field of risk management. Use a range of frequency-based <b>representations</b> of risk data to help in the interpretation and communication of absolute and relative risks.</p> <p>At year 13, <b>investigate</b> and <b>explain</b> risk management for events of low probability and high impact.</p> <p><b>Explain</b> and <b>justify</b> considerations such as:</p> <ul style="list-style-type: none"> <li>› the influence of base rates</li> <li>› how recurrence intervals describe a risk situation and the misconceptions they can cause</li> <li>› alternative ways of communicating recurrence intervals to others</li> <li>› whether it is better to use absolute risks, relative risks, or both for comparisons in a given situation</li> <li>› the interpretation of recurrence intervals.</li> </ul>

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<b>During year 12</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 13</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› make sample-to-population inferences by generating and interpreting informal confidence intervals for medians and proportions on data visualisations</li> </ul>	<ul style="list-style-type: none"> <li>› make sample-to-population inferences by generating and interpreting formal confidence intervals for means, medians, and proportions, including differences, using the bootstrap simulation-based method</li> </ul>	<p><b>Investigate</b> and use sampling variation:</p> <ul style="list-style-type: none"> <li>› for generating and interpreting medians, means, and proportions</li> <li>› to understand informal confidence intervals for medians and proportions (at year 12).</li> </ul> <p>At year 12, use informal confidence intervals to make sample-to-population inferences when comparing medians and proportions on data visualisations.</p> <p><b>Investigate</b> situations such as:</p> <ul style="list-style-type: none"> <li>› how to interpret confidence intervals in context</li> <li>› how to <b>connect</b> confidence intervals for proportions with the rules-of-thumb for margin of error.</li> </ul> <p>At year 13, <b>investigate</b>:</p> <ul style="list-style-type: none"> <li>› the modelling ideas that underpin bootstrap confidence intervals</li> <li>› using the bootstrap simulation-based method to generate confidence intervals</li> <li>› how formal confidence intervals behave.</li> </ul>
<ul style="list-style-type: none"> <li>› conduct simulation-based tests for a single proportion to assess the strength of evidence</li> </ul>	<ul style="list-style-type: none"> <li>› make experiment-to-causation inferences involving a comparison of two independent groups by considering the study design and using a simulation-based randomisation test to assess the strength of evidence</li> </ul>	<p>At year 12, <b>investigate</b> situations such as:</p> <ul style="list-style-type: none"> <li>› when a test can be conducted to see how an observed proportion compares with a just-chance model</li> <li>› generating simulated data under a just-chance model, and using the results to assess if the observed proportion is compatible with the just-chance model</li> <li>› using a simulation-based test for a single proportion to test for a change in before-and-after data.</li> </ul> <p>At year 13, <b>explain</b> and <b>justify</b>:</p> <ul style="list-style-type: none"> <li>› why random allocation to treatment groups is important</li> <li>› how randomising group labels helps assess the plausibility of the ‘chance-alone’ explanation</li> <li>› how to construct and use randomisation tests to assess strength of evidence in context (using the difference between two means or two medians).</li> </ul> <p>At year 13, <b>connect</b> randomisation test results with appropriate language to communicate findings to others.</p>

<b>During year 12</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 13</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› identify likely and unlikely outcomes within distributions, and explore the impact of conditioning on other variables, from the perspective of prediction</li> </ul>	<ul style="list-style-type: none"> <li>› develop, use, and evaluate statistical models, including regression, time-series, and classification models, to make predictions</li> </ul>	<p>At year 12, <b>connect</b> likely and unlikely outcomes with low-probability and high-probability outcomes, for distributions of numeric and categorical variables.</p> <p><b>Investigate:</b></p> <ul style="list-style-type: none"> <li>› whether the distribution of a variable of interest (numeric or categorical) changes depending on the value of an additional categorical variable, and whether this is a useful predictor (at year 12)</li> <li>› informal approaches to making predictions (at year 12)</li> <li>› ways of including information on a third variable to check if it adds predictive information</li> <li>› whether the values of a numeric variable of interest change depending on the value of an additional numeric variable, and (at year 13) whether an additional numeric variable is a useful predictor.</li> </ul> <p><b>Investigate</b> and compare data-analysis and prediction situations.</p> <p>At year 13, identify potential practical consequences when predictions are incorrect.</p> <p>At year 13, <b>explain</b> ways of evaluating:</p> <ul style="list-style-type: none"> <li>› issues with the applicability of predictions</li> <li>› the predictive accuracy of models, using representations such as confusion matrices.</li> </ul>
<ul style="list-style-type: none"> <li>› communicate findings based on statistical evidence, including               <ul style="list-style-type: none"> <li>- the context of the situation</li> <li>- limitations</li> <li>- recommendations</li> <li>- potential biases (e.g., subjectiveness)</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>› communicate findings based on statistical evidence, including               <ul style="list-style-type: none"> <li>- the context of the situation</li> <li>- assumptions and limitations</li> <li>- impacts of potential decisions and models</li> <li>- recommendations</li> <li>- new conjectures</li> <li>- uncertainties</li> <li>- potential biases (e.g., subjectiveness)</li> </ul> </li> </ul>	<p><b>Connect</b> new understandings gained from data about the context of a situation to findings of primary importance when answering the investigative question.</p> <p><b>Explain</b> and <b>justify</b> the strength and trustworthiness of answers to the investigative question by showing how assessment of limitations, including acknowledging potential biases (at year 12) and uncertainty (at year 13), supports the findings.</p> <p>Demonstrate how the impact of findings can be used for:</p> <ul style="list-style-type: none"> <li>› practical actions relevant to the context of the situation</li> <li>› generating future investigations.</li> </ul>

<b>During year 12</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 13</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› use representations such as probability tree diagrams and two-way tables to determine the probabilities of events, including joint, marginal, and informal conditional probabilities</li> </ul>	<ul style="list-style-type: none"> <li>› determine the probabilities of events using a combination of simple probability models, probability distribution models, and approaches such as conditioning and partitioning</li> </ul>	<p><b>Represent</b> information about chance-based situations using visual probability tools.</p> <p>Use a range of representations to determine probabilities, including two-way tables, probability-tree diagrams (at year 12), and bar graphs, eikosograms, pachinkograms, and Venn diagrams (at year 13).</p> <p>At year 13, <b>explain</b> how to:</p> <ul style="list-style-type: none"> <li>› reverse the order of conditioning (e.g., in screening or diagnostic tests)</li> <li>› use partitioning when solving probability problems.</li> </ul>
<ul style="list-style-type: none"> <li>› use probability distributions (including normal, binomial, and uniform) as models and to identify likely and unlikely outcomes</li> </ul>	<ul style="list-style-type: none"> <li>› select and use models of probability distributions (including Poisson, normal, binomial, and uniform) to calculate probability estimates, recognising the underlying structure and conditions of the data-generating process</li> </ul>	<p>Explore the underlying structures of formal probability distributions to justify the best model for a situation.</p> <p><b>Investigate</b> simulation and exact-calculation digital tools for obtaining probabilities.</p> <p>At year 13:</p> <ul style="list-style-type: none"> <li>› <b>explain</b> how a probability model can generate random data from a probability distribution</li> <li>› <b>connect</b> binomial and Poisson probability distribution models to relevant risk situations.</li> </ul>
<ul style="list-style-type: none"> <li>› calculate and interpret expected counts for simple probability models</li> </ul>	<ul style="list-style-type: none"> <li>› calculate and interpret expected values and standard deviations of discrete random variables</li> </ul>	<p>At year 12, use manual and digital tools to explore expected counts in context.</p> <p>At year 13, use an appropriate probability model for a discrete random variable, where the distribution is presented as a table of values with their probabilities, to calculate its expected value.</p> <p>Use formulae or digital tools to:</p> <ul style="list-style-type: none"> <li>› calculate and interpret expected values and standard deviations for known probability distributions (at year 13)</li> <li>› <b>investigate</b> standard deviation, as a measure of variation (at year 13).</li> </ul>
<ul style="list-style-type: none"> <li>› conduct large-scale, technology-supported simulations to estimate model probabilities</li> </ul>	<ul style="list-style-type: none"> <li>› use visual, technology-based informal tests to assess the goodness-of-fit of probability distribution models by comparing observed data with model-generated data</li> </ul>	<p>Use digital tools to <b>investigate</b> year-appropriate models for estimating model probabilities.</p> <p>Explain occasion-to-occasion or experimenter-to-experimenter variation in estimated probabilities, and the effect of sample size on this variation.</p>

<b>During year 12</b> <i>Informed by prior learning, teach students to:</i>	<b>During year 13</b> <i>Informed by prior learning, teach students to:</i>	<b>Teaching considerations</b>
<ul style="list-style-type: none"> <li>› explore changing the features of probability and probability distribution models and discuss the effects</li> </ul>	<ul style="list-style-type: none"> <li>› link the parameters of probability distributions to the distribution of probabilities, and discuss the effects of changing the parameters</li> </ul>	<p>At year 12, use an appropriate interactive digital tool to explore what distributions look like and to <b>connect</b> how the distribution of probabilities changes with changing parameter values.</p> <p>At year 13, <b>investigate</b> the effects of changing parameters on probability distributions.</p>
<ul style="list-style-type: none"> <li>› interpret and critique data- and model-based information and practices, data visualisations, and claims, from a variety of sources.</li> </ul>	<ul style="list-style-type: none"> <li>› interpret and critique data and model- based information, visualisations, and practices, embedded statistical information in spoken and written texts, and claims from a variety of sources.</li> </ul>	<p>At year 12, show how to use a set of critical questions to evaluate data- and model-based information, visualisations, and practices in the media.</p> <p>Critique the language and statistical information used by others when communicating claims, including:</p> <ul style="list-style-type: none"> <li>› identifying who is doing the research</li> <li>› checking for hidden agendas</li> <li>› critically evaluating a wide range of data- and model-based information in the media and from other sources (at year 13)</li> <li>› considering decisions about data-based information and the consequences of these decisions (at year 13)</li> <li>› appreciating how statistics are used to influence change for good and bad</li> <li>› recognising written and spoken communications that have embedded statistical information (at year 13)</li> <li>› interpreting informal margin of error based on poll-based reports for single proportions (at year 12) or for differences between proportions (at year 13).</li> </ul> <p><b>Investigate</b> and explain the pervasiveness of digital data collection (e.g., through interactions with social media, GPS devices, or sensors) and algorithmic model-based products (e.g., advert recommendations online or risk predictions), along with the consequences of algorithmic bias.</p>

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## The language of statistics: Phase 5

<b>Year 12</b>	<b>Year 13</b>
<p data-bbox="114 325 730 357"><i>Students will know the following new words:</i></p> <ul data-bbox="114 389 1115 564" style="list-style-type: none"><li>› absolute or relative risk</li><li>› expected counts</li><li>› informal inference</li><li>› informal, joint, or marginal conditional probability</li><li>› normal distribution</li><li>› uniform distribution</li></ul>	<p data-bbox="1122 325 1693 357"><i>Students will know the following new words:</i></p> <ul data-bbox="1122 389 2123 676" style="list-style-type: none"><li>› bootstrap</li><li>› conditioning</li><li>› discrete random variable</li><li>› expected value</li><li>› formal inference</li><li>› margin of error</li><li>› parameter (of a probability distribution)</li><li>› Poisson distribution</li><li>› randomisation test</li><li>› regression</li><li>› standard deviation</li></ul>